Model-independent characterisation of strong gravitational lenses

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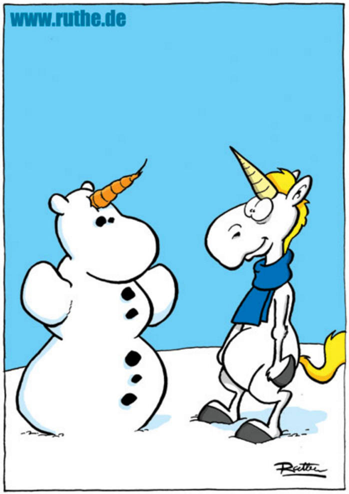
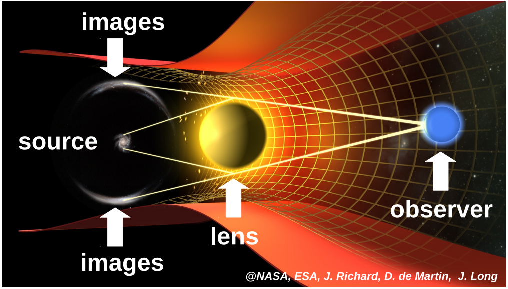
Experience biases perception and limits imagination



Gravitational lensing



source position: $\boldsymbol{y} \in \mathbb{R}^2$,

image position: $\boldsymbol{x} \in \mathbb{R}^2$,

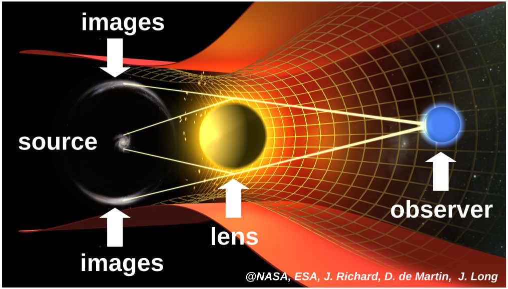
lensing potential: $\psi(\boldsymbol{x})$,

distortion matrix: $A(\boldsymbol{x})$,

lens mapping:

$$\boldsymbol{y} = \boldsymbol{x} - \nabla\psi(\boldsymbol{x}) \approx A(\boldsymbol{x})\,\boldsymbol{x}$$

Gravitational lens modelling



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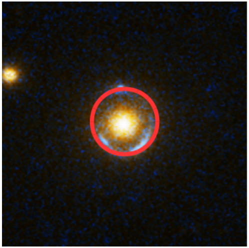
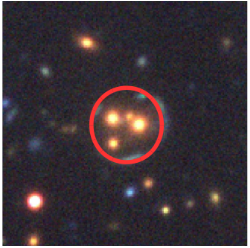
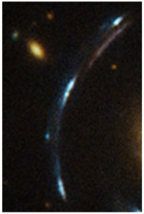
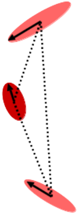




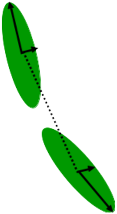
Observables of unresolved images = moments of brightness





cusp (3 images):

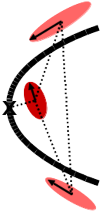
relative distances,
image ellipticities, orientations,
magnification ratios,
(time delays)



fold (2 images):

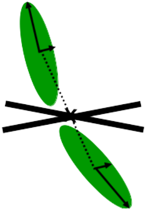
relative distances,
image ellipticities, orientations,
magnification ratios,
(time delays)

(Best case) Model-independent lens characterisation



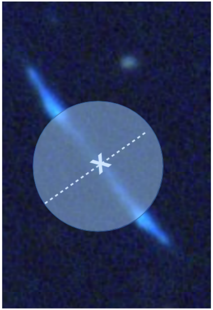
cusp (3 images):

critical cusp point,
parabolic approx. to critical curve,
local reduced shear,
(magnifications),
(source)



fold (2 images):

critical fold point,
absolute slope of critical curve,
local reduced shear,
(magnifications),
(source)



- local Taylor expansion around critical point: $\boldsymbol{y} = \boldsymbol{x} - \nabla\psi_{\mathrm{t}}(\boldsymbol{x})$

$$\begin{aligned}\psi_{\mathrm{t}}(\boldsymbol{x}) &= \psi^{(0)} + \delta\psi(\boldsymbol{x}) \\ &= \frac{1}{2}\left(1 - \psi_{11}^{(0)}\right) x_1^2 - \frac{1}{6}\psi_{111}^{(0)} x_1^3 - \frac{1}{2}\psi_{112}^{(0)} x_1^2 x_2 \\ &\quad - \frac{1}{2}\psi_{122}^{(0)} x_1 x_2^2 - \frac{1}{6}\psi_{222}^{(0)} x_2^3\end{aligned}$$

  $\rightarrow$ **proximity to critical curve vs. accuracy**

- coefficients determined by observables (brightness moments)

  $\rightarrow$ **precision of moments vs. number of coefficients**

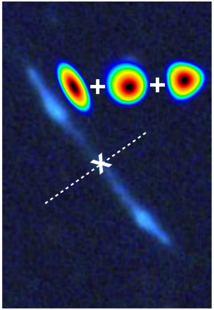
- images from the same source to eliminate $\boldsymbol{y}$

  $\rightarrow$ **number of multiple images vs. number of equations**

- system of equations subject to degeneracies

  $\rightarrow$ **degeneracy of observables vs. entanglement of coefficients**

Principle



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$$\psi_{\mathrm{t}}(\boldsymbol{x}) = \psi^{(0)} + \delta\psi(\boldsymbol{x})$$
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$$- \frac{1}{2}\psi_{122}^{(0)} x_1 x_2^2 - \frac{1}{6}\psi_{222}^{(0)} x_2^3$$

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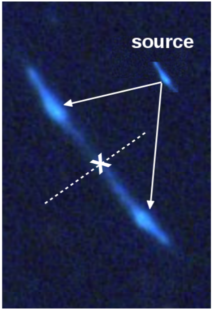

- local Taylor expansion around critical point: $\boldsymbol{y} = \boldsymbol{x} - \nabla\psi_{\mathrm{t}}(\boldsymbol{x})$

$$\begin{aligned}\psi_{\mathrm{t}}(\boldsymbol{x}) &= \psi^{(0)} + \delta\psi(\boldsymbol{x}) \\ &= \frac{1}{2}\left(1 - \psi^{(0)}_{11}\right) x_1^2 - \frac{1}{6}\psi^{(0)}_{111} x_1^3 - \frac{1}{2}\psi^{(0)}_{112} x_1^2 x_2 \\ &\quad - \frac{1}{2}\psi^{(0)}_{122} x_1 x_2^2 - \frac{1}{6}\psi^{(0)}_{222} x_2^3\end{aligned}$$

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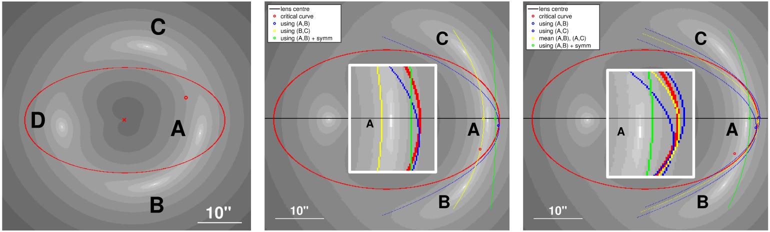
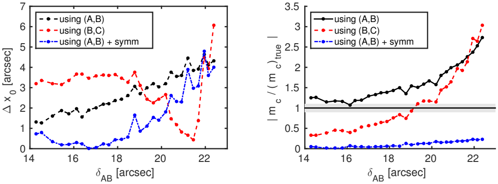
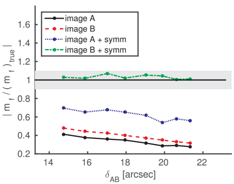
Accuracy limits for a simulated singular isothermal ellipse



cusp configuration (A,B,C)

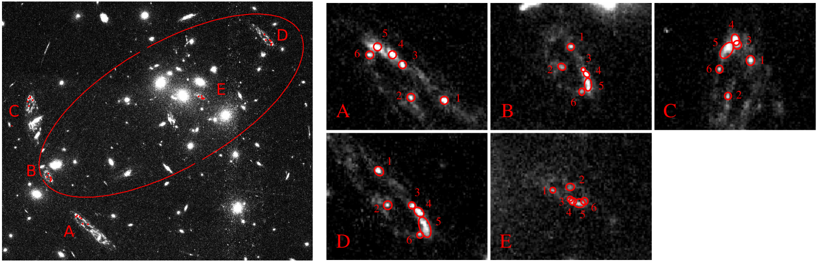


fold config. (A,B)



→ reconstruction within tolerable range of accuracy

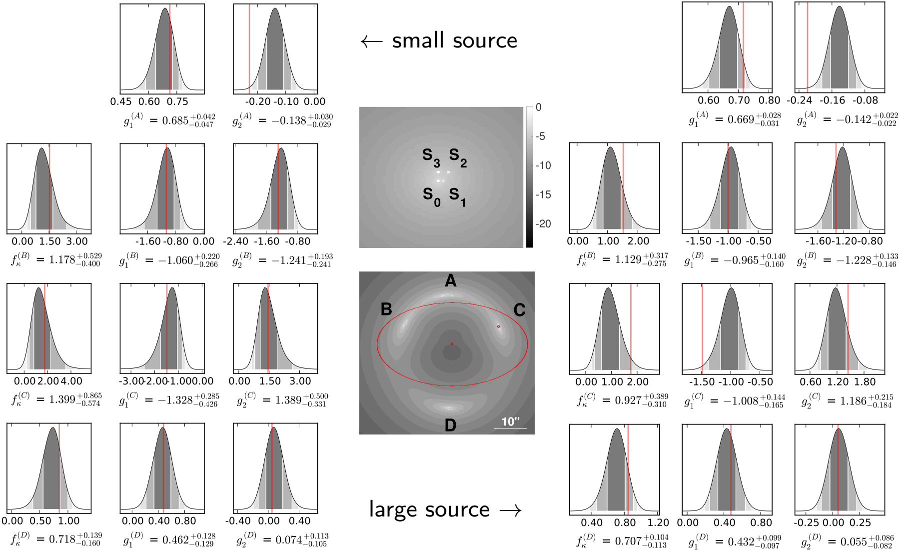
Alternative to moments for resolved images (with N. Tessore)



linear transformation between images instead of moments

- encodes same information as quadrupole moments
- yields the same results close to the critical curve (up to the parametrisation)
- recovers properties at images D, E

Accuracy for a simulated resolved source in an SIE lens



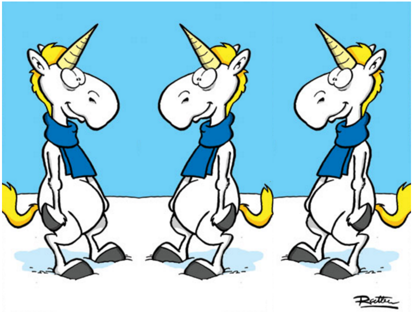
→ **works well until image extension ≈ 10% distance between images**

Conclusion

- **purely data-driven approach:**
  - is based on general mathematical properties of the lens potential
  - system of equations directly yields (ratios of) potential derivatives
  - no model fitting, no fine-tuning, no degeneracies due to model assumptions
  - galaxies or galaxy clusters as lenses treated by same set of equations
- **local information retrieval:**
  - reconstruction of critical curve, potential derivatives close to multiple images
  - special configurations (fold, cusp, small resolved images) required
- **applications in the cluster regime:**
  - locally characterise a lensing region of interest (small-scale properties of dark matter),
  - include constraints into a full galaxy cluster reconstruction to resolve central regions,
  - reconstruct magnified source galaxies to study galaxy evolution.

Model-independent information reduces assumptions/ biases

Thank you for your attention!



I gratefully acknowledge

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- funding by the DFG (WA3547/1-1)
- and the opportunity to present my results here!

Further information:
www.zah.uni-heidelberg.de/staff/jwagner **j.wagner@uni-heidelberg.de**

