

On the assumptions about dynamical mass estimation

Yan-Chuan Cai

Institute for Astronomy, University of Edinburgh

The basic of cluster cosmology

$$f(\sigma_M, z) = \sqrt{\frac{2a}{\pi}} C \left[1 + \left(\frac{\sigma_M^2}{a\delta_c^2} \right)^q \right] \frac{\delta_c}{\sigma_M} \exp\left(-\frac{a\delta_c^2}{2\sigma_M^2}\right)$$

$$f(\sigma_M, z) = \frac{M}{\bar{\rho}} \frac{dn(M, z)}{d \ln \sigma_M^{-1}}$$

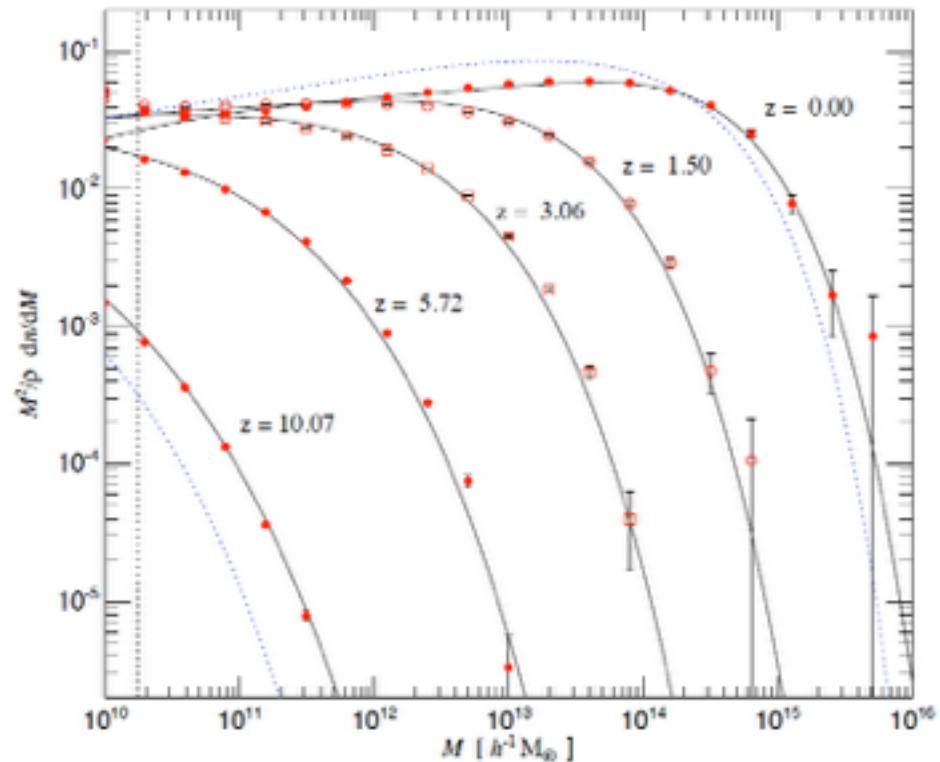
Press & Schechter 1974

Sheth & Tormen 1999

Jenkins et al. 2001

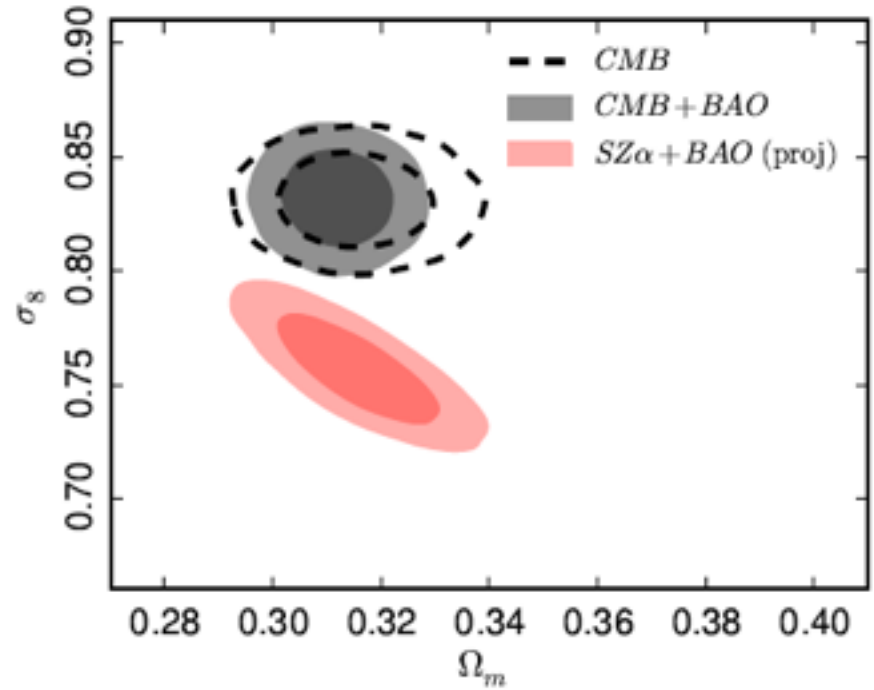
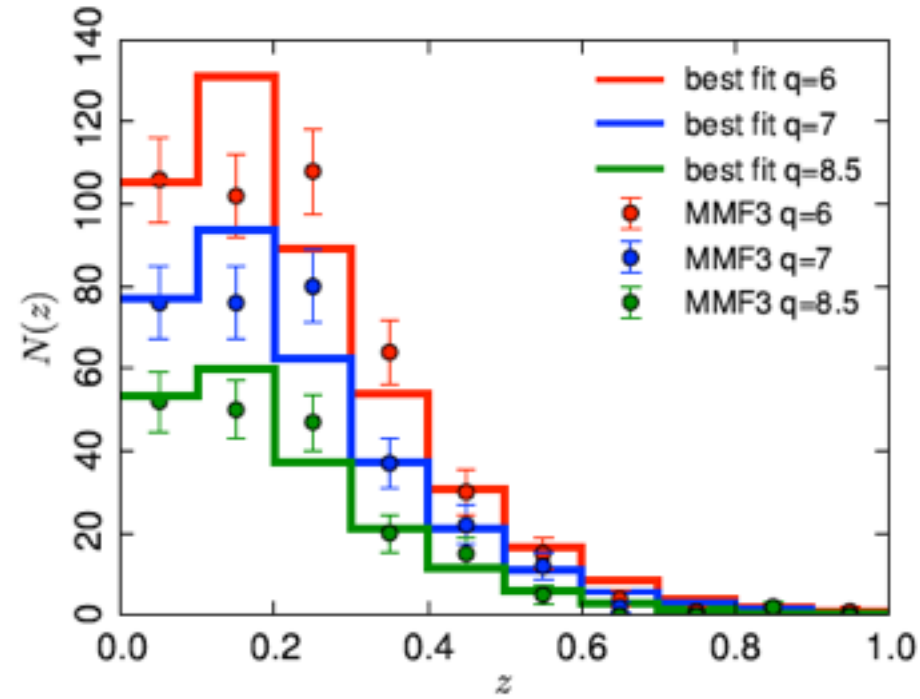
Tinker et al. 2008

...



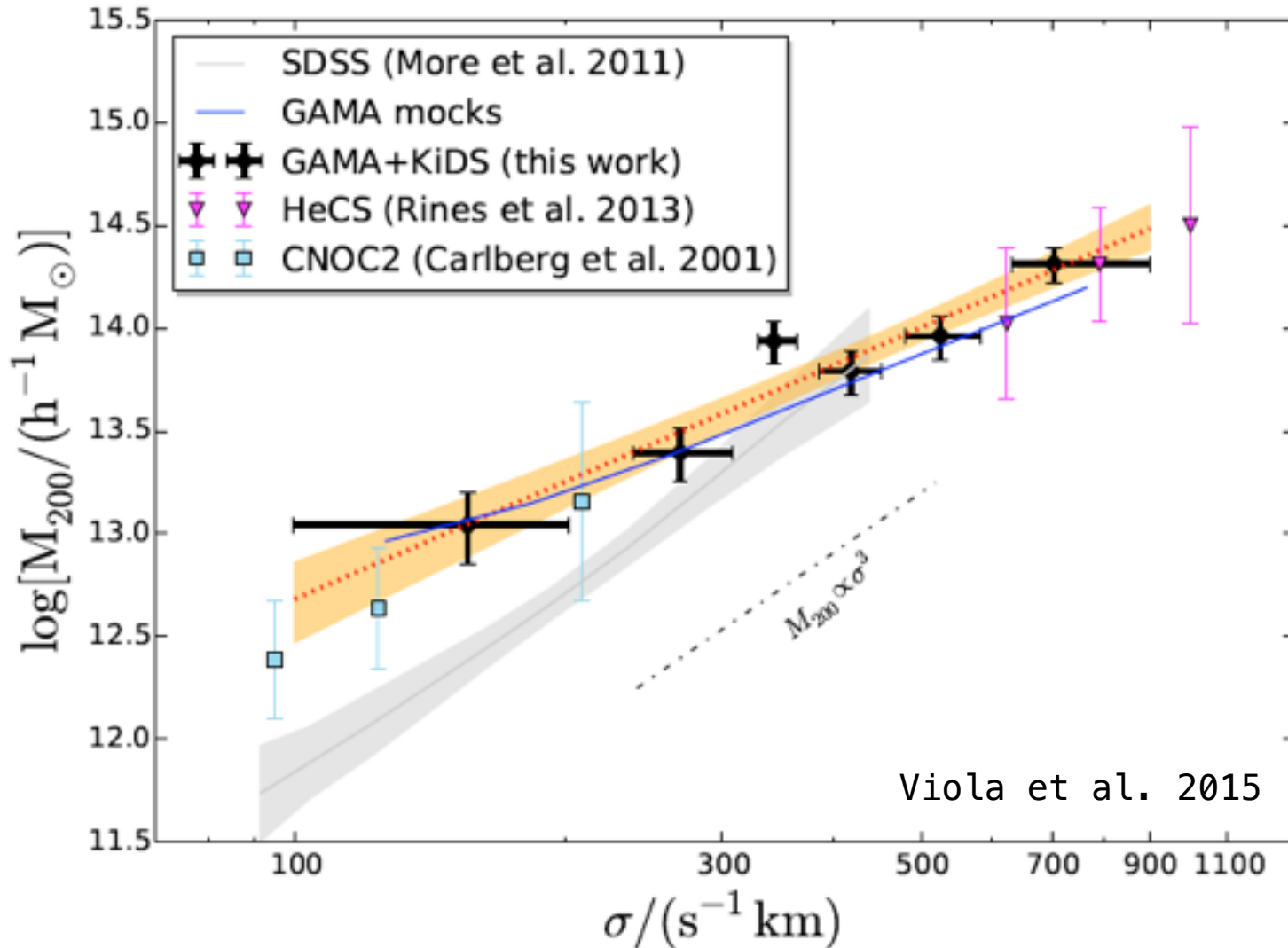
Springel et al. 2005

A tension?

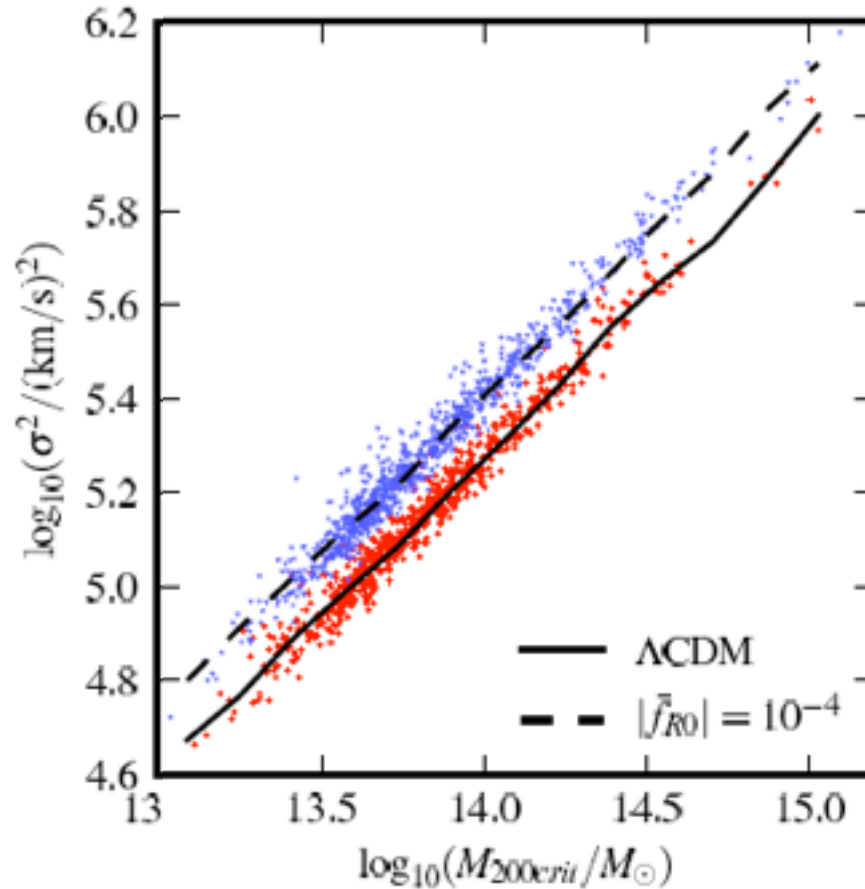


Planck collaboration 2015 XXIV

Dynamical mass vs. lensing mass

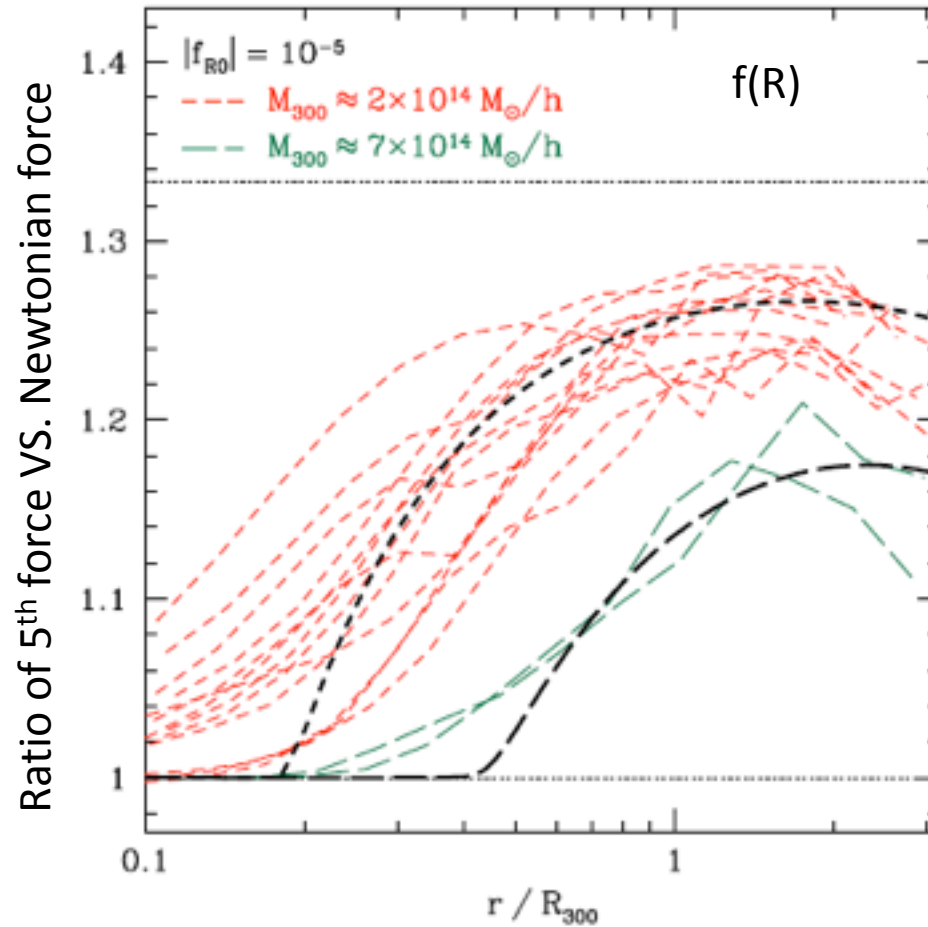


Dynamical mass vs. lensing mass



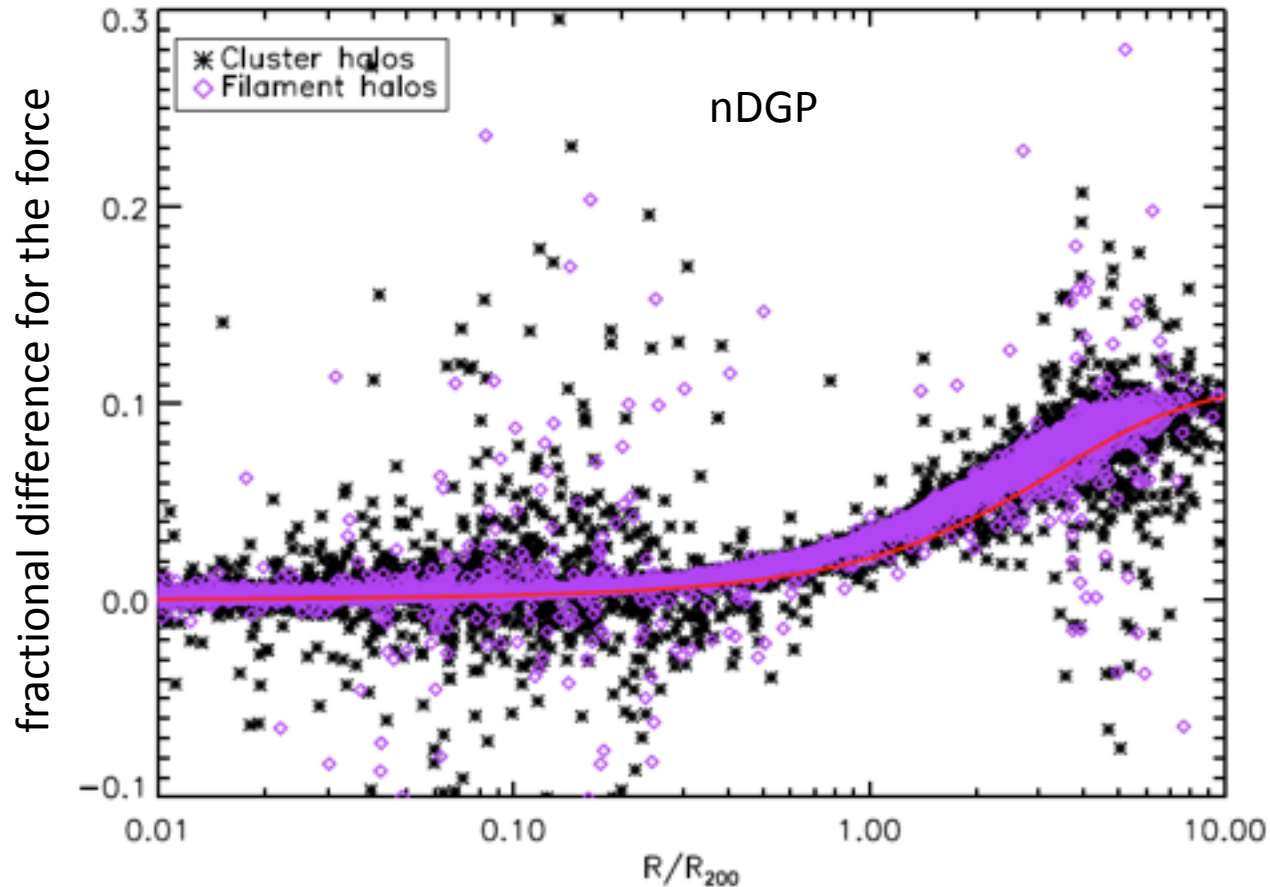
Arnold, Puchwein & Springel 2014

Acceleration at the outskirts



Schmidt 2003

Acceleration at the outskirts



Falck et al. 2014; 2015

The Jeans equation

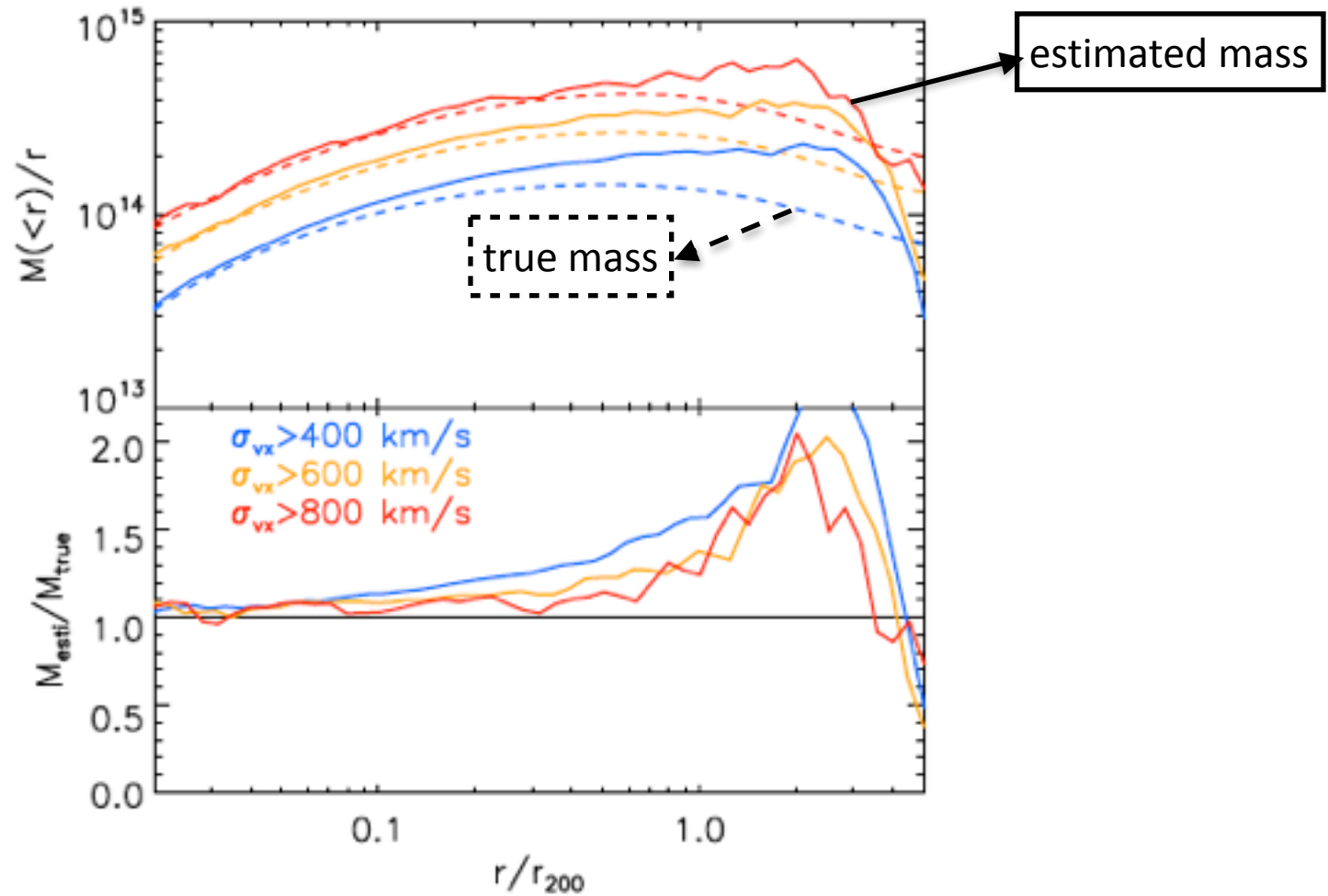
CBE: $\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \vec{\nabla} \Phi \cdot \frac{\partial f}{\partial \vec{v}} = 0$

$$\partial_t n \langle v_i \rangle + \partial_j (n \langle v_i v_j \rangle) + n \partial_i \Phi = 0$$

evolution of
mean velocity

gradient of
pressure tensor

gradient of
potential



- Mass bias up to a factor of 1.5-2 at 1-2 r_{200}

Cai, Kaiser & Cole 2017, in prep.

The Jeans equation

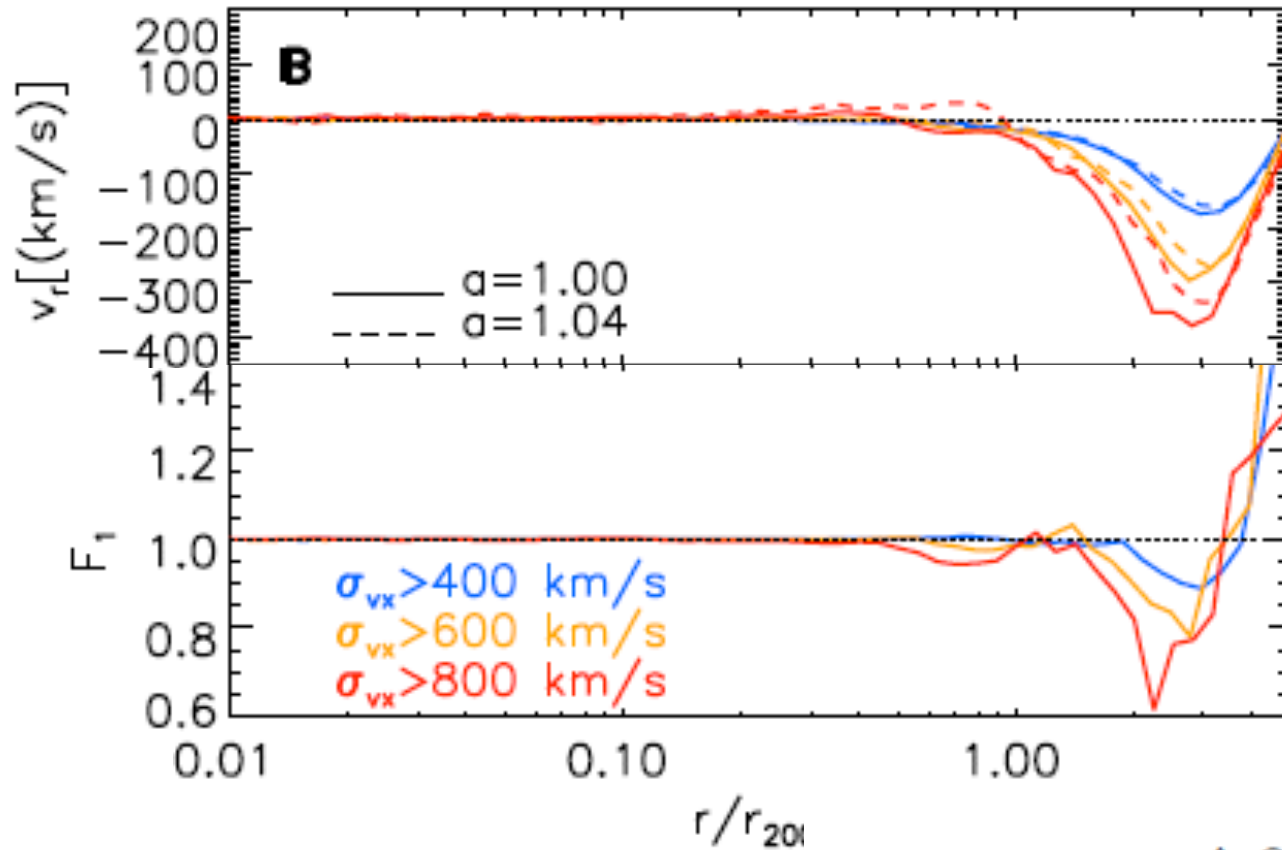
$$\partial_t n \langle v_i \rangle + \partial_j (n \langle v_i v_j \rangle) + n \partial_i \Phi = 0$$

$$M(< r) = -F_1 r^2 \partial_j (n \langle v_i v_j \rangle) / (nG)$$

$$F_1(r) = 1 + \frac{\hat{r}_i \overline{\partial_t n \langle v_i \rangle}}{\hat{r}_i \overline{\partial_j n \langle v_i v_j \rangle}}$$

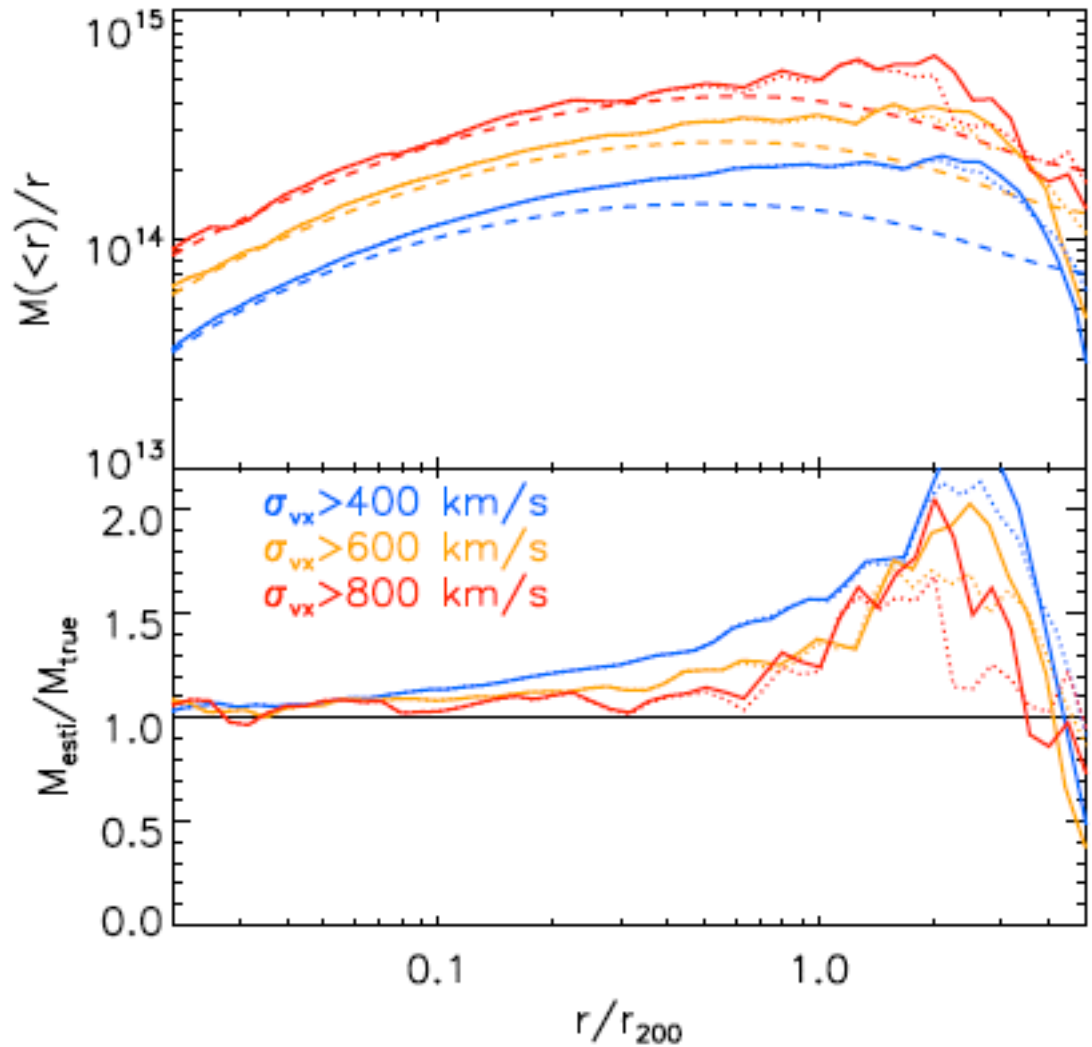
The evolution of infall

Cai, Kaiser & Cole 2017, in prep.



$$M(< r) = -F_1 r^2 \partial_j (n \langle v_i v_j \rangle) / (nG)$$

$$F_1(r) = 1 + \frac{\hat{r}_i \partial_t \overline{n \langle v_i \rangle}}{\hat{r}_i \partial_j \overline{n \langle v_i v_j \rangle}}$$

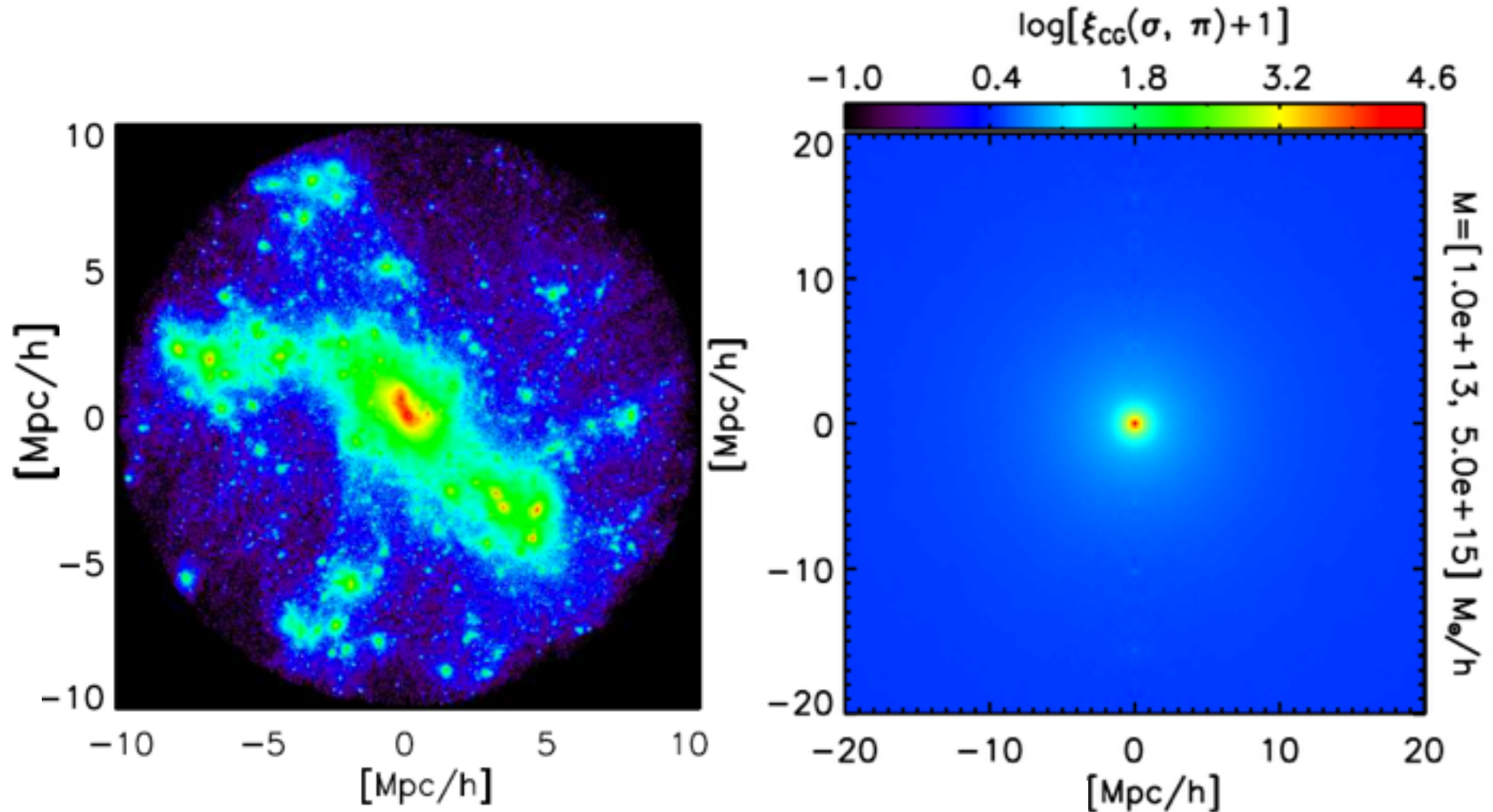


- Mass infall can not fully account for the bias

$$M(< r) = -F_1 r^2 \partial_j (n \langle v_i v_j \rangle) / (nG)$$

Cai, Kaiser & Cole 2017, in prep.

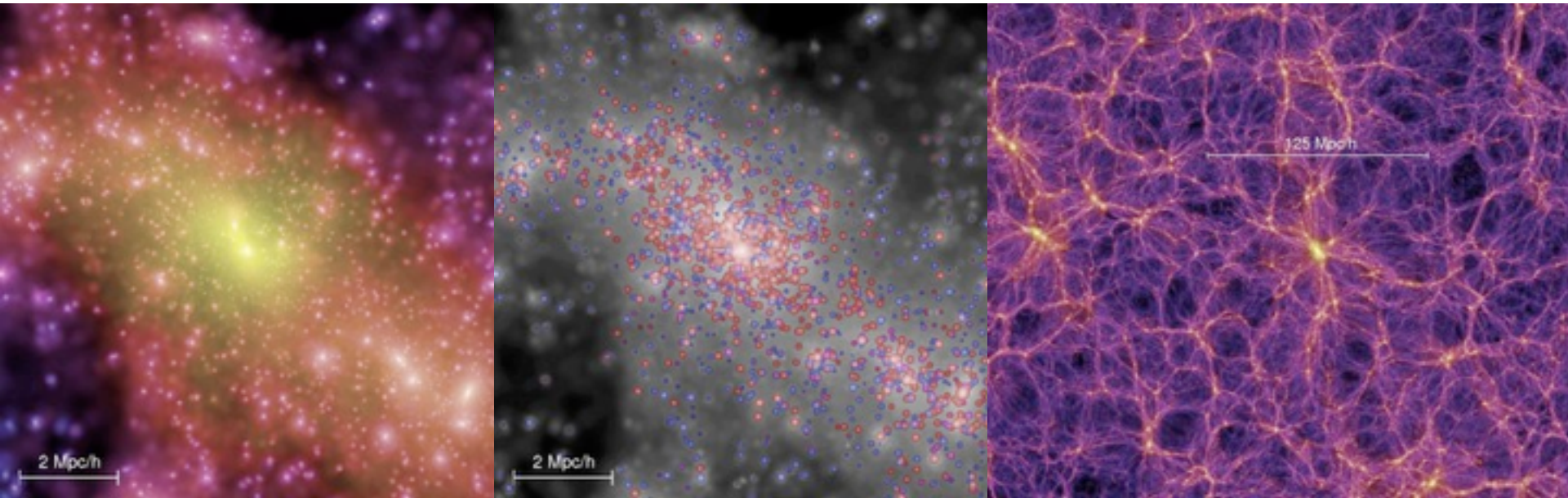
Spherical symmetry



Individual halo

stacked sample

Static centre



Springel et al. (2005)

The extended Jeans equation

$$\partial_t n \langle v_i \rangle + \partial_j (n \langle v_i v_j \rangle) + n \partial_i \Phi = 0$$

$$\partial_t n \langle v_i \rangle + \partial_j (n \langle v_i v_j \rangle) + n a_{0i} + n \partial_i \Phi = 0$$

evolution of
mean velocity

gradient of
pressure tensor

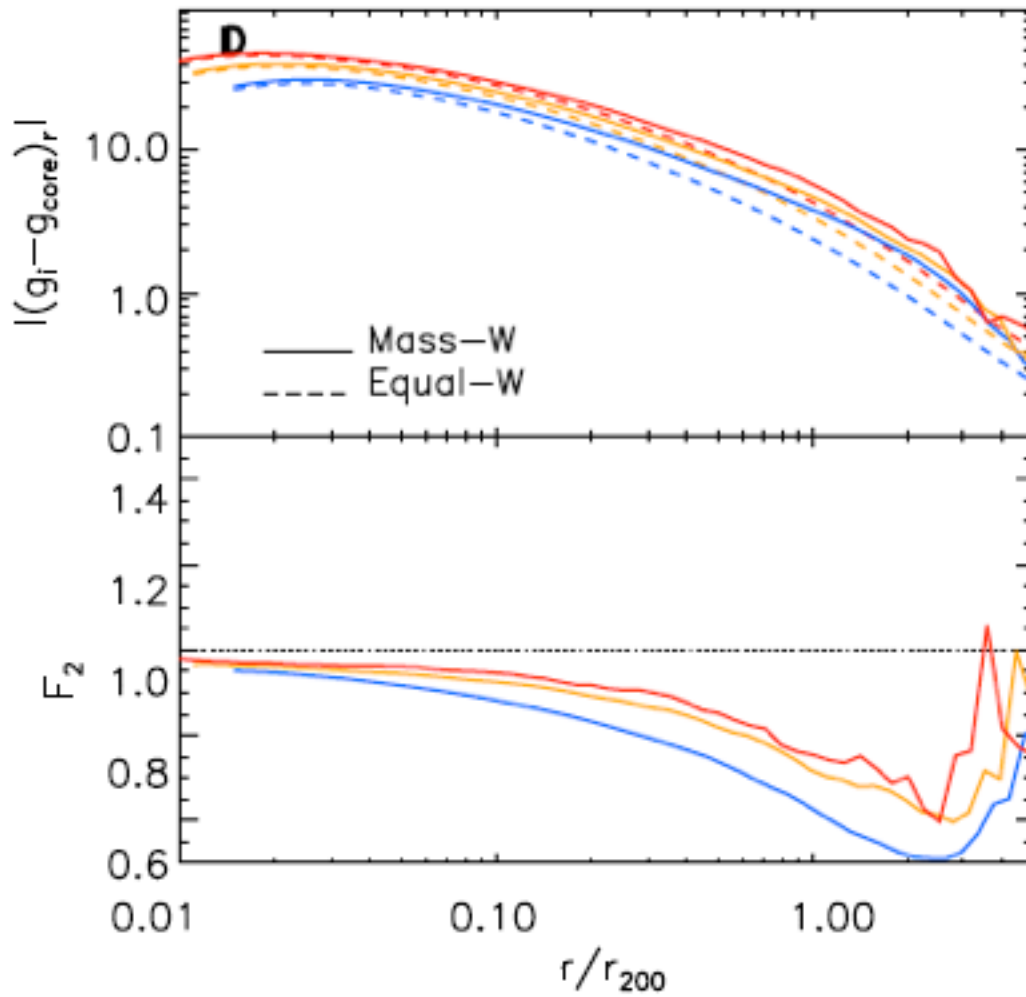
acceleration
of the centre

gradient of
potential

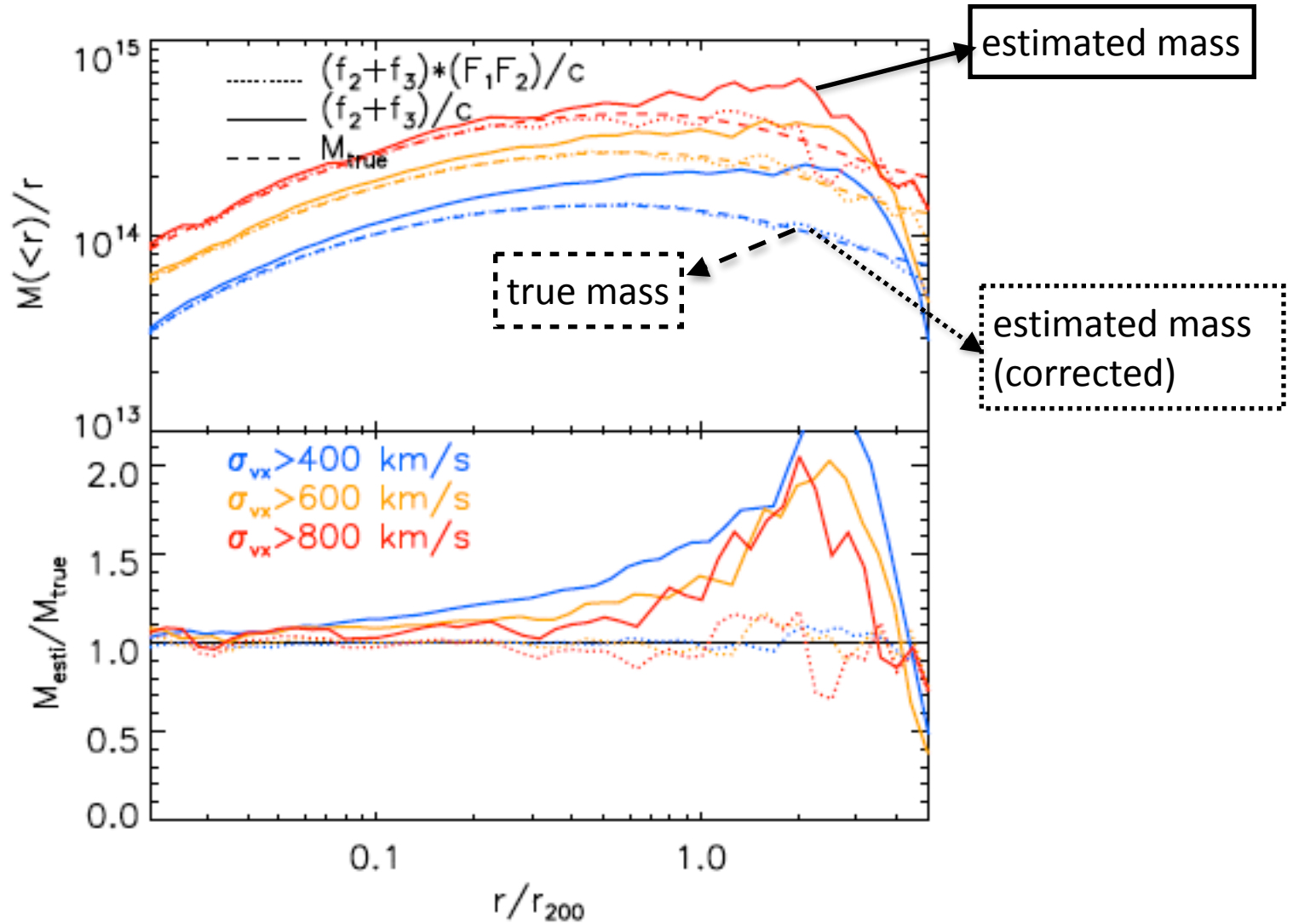
$$F_2(r) = \frac{\overline{\int \hat{r} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] d\Omega}}{\bar{n}(r)^{-1} \overline{\int \hat{r} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] \mathbf{n}(\mathbf{r}) d\Omega}}$$

$$M(< r) = -F_1 F_2 r^2 \partial_j (n \langle v_i v_j \rangle) / (nG)$$

$$F_2(r) = \frac{\int \hat{\mathbf{r}} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] d\Omega}{\bar{n}(r)^{-1} \int \hat{\mathbf{r}} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] \mathbf{n}(\mathbf{r}) d\Omega}$$



$$M(< r) = -F_1 F_2 r^2 \partial_j (n \langle v_i v_j \rangle) / (n G)$$



Cai, Kaiser & Cole 2017, in prep.

The corrected mass estimator

$$\partial_t n \langle v_i \rangle + \partial_j (n \langle v_i v_j \rangle) + n a_{0i} + n \partial_i \Phi = 0$$

$$M(< r) = -F_1 F_2 r^2 \partial_j (n \langle v_i v_j \rangle) / (n G)$$

$$F_1(r) = 1 + \frac{\hat{r}_i \overline{\partial_t n \langle v_i \rangle}}{\hat{r}_i \overline{\partial_j n \langle v_i v_j \rangle}}$$

$$F_2(r) = \frac{\overline{\int \hat{\mathbf{r}} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] d\Omega}}{\bar{n}(r)^{-1} \overline{\int \hat{\mathbf{r}} \cdot [\mathbf{g}(\mathbf{r}) - \mathbf{a}_0] \mathbf{n}(\mathbf{r}) d\Omega}}$$

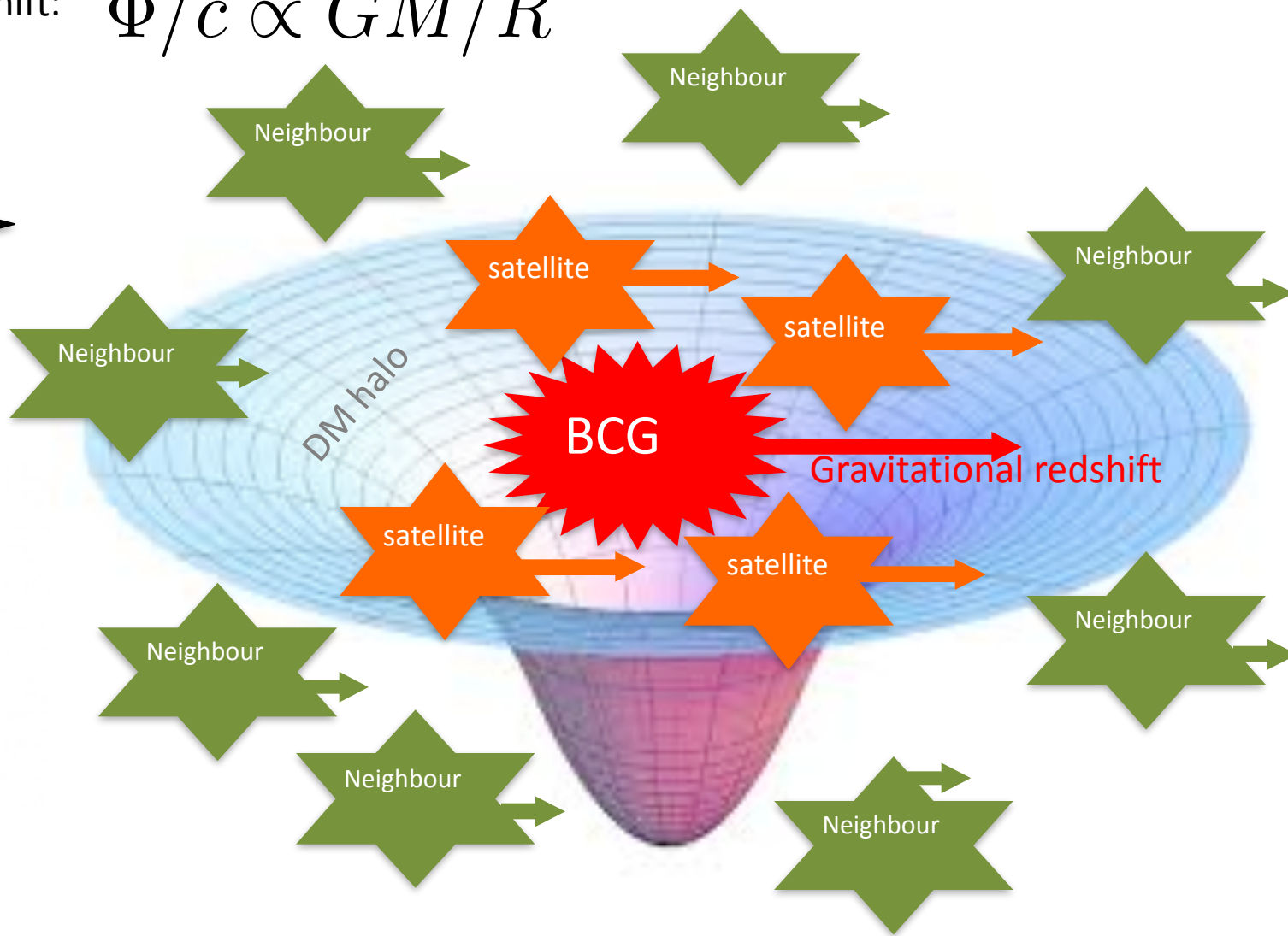
Summery so far

- The conventional Jeans equation is biased because:
- static assumption: no bulk acceleration
- spherical symmetry assumption

Further implications —
Gravitational redshift from stacked clusters

Gravitational Redshift: $\Phi/c \propto GM/R$

Line of sight 



The observed redshift

To the lowest order of the potential and peculiar velocity

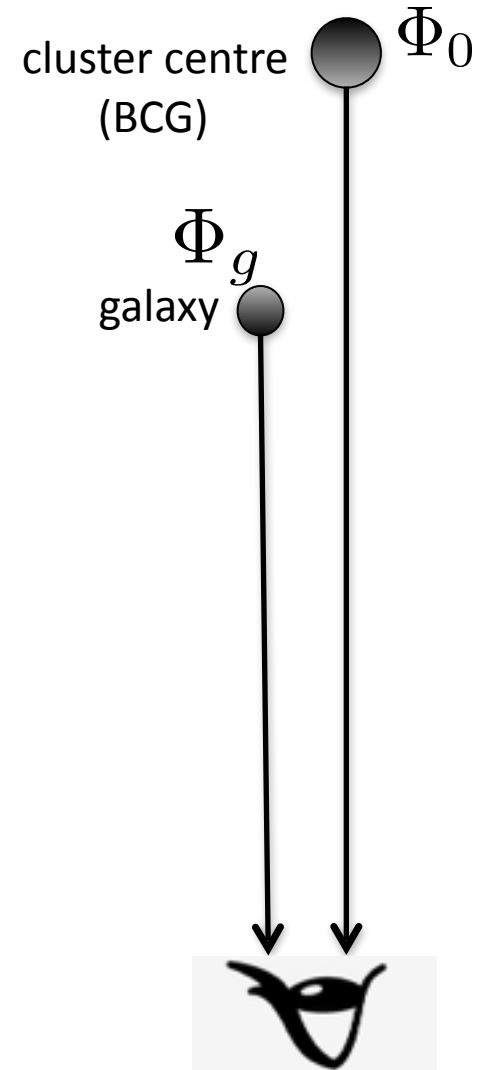
$$cz = Hx + v_x$$

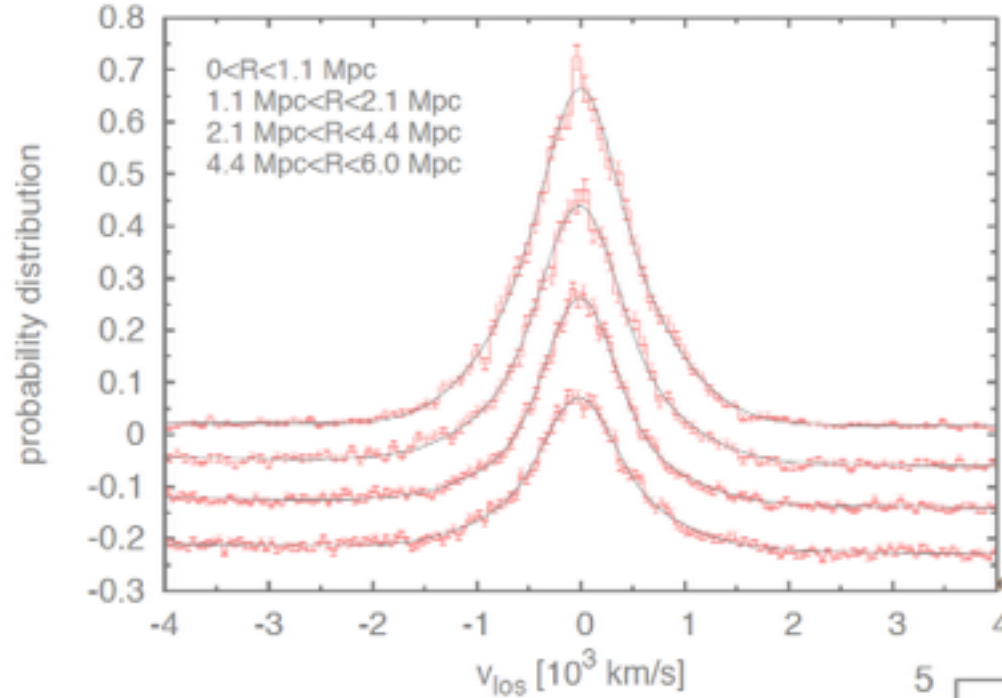
In the weak field limit in GR

$$cz = Hx + v_x - \Phi/c$$

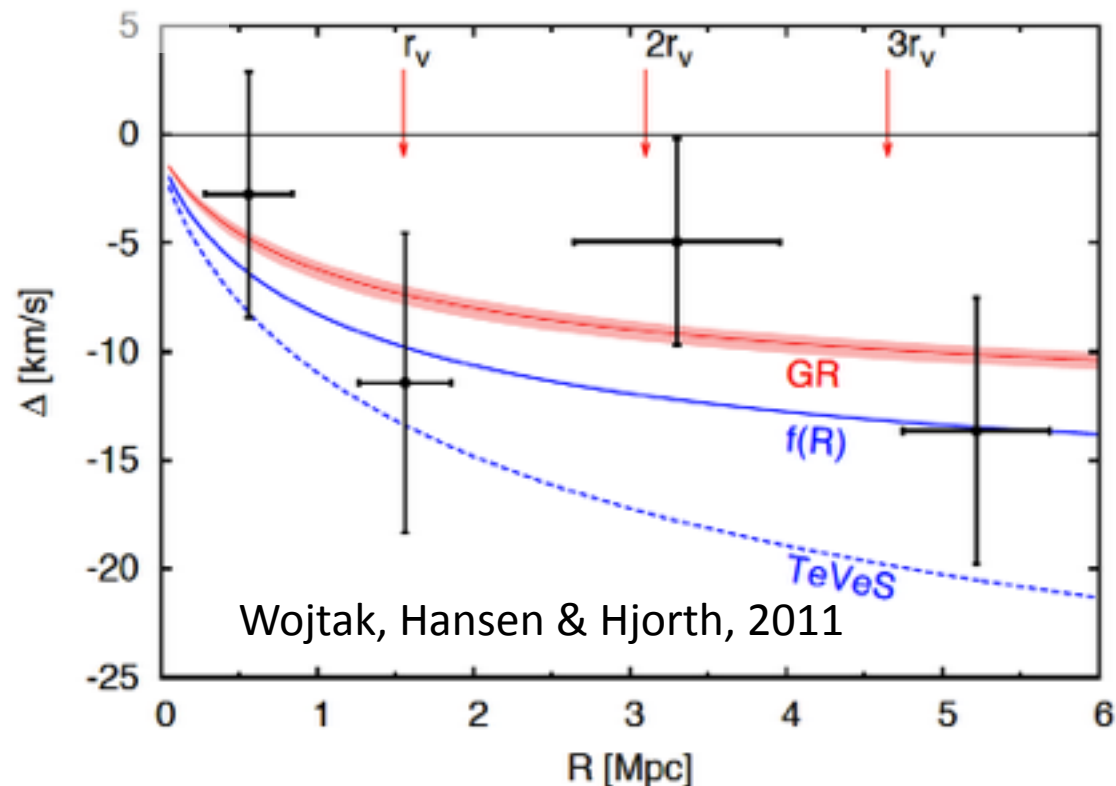
Stacking to beat velocity dispersion

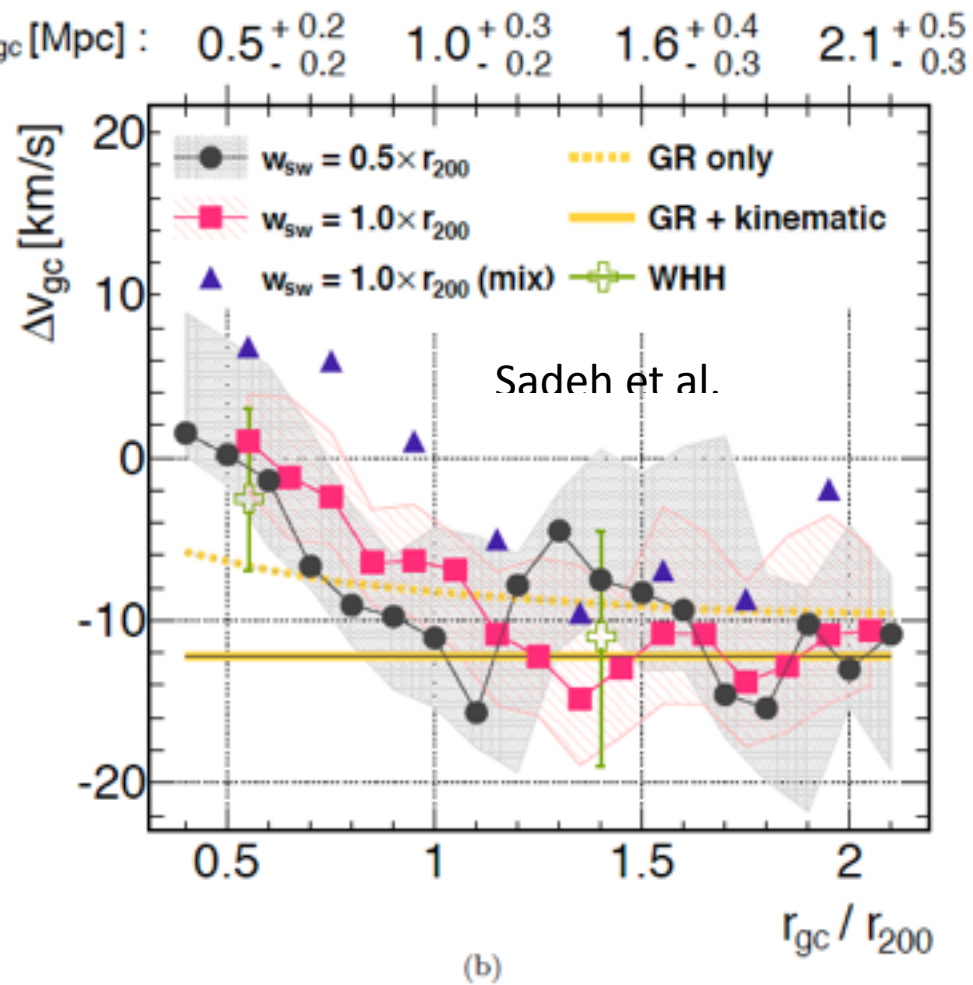
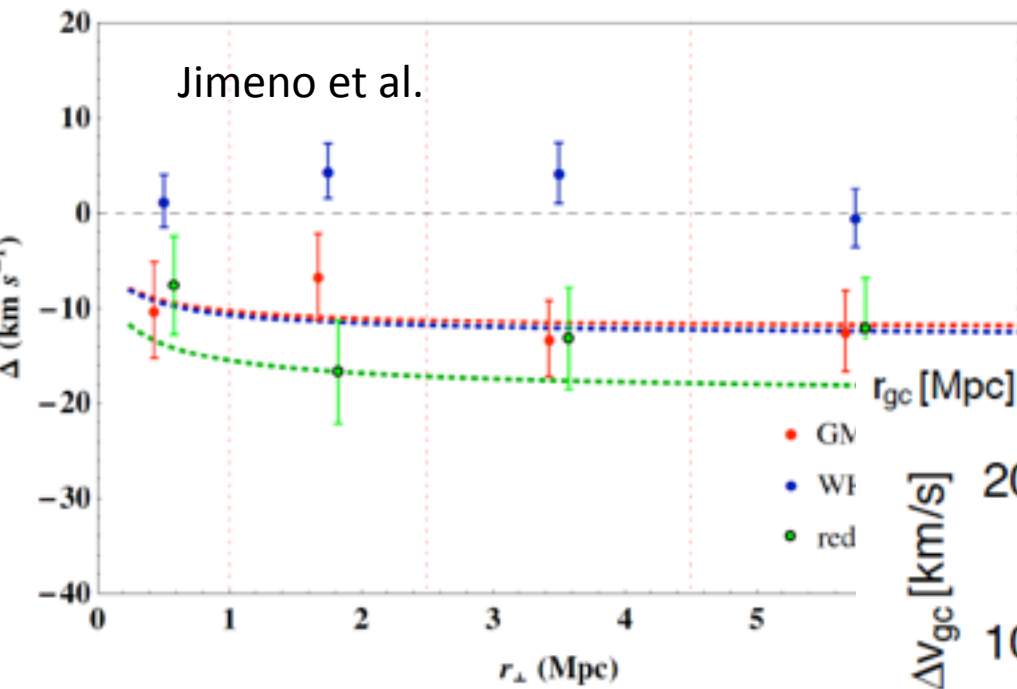
$$\langle cdz \rangle = \langle cz_g - cz_0 \rangle = \langle \Phi_0 - \Phi_g \rangle / c$$

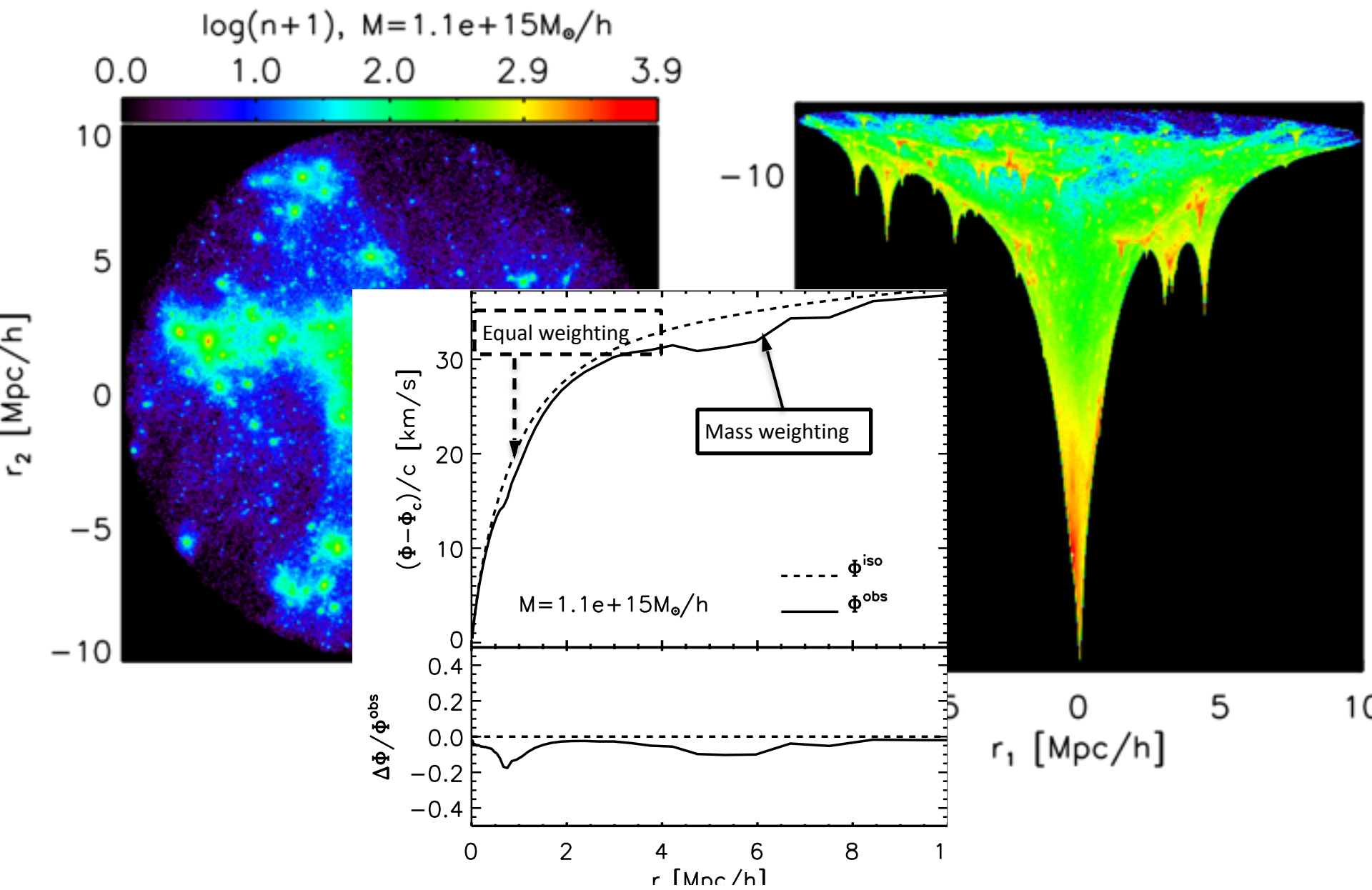




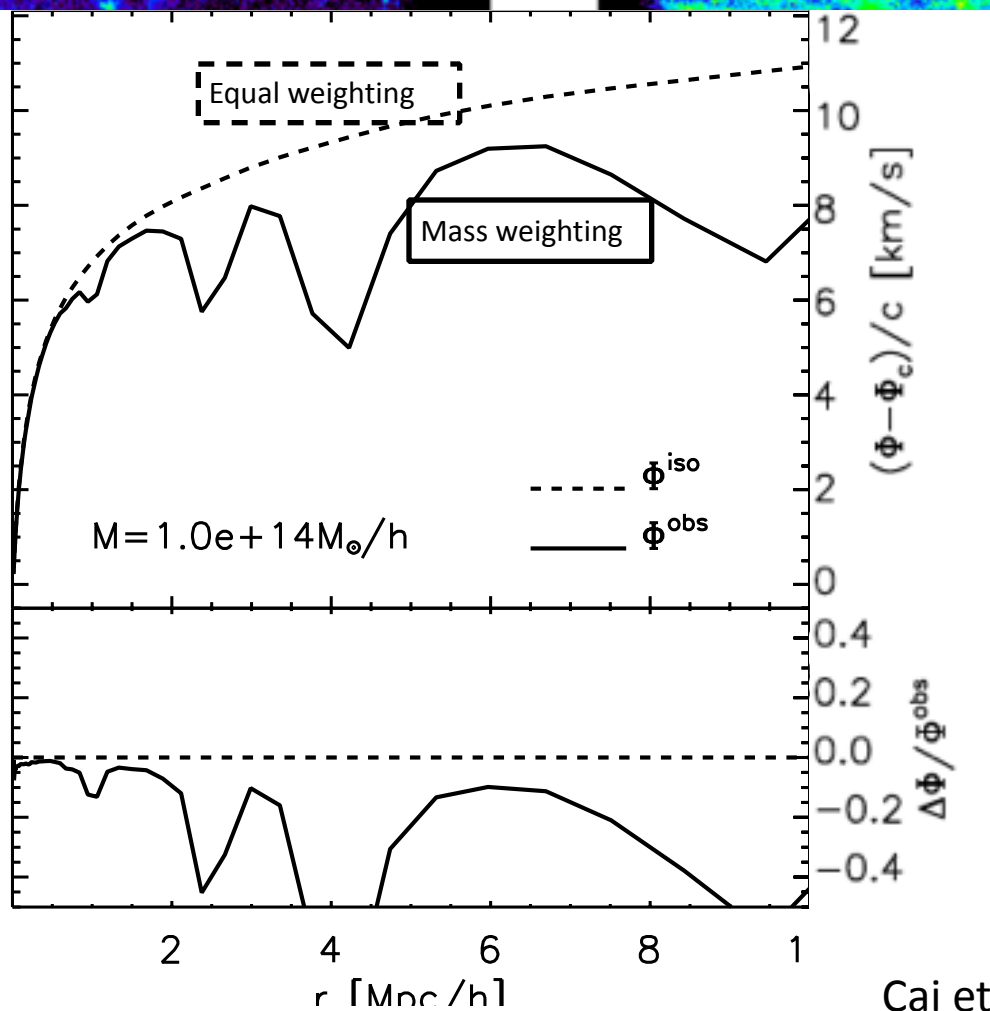
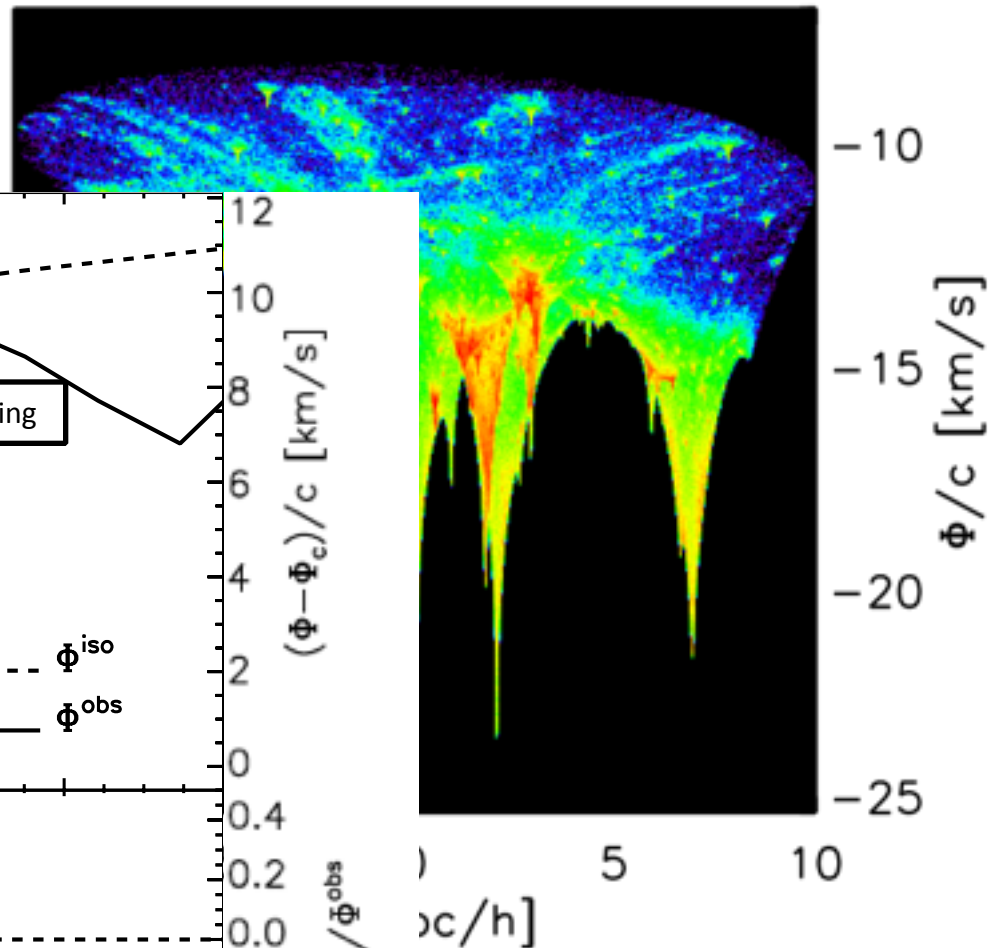
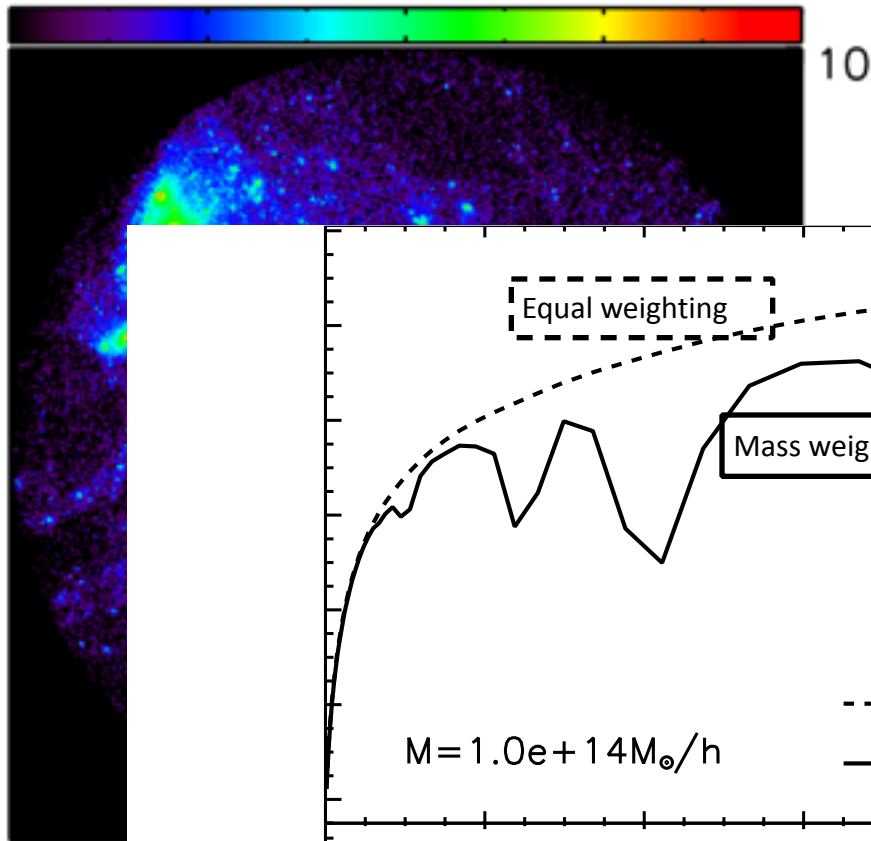
$$\bar{\Phi}^{\text{iso}}(r) = \frac{\int (dN_c/dM) n_{c,g}(r) [\Phi_{\text{core}} - \Phi(r)] dM}{\int (dN_c/dM) n_{c,g}(r) dM}$$

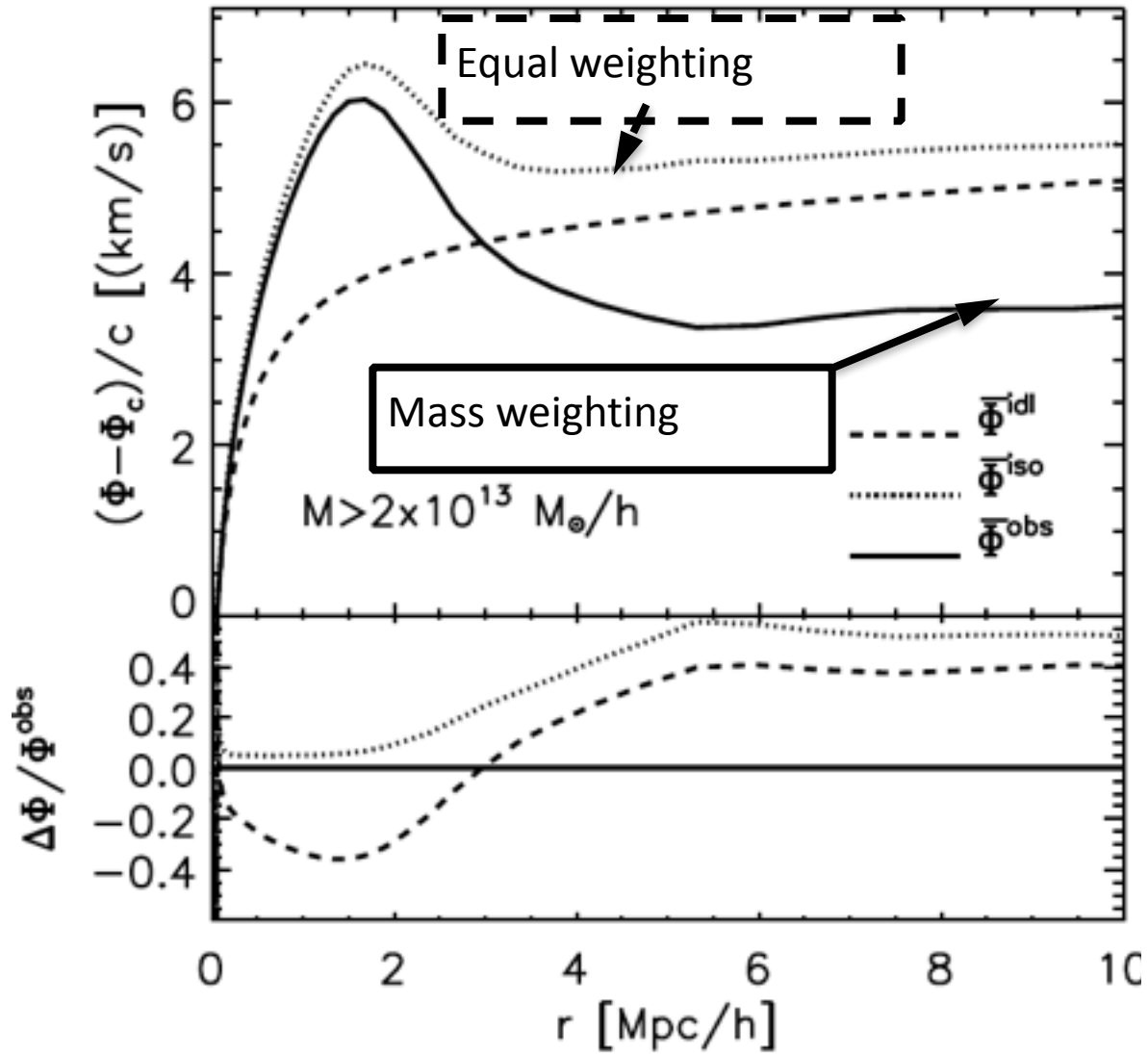






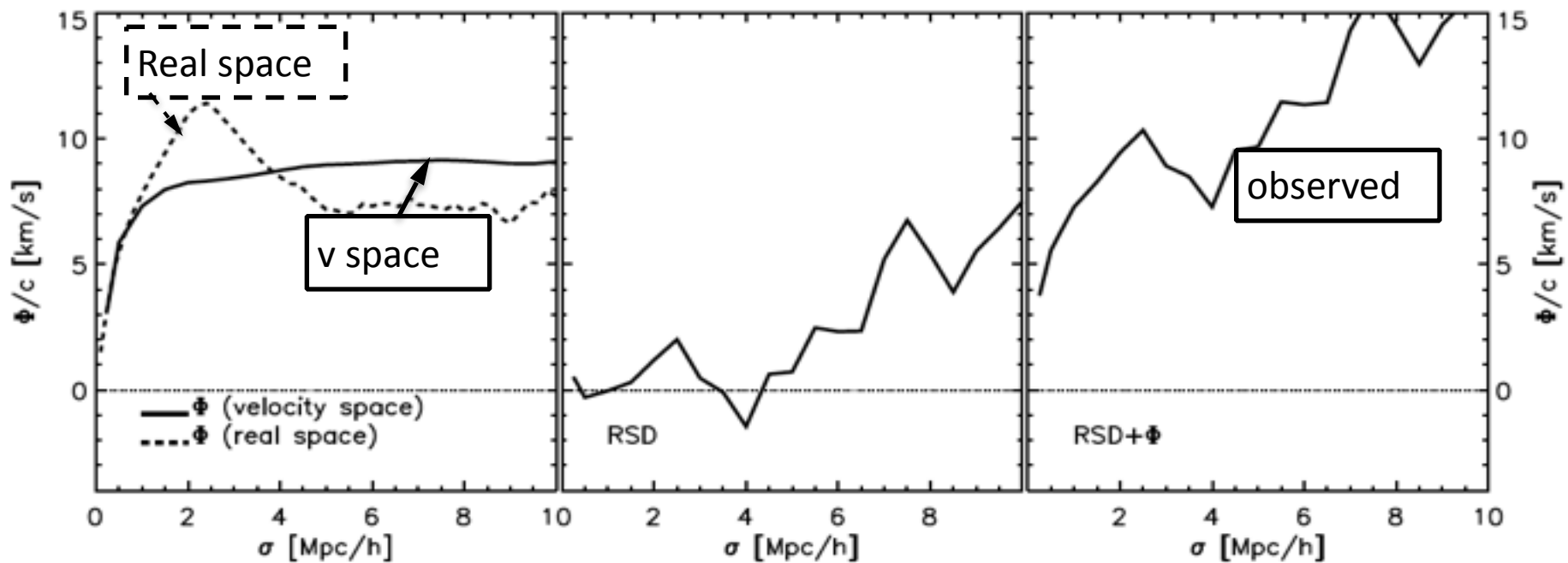
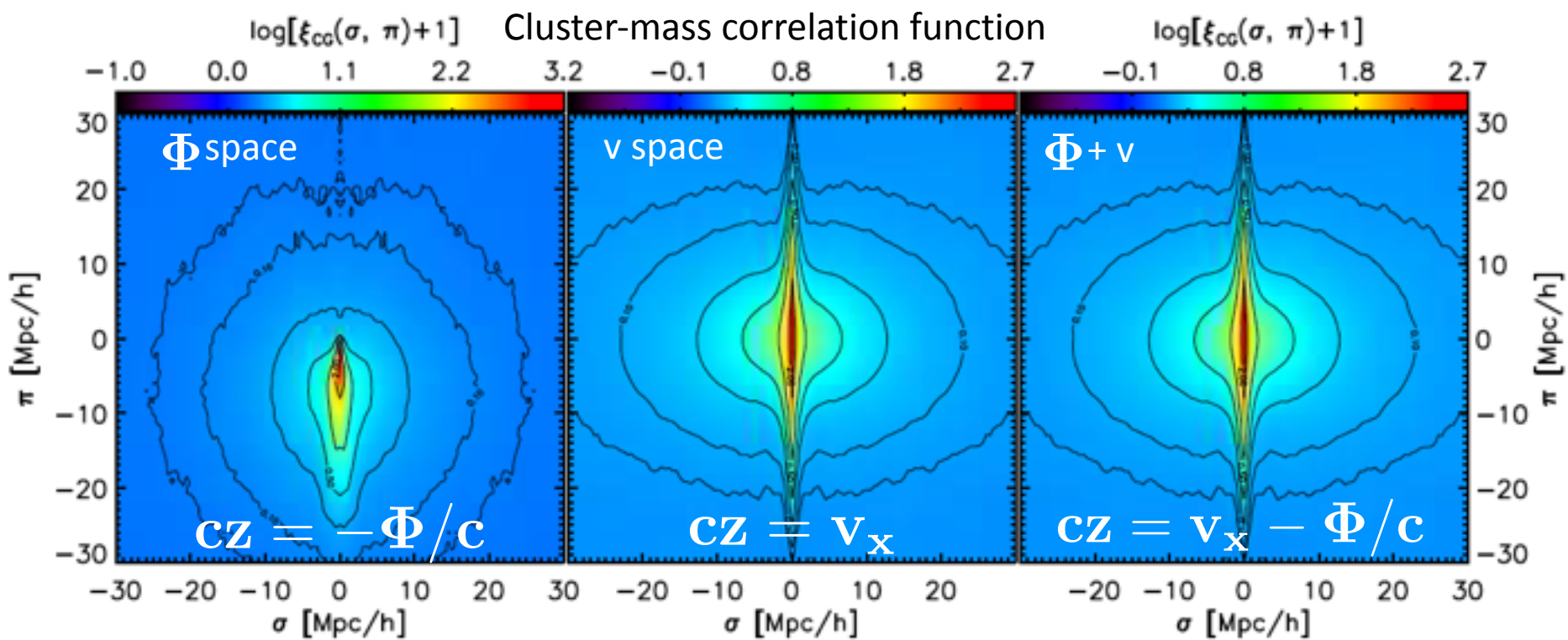
$\log(n+1), M=1.0e+14M_{\odot}/h$
 0.9 1.8 2.7 3.6





real space vs. v-space

Cluster-mass correlation function



Redshift in the past light cone

Photons emitted at time η_0 and at η are received at the same time

$$\eta = \eta_0 - \hat{\mathbf{x}} \cdot \mathbf{r}(\eta) + \dots$$

Galaxy moves: the trajectory of a galaxy

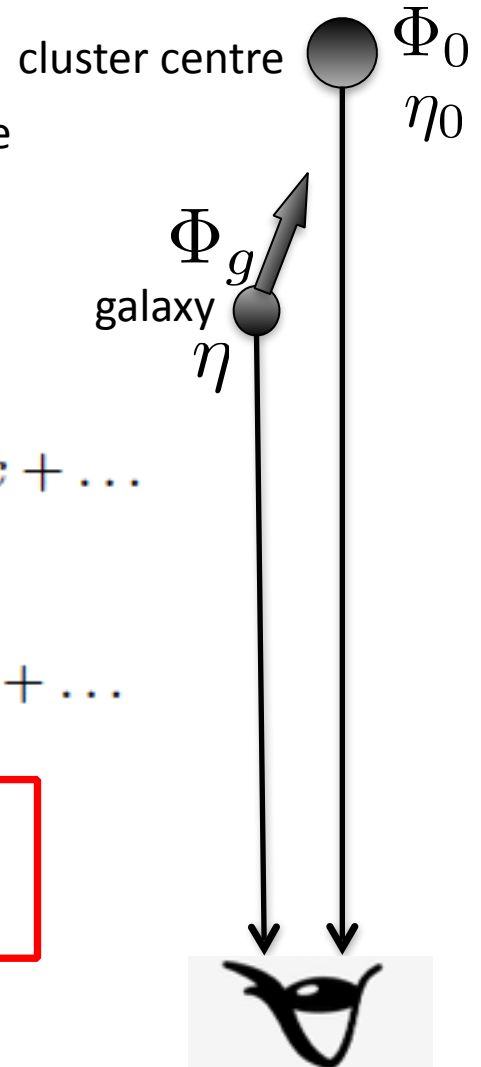
$$\mathbf{r}(\eta) = \mathbf{r} + (\eta - \eta_0)\dot{\mathbf{r}} + \dots$$

Conformal time interval $\Delta\eta = -\hat{\mathbf{x}} \cdot \mathbf{r}/(1 + \hat{\mathbf{x}} \cdot \dot{\mathbf{r}}) = -x + x\dot{x} + \dots$

The Universe expand: expand off the redshift around η_0

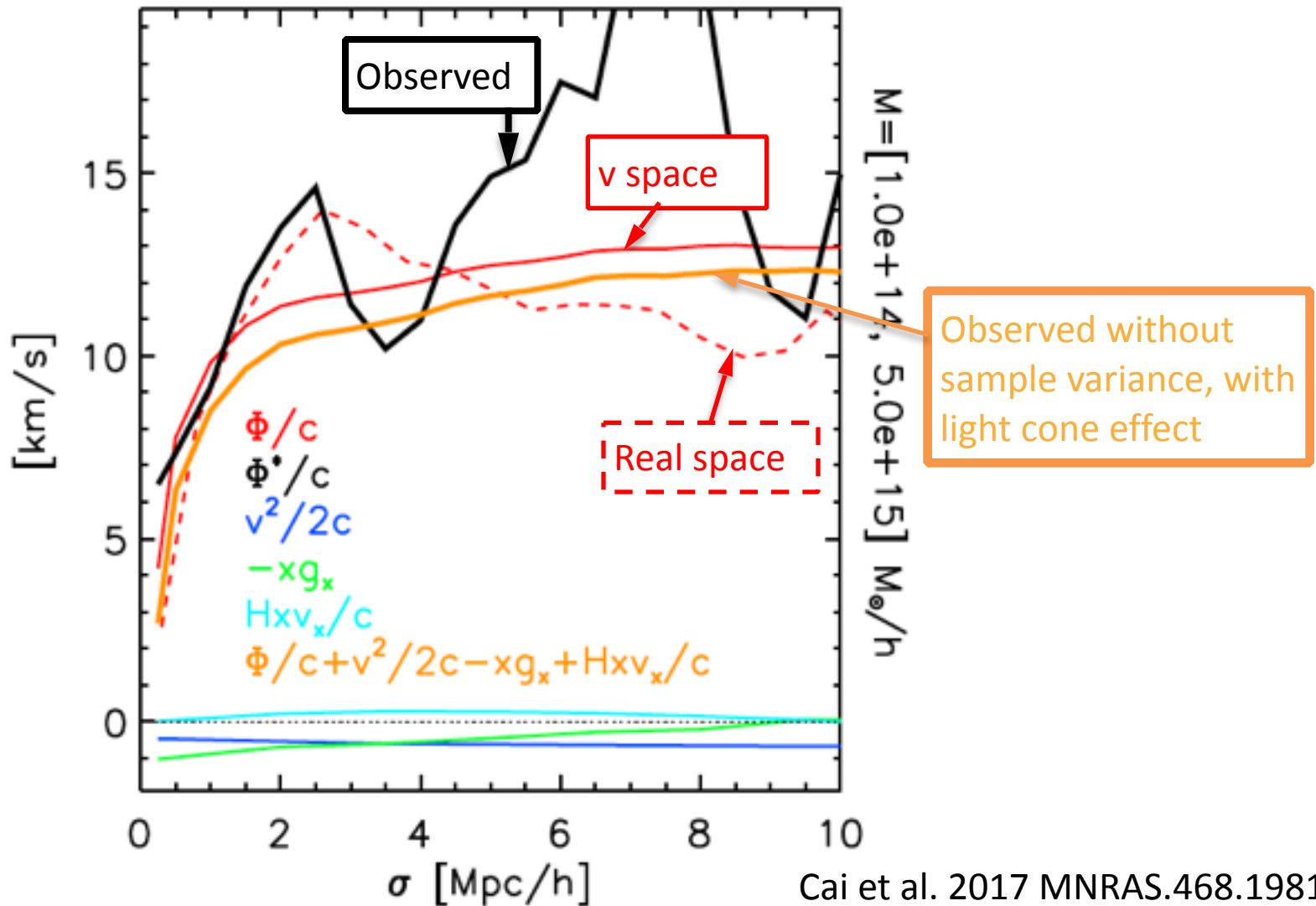
$$(1+z)^{-1} = \frac{a(\eta)}{a(\eta_0)} = 1 + \frac{\dot{a}}{a}\Delta\eta + \frac{1}{2}\frac{\ddot{a}}{a}(\Delta\eta)^2 + \dots$$

$$cz = Hx + v_x + v^2/2c - \Phi/c - xg_x + Hxv_x/c + [H^2 - \ddot{a}/(2a^2)]x^2/c,$$



stationary observer relative to the cluster centre in conformal coordinates

The observed potential profile



SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION*

R. A. SUNYAEV and YA. B. ZELDOVICH

Institute of Applied Mathematics, Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.

(Received 11 September, 1969)

Abstract. Perturbations of the matter density in a homogeneous and isotropic cosmological model which leads to the formation of galaxies should, at later stages of evolution, cause spatial fluctuations of relic radiation. Silk assumed that an adiabatic connection existed between the density perturbations at the moment of recombination of the initial plasma and fluctuations of the observed temperature of radiation $\delta T/T = \delta \rho_m/3\rho_m$. It is shown in this article that such a simple connection is not applicable due to:

- (1) The long time of recombination;
- (2) The fact that when regions with $M < 10^{15} M_\odot$ become transparent for radiation, the optical depth to the observer is still large due to Thompson scattering;
- (3) The spasmodic increase of $\delta \rho_m/\rho_m$ in recombination.

As a result the expected temperature fluctuations of relic radiation should be smaller than adiabatic fluctuations. **In this article the value of $\delta T/T$ arising from scattering of radiation on moving electrons is calculated;** the velocity field is generated by adiabatic or entropy density perturbations. Fluctuations of the relic radiation due to secondary heating of the intergalactic gas are also estimated. **A detailed investigation of the spectrum of fluctuations may, in principle, lead to an understanding of the nature of initial density perturbations since a distinct periodic dependence of the spectral density of perturbations on wavelength (mass) is peculiar to adiabatic perturbations. Practical observations are quite difficult due to the smallness of the effects and the presence of fluctuations connected with discrete sources of radio emission.**

* Translated from the Russian by D. F. Smith.

Summary

- Biases for modelling gravitational redshift:
 - (1) spherical symmetry assumption
 - (2) peculiar velocity
 - (3) past light cone effects

Thank you!