Effects of Entanglement during Inflation on Cosmological Observables

Nadia Bolis [1]

Andreas Albrecht [1] Rich Holman [2]

[1]University of California Davis

[2]Carnegie Mellon University

September 5, 2015

Initial State of Inflation?

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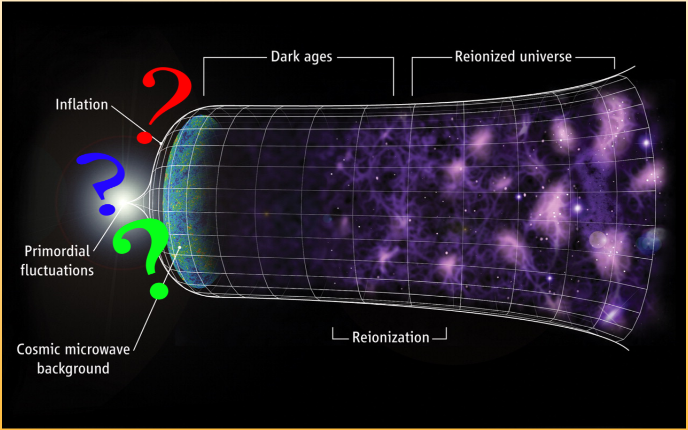

[http://www.giantfreakinrobot.com/wp-content/uploads/2014/03/cosmic-inflation.jpg]

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- Assuming finite inflation, from a phenomenological perspective, it is plausible that the initial state is non-Bunch Davies.
- We choose a particular type of excited state to analyze: an entangled state.

Entangled State

Entangled Gaussian state:

$$\Psi_{\vec{k}}\left[\zeta_{\vec{k}}\,,\gamma_{ij,\vec{k}}\,;\tau\right]=$$

$$N_k(\tau)e^{-\frac{1}{2}\left(A_k(\tau)\zeta_{\vec{k}}\zeta_{-\vec{k}}+B_k(\tau)\gamma_{ij,\vec{k}}\gamma^{ij}_{-\vec{k}}+C_{k,ij}(\tau)\left[\zeta_{\vec{k}}\gamma^{ij}_{-\vec{k}}+\gamma^{ij}_{\vec{k}}\zeta_{-\vec{k}}\right]\right)}$$

where $C_{k,ij}(\tau)$ is the entanglement coefficient between the gauge invariant scalar inflaton fluctuation $\zeta_{\vec{k}}$ and the metric fluctuation $\gamma^{ij}_{\vec{k}}$.

- Gaussian coefficients $A_k(\tau)$ and $B_k(\tau)$ $\rightarrow$ scalar $f_k(\tau)$ and tensor $g_k(\tau)$ mode functions.

Schrödinger Picture QFT

Schrödinger picture QFT $\rightarrow$ equations of motion for A_k, B_k, C_k (on a slow roll background) $\rightarrow$ fluctuation mode functions.
The functional Schrödinger equation is:

$$i\partial_\tau \Psi_{\vec{k}} \left[\zeta_{\vec{k}}\,, \gamma_{ij,\vec{k}}\,; \tau\right] = \left(H_{\zeta\vec{k}} + H_{\gamma\vec{k}}\right) \Psi_{\vec{k}} \left[\zeta_{\vec{k}}\,, \gamma_{ij,\vec{k}}\,; \tau\right]$$

Note, the Hamiltonian's, $H_{\zeta\vec{k}}$ and $H_{\gamma\vec{k}}$, for the scalar and tensor perturbations are decoupled.

Un-Entangled State

If the entanglement parameter $C_{k,ij}(\tau) = 0$ the gaussian state becomes,

$$\Psi_{\vec{k}}\left[\zeta_{\vec{k}}\,,\gamma_{ij,\vec{k}}\,;\tau\right] = N_k(\tau)e^{-\frac{1}{2}\left(A_k(\tau)\zeta_{\vec{k}}\zeta_{-\vec{k}}+B_k(\tau)\gamma_{ij,\vec{k}}\gamma^{ij}_{-\vec{k}}\right)}$$

$$\downarrow$$

Equations of motion of $A_k(\tau)$ and $B_k(\tau)$ result in Bunch-Davies mode functions $f_k^{BD}(\tau)$ and $g_k^{BD}(\tau)$.

- Initial conditions of the mode functions set their BD value:

$$A_k(\tau_0) = A_k^{BD} \rightarrow f_k^{BD}(\tau) \text{ and } B_k(\tau_0) = B_k^{BD} \rightarrow g_k^{BD}(\tau).$$

Entangled State: Closer Look

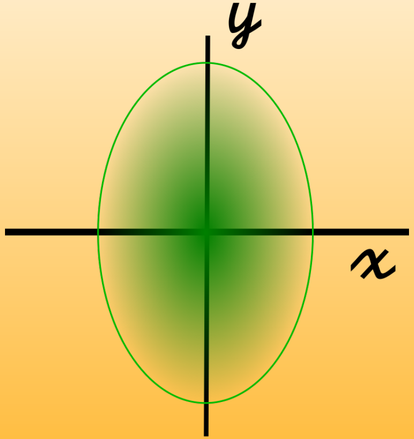
How to think of this state?

The plot a 2D Gaussian of the form,

$$\psi = Ne^{-\frac{1}{2}(Ax^2+By^2)}$$

is an ellipse with axis determined by A and B.

Here there is no entanglement between x and y.



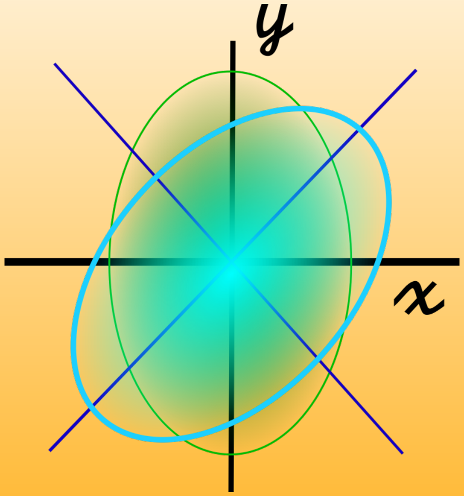
Entangled State: Closer Look

Our state of the form

$$\psi = Ne^{-\frac{1}{2}(Ax^2+By^2+2Cxy)}$$

is a tilted ellipse with respect to the x and y coordinates.

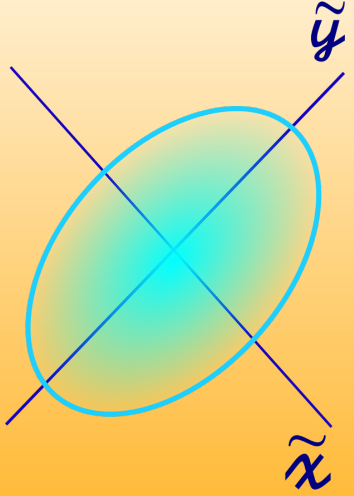
This is an entangled state in the x and y coordinates.



Entangled State: Closer Look

We could redefine the coordinates $\tilde{x}$, $\tilde{y}$ such that the ellipse would no longer be tilted.

In these coordinates the state is not entangled, however the Hamiltonians would have a coupling term between $\tilde{x}$ and $\tilde{y}$.



Observational effects of Entanglement

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- three point functions, i.e. in the level of observable non-gaussianity in the CMB (in progress).

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- three point functions, i.e. in the level of observable non-gaussianity in the CMB (in progress).

This will help:

- constrain levels of allowed entanglement.
- It might help explain the large scale (low l) anisotropy anomaly that appeared in the recent Planck data.

Angular Power Spectra

The final goal is to calculate the temperature fluctuation angular power spectrum,

$$C^{TT}_{lm,l'm'} = \sum_{s,s'} \mathcal{I}_{ss'} = 4\pi \int \frac{dk}{k} \sum_{s,s'} \Delta^T_{l,s}(k,\eta_0) \Delta^T_{l',s'}(k,\eta_0)$$
$$\int d\Omega_{\hat{\mathbf{k}}} P^{ss'}(\mathbf{k})\, {}_{-s}Y^*_{lm}(\hat{\mathbf{k}},\mathbf{e})\, {}_{-s'}Y_{l'm'}(\hat{\mathbf{k}},\mathbf{e})^1$$

where $s = 0, \pm 2$, indicates the spin of the perturbation, the primordial power spectrum is $P^{ss'}$, and $\Delta^T_{l,s}(k,\eta_0)$ is the transfer function that encodes the evolution of the perturbation after the end of inflation until today.

[1] Watanabe, Kanno and Soda: arXiv:1011.3604v3 [astro-ph.CO]

Angular Power Spectra Differences

- In the regular picture there are no cross scalar-tensor terms in the angular power spectrum.
- In our picture, however, the non zero scalar-tensor two point functions lead to extra terms in the C_l's.
- In terms of each spin integral $\mathcal{I}_{ss'}$ (s.t. $C^{TT}_{lm,l'm'} = \sum_{s,s'} \mathcal{I}_{ss'}$):

$$\begin{aligned} C^{TT}_{lm,l'm'} &= \mathcal{I}_{00} + (\mathcal{I}_{22} + \mathcal{I}_{-2-2}) \\ &\quad + (\mathcal{I}_{2-2} + \mathcal{I}_{-22}) + (\mathcal{I}_{02} + \mathcal{I}_{20} + \mathcal{I}_{0-2} + \mathcal{I}_{-20}) \end{aligned}$$

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\begin{aligned}
C^{TT}_{lm,l'm'} &= \overbrace{\mathcal{I}_{00} + (\mathcal{I}_{22} + \mathcal{I}_{-2-2})}^{\propto\, \delta_{ll'}\ \delta mm'} \\
&\quad \underbrace{+(\mathcal{I}_{2-2} + \mathcal{I}_{-22}) + (\mathcal{I}_{02} + \mathcal{I}_{20} + \mathcal{I}_{0-2} + \mathcal{I}_{-20})}_{\propto\, \delta_{mm'} \text{ but } \not\propto\, \delta_{ll'}}
\end{aligned}
$$

Two Entanglement Parameters

We parametrize the entanglement in our state with two entanglement constants:

- *Entanglement amplitude constant* $\lambda_k \rightarrow$ "strength of entanglement"
- *Polarization entanglement constant* $\phi_k \rightarrow$ "amount of h_+ or $h_\times$ polarization contribution to the entanglement"

Two Entanglement Parameters

Schrödinger equation $\rightarrow$

$$C_{k,ij}C_k^{ij} = \frac{1}{2}\frac{(C_{+0}^2 + C_{\times 0}^2)}{(f_k(\tau)g_k(\tau))^2}\frac{a(\tau)^4 \epsilon M_{pl}^4}{4}.$$

$$\downarrow$$

$$C_{+0} = \sqrt{2}|\lambda_k|\cos\phi_k \quad C_{\times 0} = \sqrt{2}|\lambda_k|\sin\phi_k$$

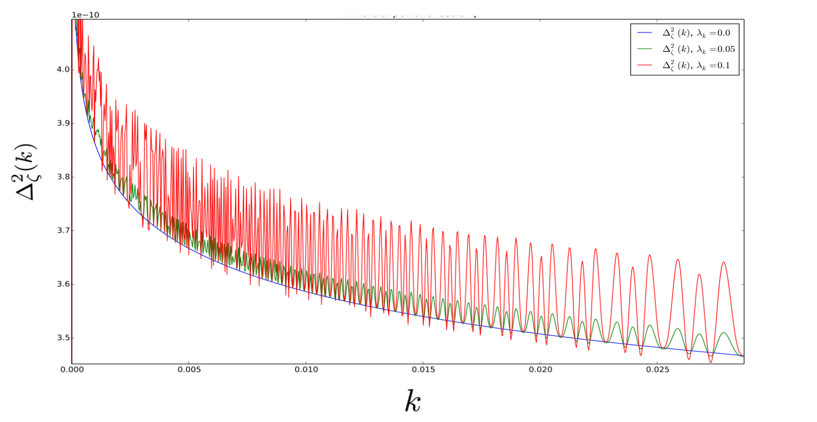
(Note: $\phi_k = n\pi$ or $\phi_k = n\frac{\pi}{2}$ where $n = 0, 1, 2, \ldots$)

$$\downarrow$$

$$C_{k,ij}C_k^{ij} = \frac{|\lambda_k|^2}{(f_k(\tau)g_k(\tau))^2}\frac{a(\tau)^4 \epsilon M_{pl}^4}{4}.$$

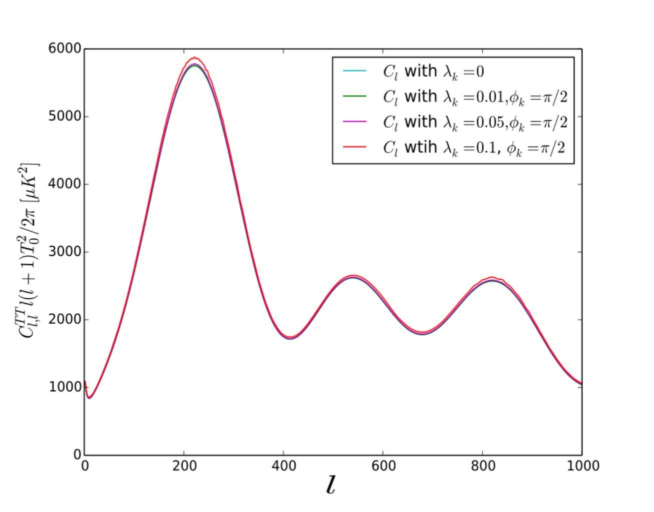
Primordial Power

Primordial power of scalar ζ_k for different values of λ_k : $\Delta^2_\zeta(k)$



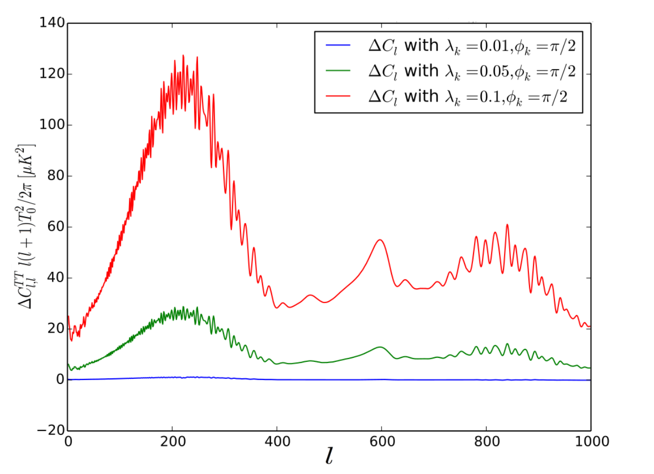
Angular Power Spectra ($l = l'$)

Temperature angular power spectrum for different values of λ_k



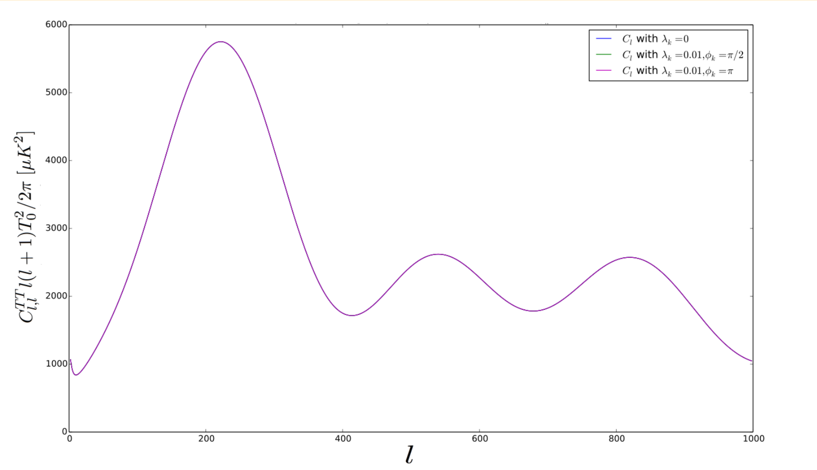
Δ Angular Power Spectra ($l = l'$)

Difference between zero entanglement $C_l^{TT}(\lambda_k = 0)$ and $C_{l,l}^{TT}$ for different values of λ_k



Angular Power Spectra ($l = l'$)

Temperature angular power spectrum for different values of ϕ_k



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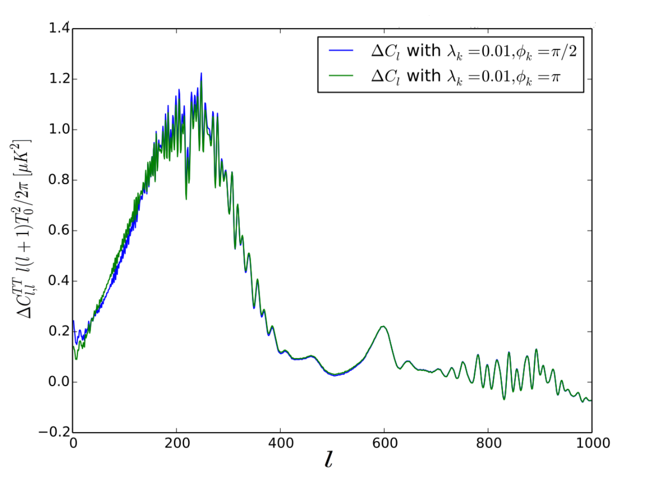
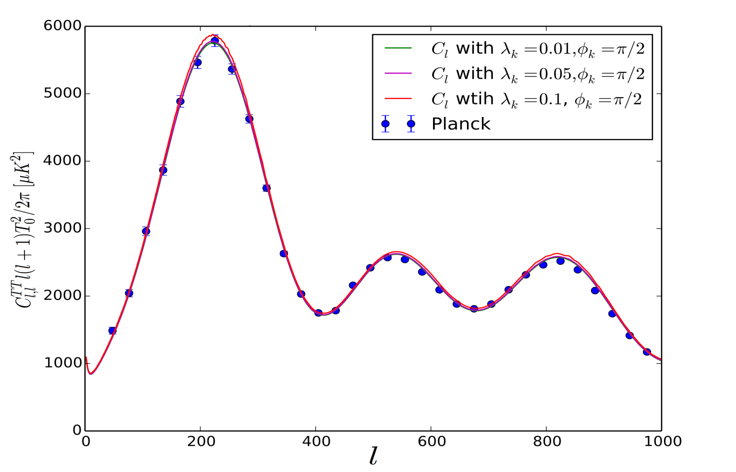


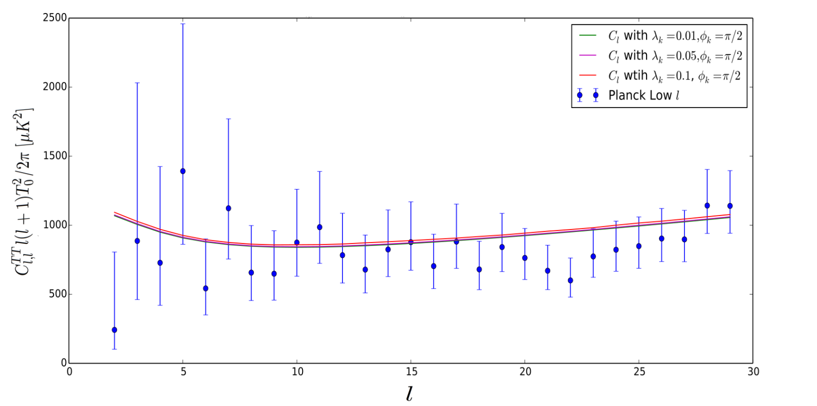
Angular Power Spectrum ($l = l'$) with Planck

$C_{l,l}^{TT}$ for different values of λ_k with Planck. Note: our C_l's do not yet include lensing so higher l amplitudes are less damped.



Angular Power Spectra ($l = l'$). Low l

$C_{l,l}^{TT}$ at low l for different values of λ_k with Planck.



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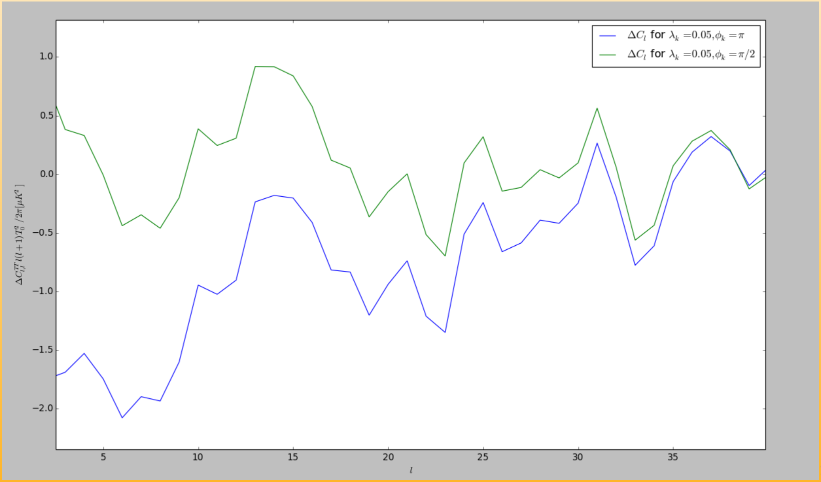
How does address the low l anomaly?

- Note, so far I assumed same scale of inflation for all curves
- However, scale of inflation is not yet experimentally set so it can be treated as a free parameter

Shifted Inflation Scale C_l

Difference between zero entanglement $C_l^{TT}(\lambda_k = 0)$ and $C_{l,l}^{TT}$ (with lower inflation scale) for different values of ϕ_k for low l.



Future Work and Improvements

- add lensing
- find constraints on the entanglement parameters λ_k and ϕ_k from the Planck data.
- calculate the f_{NL}'s to constrain the parameters based non-gaussianity data constraints.
- Find best fit parameters (entanglement and inflation scale) that may give a C_l that fits the low l power deficit (very speculative)

The End

The End/La Fine

BackUp

Metric and Scalar Perturbations

- The ζ is the gauge invariant scalar perturbation to the inflaton.
- The metric perturbation γ_{ij} is also a physical degree of freedom, and in terms of the cross $\times$ and plus $+$ polarization fluctuations it can be expressed as:

$$\gamma_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- The z-direction is in the line of sight and γ_{ij} is both traceless $\gamma_{ii} = 0$ and divergentless $\partial_i \gamma_{ij} = 0$.

Actions

The actions for the gauge invariant scalar perturbations and the 3-metric perturbation are:

$$
\begin{aligned}
S_\zeta &= \int d^4x\, a(t)^3 \left[\frac{\epsilon M_{pl}^2}{2}\partial_\mu\zeta\partial^\mu\zeta\right], \\
S_\gamma &= \int d^4x\, a(t)^3 \left[\frac{M_{pl}^2}{8}\partial_\mu\gamma_{ij}\partial^\mu\gamma^{ij}\right]'
\end{aligned}
$$

where $a(t)$ is the scale factor, ϵ the slow roll parameter and M_{pl} the Plank mass.

Momentum Space Hamiltonians

In momentum space the Hamiltotinans per (decoupled) k mode are:

$$H_{\vec{k},\zeta} = \frac{\Pi_{\vec{k}\zeta}\Pi_{-\vec{k}\zeta}}{2\alpha^2} + \frac{k^2\alpha^2}{2}\zeta_{\vec{k}}\zeta_{-\vec{k}}, \quad H_{\vec{k},\gamma} = \frac{\Pi_{ij,\vec{k}}\Pi^{ij}_{-\vec{k}}}{2\beta^2} + \frac{k^2\beta^2}{2}\gamma_{ij,\vec{k}}\gamma^{ij}_{-\vec{k}}$$

where

$$\alpha^2 = a(\tau)^2\epsilon M_{pl}^2, \qquad \beta^2 = \frac{a(\tau)^2 M_{pl}^2}{4}$$

and the canonical momentum is defined in the usual way as,

$$\Pi_{\vec{k},\zeta} = \frac{\delta\mathcal{L}}{\delta\dot{\zeta}} = a(\tau)^2\epsilon M_{pl}^2\zeta_{\vec{k}}, \qquad \Pi_{ij,\vec{k}} = \frac{\delta\mathcal{L}}{\delta\dot{\gamma}^{ij}} = \frac{a(\tau)^2 M_{pl}^2}{4}\gamma_{ij,\vec{k}}.$$

Mode Equations

Using the Schrödinger equation and doing the change of variables:

$$
\begin{aligned}
iA_k(\eta) &= \alpha^2(\tau)\left(\frac{f_k'(\tau)}{f_k(\tau)} - \frac{a'(\tau)}{a(\tau)}\right), \\
iB_k(\eta) &= \beta^2(\tau)\left(\frac{g_k'(\tau)}{g_k(\tau)} - \frac{a'(\tau)}{a(\tau)}\right).
\end{aligned}
$$

we get the following equations of motion for the mode functions of ζ and γ_{ij} :

$$
\zeta : \quad f_k'' + \left(k^2 - \frac{\alpha''(\tau)}{\alpha(\tau)}\right) f_k = \frac{C_{ij,k} C_k^{ij}}{\alpha^2 \beta^2} f_k
$$

$$
\gamma : \quad g_k'' + \left(k^2 - \frac{\beta''(\tau)}{\beta(\tau)}\right) g_k = \frac{C_{ij,k} C_k^{ij}}{\alpha^2 \beta^2} g_k
$$

Mode Equations

Furthermore, the equation for C_k yields the relation,

$$\frac{C_{ij,k}(\tau)C_k^{ij}(\tau)}{\alpha^2\beta^2} = \frac{\lambda_k^2}{(f_k(\tau)g_k(\tau))^2},$$

where λ_k is a constant that parametrizes the entanglement.

Bunch-Davies Initial Conditions

To later compare to the standard Bunch-Davies initial state picture, we set our initial state value to be BD:

$$iA_k^{BD}(\tau) = \alpha^2(\tau)\left(\frac{f_k^{BD\prime}(\tau)}{f_k^{BD}(\tau)} - \frac{a'(\tau)}{a(\tau)}\right),$$

with the Bunch-Davies state being:

$$f_k^{\mathrm{BD}}(\tau) = \frac{\sqrt{-\tau\pi}}{2} H_{\nu_\zeta}^{(1)}(-k\tau), \quad \nu_\varphi = \sqrt{\frac{9}{4} - \frac{m_\zeta^2}{H_I^2}}$$

where $H_{\nu_\zeta}^{(1)}$ is a Hankel function of the first kind.

Equations of Motion and Initial Conditions

The final form of the equations of motions are:

$$\zeta \text{ equation:} \quad f_k'' + \left(k^2 + \frac{\nu_\zeta^2 - \frac{1}{4}}{\tau^2}\right) f_k = \frac{\lambda_k^2}{f_k g_k^2},$$

$$\gamma \text{ equation:} \quad g_k'' + \left(k^2 + \frac{\nu_\gamma^2 - \frac{1}{4}}{\tau^2}\right) g_k = \frac{\lambda_k^2}{f_k^2 g_k},$$

subject to the initial conditions

$$f_k(\tau_0) = f_k^{\mathrm{BD}}(\tau_0), \quad f_k'(\tau_0) = f_k^{\mathrm{BD}\prime}(\tau_0)$$
$$g_k(\tau_0) = g_k^{\mathrm{BD}}(\tau_0), \quad g_k'(\tau_0) = g_k^{\mathrm{BD}\prime}(\tau_0).$$

with $\nu_\zeta^2 = \frac{3}{2}(1 - n_s) + \frac{9}{4}$ in terms of the spectral index n_s and $\nu_\gamma = \frac{3}{2}$ indicating that the tensor perturbations are massless.

The Second Entanglement Parameter

As we will see the two-point functions will be expressed in terms of 2 entanglement parameters. The second parameter is defined by the following:

- In terms of the polarizations, the matrix

$$\gamma_{ij} = h_+ \hat{e}^+_{ij} + h_\times \hat{e}^\times_{ij}$$

  where $\hat{e}^{+,\times}_{ij}$ are unit matrices that obey $\hat{e}^\sigma_{ij}\hat{e}^{\sigma'}_{ij} = \delta^{\sigma\sigma'}$.
- We define $C_+ = C^{ij}\hat{e}^+_{ij}$ and similarly $C_\times = C^{ij}\hat{e}^\times_{ij}$.
- We also define

$$C_+ = \frac{C_{+0}}{\sqrt{2}}\frac{\alpha\beta}{f_k g_k} \quad C_\times = \frac{C_{\times 0}}{\sqrt{2}}\frac{\alpha\beta}{f_k g_k}$$

  in terms of constants C_{+0} and $C_{\times 0}$ such that

$$C_{ij}C^{ij} = \frac{1}{2}(C_+^2 + C_\times^2) = \lambda_k^2 \frac{\alpha^2\beta^2}{f_k^2 g_k^2}.$$

The Second Entanglement Parameter

- This leads to

$$C_{+0} = \sqrt{2}|\lambda_k| \cos\phi_k \quad C_{\times 0} = \sqrt{2}|\lambda_k| \sin\phi_k$$

  where ϕ_k is the second entanglement parameter that parametrized the amount of each polarization affected.
- Moreover we will write the mode functions f_k and g_k in terms of their magnitudes and phases:

$$f_k = |f_k| e^{i\theta_{fk}} \quad g_k = |g_k| e^{i\theta_{gk}}.$$

Two Entanglement Parameters

We parametrize the entanglement in our state with two entanglement constants λ_k and ϕ_k.

- In terms of the polarizations, the metric fluctuation matrix $\gamma_{ij} = h_+ \hat{e}^+_{ij} + h_\times \hat{e}^\times_{ij}$, we define $C_+ = C^{ij}\hat{e}^+_{ij}$ and $C_\times = C^{ij}\hat{e}^\times_{ij}$ such that

$$C_{k,ij}C^{ij}_k = \frac{1}{2}(C^2_+ + C^2_\times) = \frac{\lambda_k^2}{(f_k(\tau)g_k(\tau))^2}\frac{a(\tau)^4 \epsilon M_{pl}^4}{4}.$$

where the entanglement parameter $C_{k,ij}$ can be written in terms of the scalar and tensor mode functions and the first entanglement parameter λ_k.

Two Entanglement Parameters

- In addition we define

$$C_+ = \frac{1}{\sqrt{2}} \frac{C_{+0}}{f_k g_k} \frac{a(\tau)^2 \sqrt{\epsilon} M_{pl}^2}{2} \qquad C_\times = \frac{1}{\sqrt{2}} \frac{C_{\times 0}}{f_k g_k} \frac{a(\tau)^2 \sqrt{\epsilon} M_{pl}^2}{2}$$

in order to define the second entanglement parameter ϕ_k with:

$$C_{+0} = \sqrt{2}|\lambda_k| \cos\phi_k \quad C_{\times 0} = \sqrt{2}|\lambda_k| \sin\phi_k$$

- The second entanglement constant ϕ_k sets the amount of $+$ or $\times$ polarization contribution. In this case, the equations of motion from the Shrödinger equation, set the values to either $\phi_k = n\pi$ or $\phi_k = n\frac{\pi}{2}$ where $n = 0, 1, 2, ...$

Two Point Function and Density Matrix

- The primordial power spectrum of the inflaton scalar perturbations ζ and similarly for the tensor (polarization) perturbations h_+ and $h_\times$ are related to the two point function by:

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\rangle = (2\pi)^3 P_\zeta(k)\delta^3(\vec{k}-\vec{k}')$$

- The two point function of an operator $\mathcal{O}$ is defined as $\langle \mathcal{O}^2 \rangle = Tr\left[\rho\, \mathcal{O}^2\right]$ where ρ is the density matrix of the system.
- The density matrix needed is the reduced density matrix of ζ that has the degrees of freedom related to the other fields h_+ and $h_\times$ traced out.

Two Point Functions

The two point functions for the perturbations are:

$$\langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle = \frac{|f_k|^2}{a^2 \epsilon M_{pl}^2} \frac{1}{1 - 4|\lambda_k|^2 \cos^2(\theta_{fk} + \theta_{gk})},$$

$$\langle h_{\vec{k}+} h_{-\vec{k}+} \rangle = \frac{4|g_k|^2}{a^2 M_{pl}^2} \frac{1 - 4|\lambda_k|^2 \sin^2(\phi_k) \cos^2(\theta_{fk} + \theta_{gk})}{1 - 4|\lambda_k|^2 \cos^2(\theta_{fk} + \theta_{gk})},$$

$$\langle h_{\vec{k}\times} h_{-\vec{k}\times} \rangle = \frac{4|g_k|^2}{a^2 M_{pl}^2} \frac{1 - 4|\lambda_k|^2 \cos^2(\phi_k) \cos^2(\theta_{fk} + \theta_{gk})}{1 - 4|\lambda_k|^2 \cos^2(\theta_{fk} + \theta_{gk})},$$

Two Point Functions

$$\langle \zeta_{\vec{k}} h_{-\vec{k}+} + h_{\vec{k}+} \zeta_{-\vec{k}} \rangle = -\frac{2}{a^2 \sqrt{\epsilon} M_{pl}^2} \frac{4|f_k||g_k||\lambda_k| \cos(\phi_k)}{1 - 4|\lambda_k|^2 \cos^2(\theta_{fk} + \theta_{gk})},$$

$$\langle \zeta_{\vec{k}} h_{-\vec{k}\times} + h_{\vec{k}\times} \zeta_{-\vec{k}} \rangle = -\frac{2}{a^2 \sqrt{\epsilon} M_{pl}^2} \frac{4|f_k||g_k||\lambda_k| \sin(\phi_k)}{1 - 4|\lambda_k|^2 \cos^2(\theta_{fk} + \theta_{gk})}.$$

The primordial power spectrum for each perturbation is of the form:

$$\Delta_\varphi^2(k) \equiv \frac{k^3}{2\pi^2} \langle \varphi_{\vec{k}} \varphi_{-\vec{k}} \rangle \, |_{\tau \to 0^-}$$