Holography for the Pseudo-Conformal Universe

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KH, Justin Khoury arXiv:1106.1428 (JCAP 070 1011)
KH, James Stokes, Mark Trodden arXiv:1408.1955 (JHEP 073 1014)

Cosmo Cruise, Sept. 8, 2015

Inflation

Dominant paradigm for the very early universe

Inflation ≈ exponential expansion ≈ de Sitter space

$$ds^2 = -dt^2 + a(t)^2 dx^2 \ , \qquad a(t) = e^{Ht}$$

Driven by vacuum energy with w ≈ -1, $\rho \sim a^{-3(1+w)}$

$$3M_P^2 H^2 = \rho_V \longleftarrow \text{constant}$$

Smoothness, flatness, monopole problems:
Other possible components with w > -1 are emptied out

$$3M_P^2 H^2 = \underset{\text{constant}}{\rho_V} - \underbrace{\overset{\text{curvature}}{\frac{3M_P^2 k}{a^2}} + \overset{\text{matter}}{\frac{\rho_{m,0}}{a^3}} + \overset{\text{radiation}}{\frac{\rho_{r,0}}{a^4}} + \overset{\text{anisotropies}}{\frac{\rho_{A,0}}{a^6}}}_{\to 0}$$

Inflation is rooted in symmetries

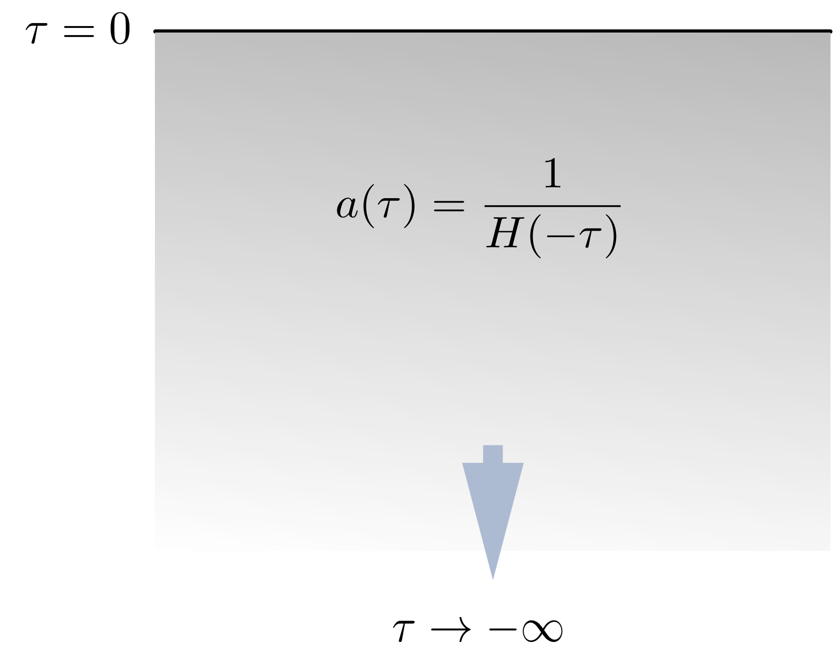
de Sitter space is maximally symmetric:
There are 10 Killing vectors:

- 3 spatial translations and 3 spatial rotations, forming *iso(3)*

- Plus a dilation and 3 special generators:

$$D = \tau\partial_\tau + x^i\partial_i$$

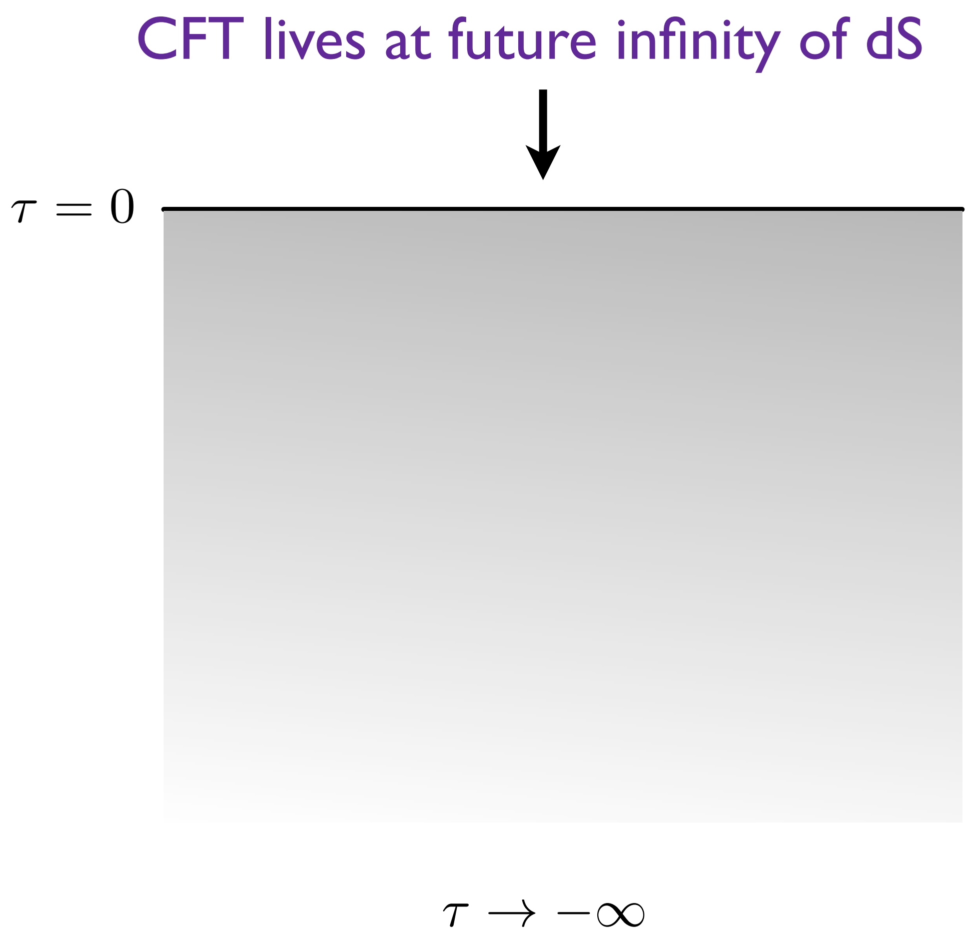
$$K_i = 2x_i\tau\partial_\tau - \left(-\tau^2 + x^2\right)\partial_i + 2x_i x^j\partial_j$$



Symmetry algebra is: $so(4,1)$

Symmetry accounts for the observed scale invariance of fluctuations, and relations among observables

Strong coupling: dS/CFT realization of inflation



cosmological correlation functions $\longleftrightarrow$ boundary CFT correlators

- Interested in bulk cosmology: boundary theory is a mathematical trick to calculate bulk observables

Pseudo-conformal scenario

- Non-inflationary scenario
- Gravity is relatively unimportant: spacetime is approximately flat
- More symmetric than inflation: CFT with so(4,2) symmetry
- Spontaneously broken: so(4,2) $\rightarrow$ so(4,1)
- Essential physics is fixed by the symmetry breaking pattern, independently of the specific realization or microphysics
- Many possible realizations:

$$\left\{ \begin{array}{l} \text{Rubakov's U(1) model} \\ \text{Galilean Genesis} \\ \phi^4 \text{ model} \\ \qquad\vdots \end{array} \right.$$

V. Rubakov, 0906.3693; 1007.3417; 1007.4949; 1105.6230

Creminelli, Nicolis & Trincherini, 1007.0027

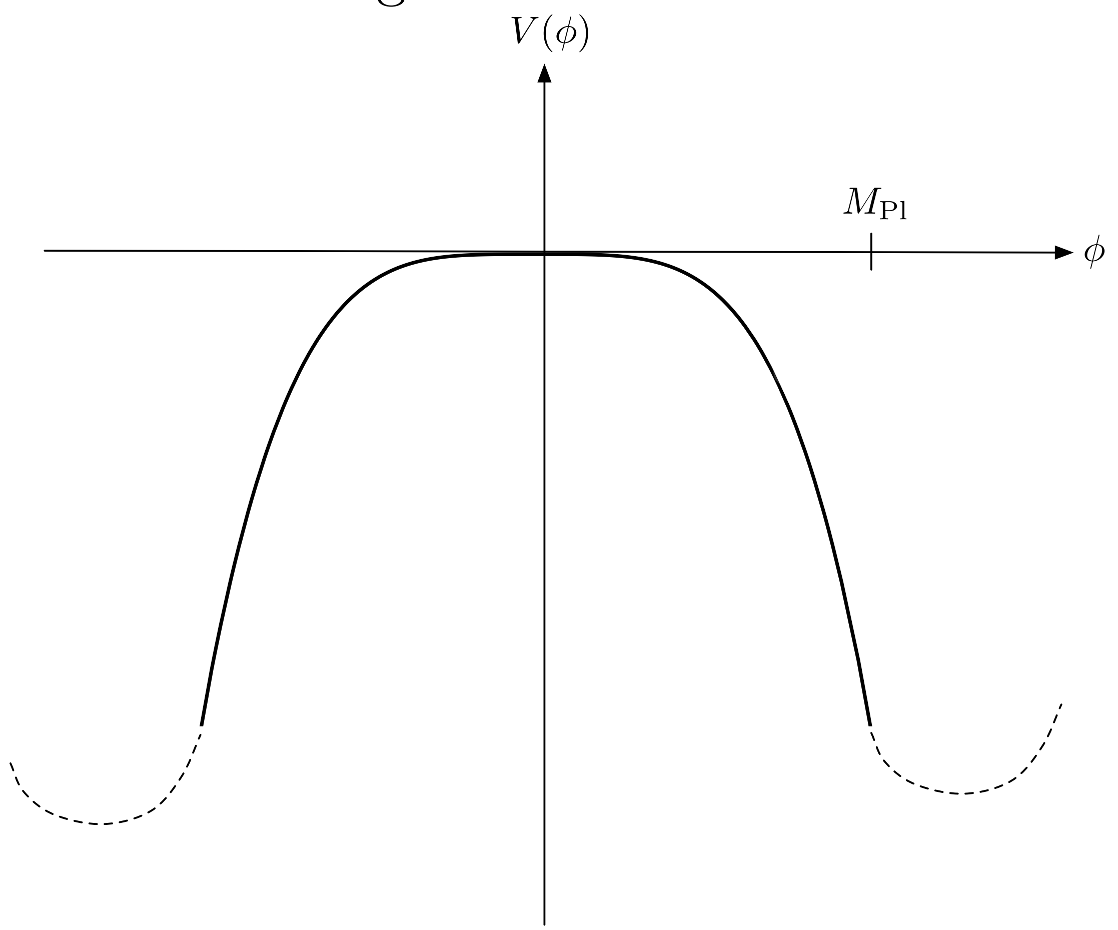
KH, Justin Khoury arXiv:1106.1428

Simplest example: negative quartic

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4$$

KH, Justin Khoury arXiv:1106.1428

Classically, this is a conformal field theory, with a field of weight $\Delta = 1$



There is a solution where the field rolls down the negative quartic potential:

$$\bar{\phi} = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}, \quad t \in (-\infty, 0)$$

- Solution has zero energy $\rho_\phi = 0$

- Pressure is non-zero and positive (satisfies the NEC, infinite equation of state) $p_\phi = \frac{2}{\lambda t^4}, \quad w = \infty$

- Solution is an attractor

- Symmetry breaking pattern: $so(4,2) \to so(4,1)$

Adding gravity

Couple minimally to gravity (breaks conformal invariance at $1/M_P$ level):

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\lambda}{4} \phi^4 \right)$$

Solution has zero energy $\Rightarrow$ spacetime approximately flat
Solve the Friedman equations in powers of $1/M_P$

$$\phi(t) \approx \frac{\sqrt{2}}{\sqrt{\lambda}(-t)} \quad , \quad H(t) \approx \frac{1}{3\lambda t^3 M_{\rm P}^2} \quad , \quad a(t) \approx 1 - \frac{1}{6\lambda t^2 M_{\rm P}^2}$$

Solution is a slowly contracting universe

Approximation is valid in the range $-\infty < t < t_{\rm end}$

$$t_{\rm end} \sim -\frac{1}{\sqrt{\lambda} M_{\rm Pl}} \, , \quad \phi_{\rm end} \sim M_{\rm Pl} \, .$$

The field forms a very stiff fluid

$$\rho_\phi \approx \frac{1}{3\lambda^2 t^6 M_{\rm P}^2} \, , \quad p_\phi \approx \frac{2}{\lambda t^4} \, , \quad w \approx 6\lambda t^2 M_{\rm P}^2$$

w goes from $\gg 1$ to $O(1)$ as t ranges from $-\infty$ to $t_{\rm end}$

Solution to flatness, smoothness problems

There is now a scalar field component with extremely stiff equation of state $w \gg 1$

$$3M_P^2 H^2 = \underbrace{-\frac{3M_P^2 k}{a^2} + \frac{\rho_{m,0}}{a^3} + \frac{\rho_{r,0}}{a^4} + \frac{\rho_{A,0}}{a^6}}_{\approx \text{ constant}} + \rho_\phi$$

rapidly *increasing* $\sim \dfrac{1}{t^6}$

Homogeneous energy density of the scalar washes out everything else

Similar to ekpyrotic cosmology (contracting universe with $w >> 1$)

Khoury, Ovrut, Steinhardt, Turok (2001);
Gratton, Khoury, Steinhardt, Turok (2003);
Erickson, Wesley, Steinhardt, Turok (2004).

General framework

KH, Justin Khoury arXiv:1106.1428

Start with any CFT with scalar primary operators:

$$\phi_I, \quad I = 1, \ldots, N. \quad \text{conformal weight } \Delta_I$$

These need not be fundamental fields or degrees of freedom, and a conformal invariant stable ground state need not exist.

Dynamics must be such that the operators get a VEV:

$$\bar{\phi}_I(t) = \frac{c_I}{(-t)^{\Delta_I}} \, ,$$

VEV preserves an $so(4,1)$ subgroup of $so(4,2)$:

Symmetry breaking pattern for pseudo-conformal scenario is: $so(4,2) \to so(4,1)$

Distinguishable from inflation?

- Detailed predictions (spectral index, non-gaussianity, etc...) will depend on the realization.
- Pseudo-conformal scenario is more symmetric than inflation
- Symmetries $\rightarrow$ Ward identities $\rightarrow$ constraints/relations on observables

KH, Austin Joyce, Justin Khoury arXiv:1202.6056

- Gravity waves:

Spacetime is not doing very much

$\Downarrow$

Primordial gravity wave amplitude is exponentially suppressed

$\Downarrow$

Predicts scalar/tensor ratio: $r \sim 0$

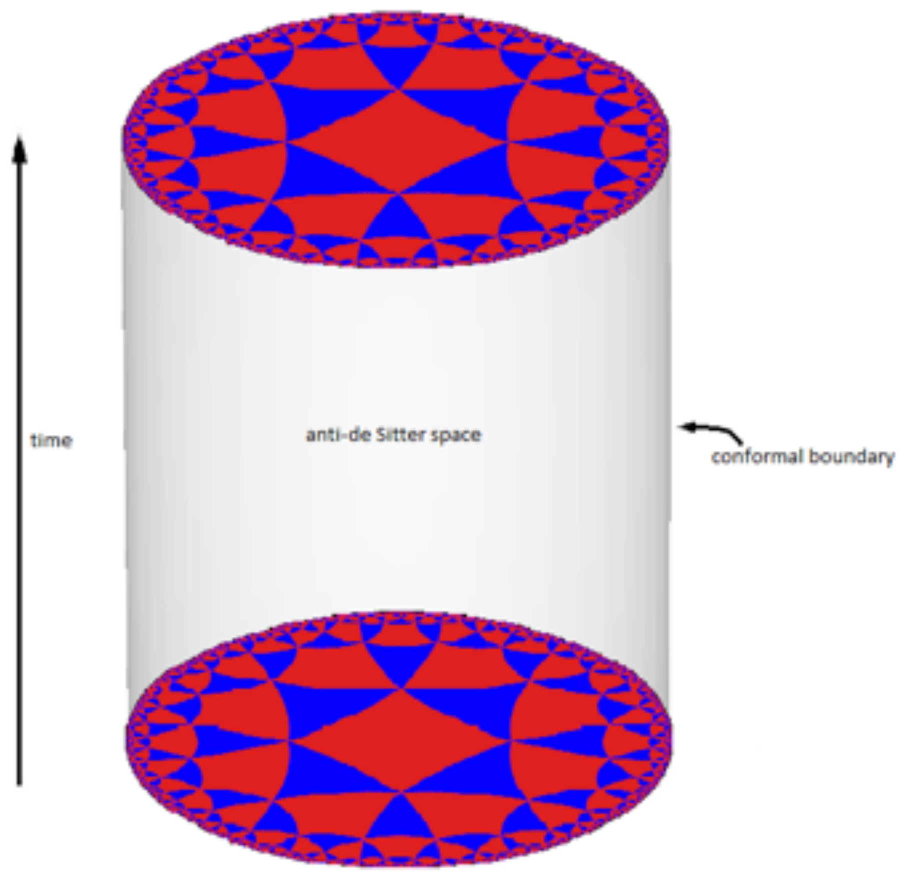
$\Downarrow$

If Bicep successors find anything, theory is ruled out

AdS/CFT realization

KH, Stokes, Trodden (arXiv:1408.1955)

- Realization in a true (strongly coupled) CFT
- Cosmological application of AdS/CFT where we are interested in the boundary.



AdS/CFT generalities

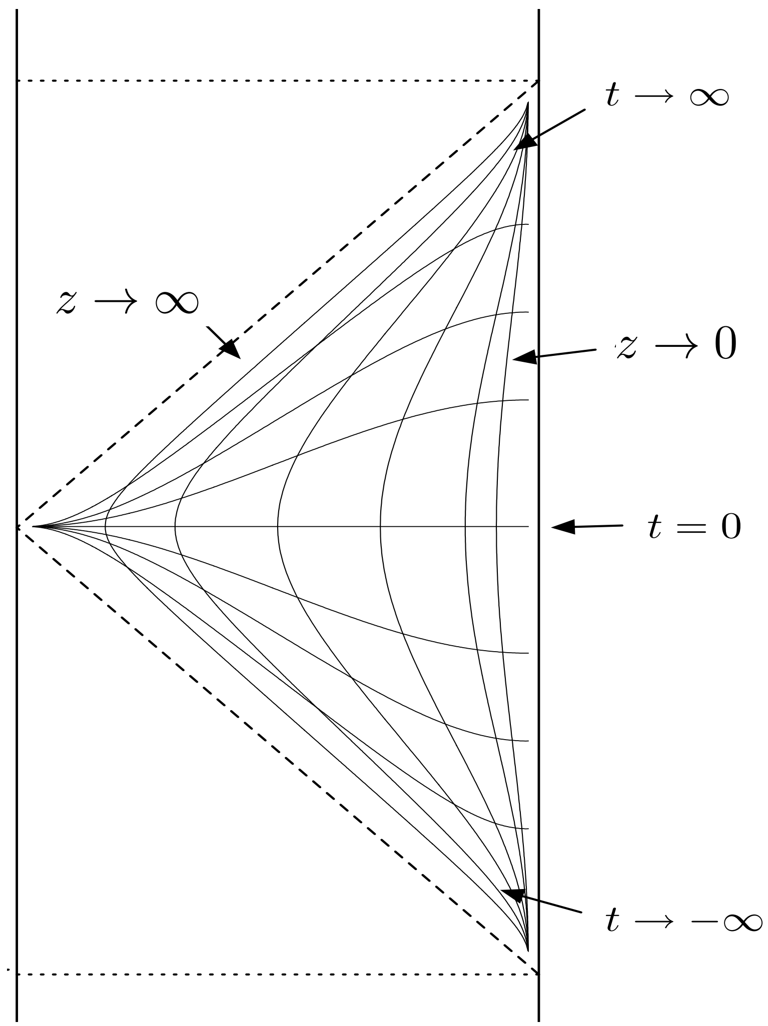
Poincare coordinates:

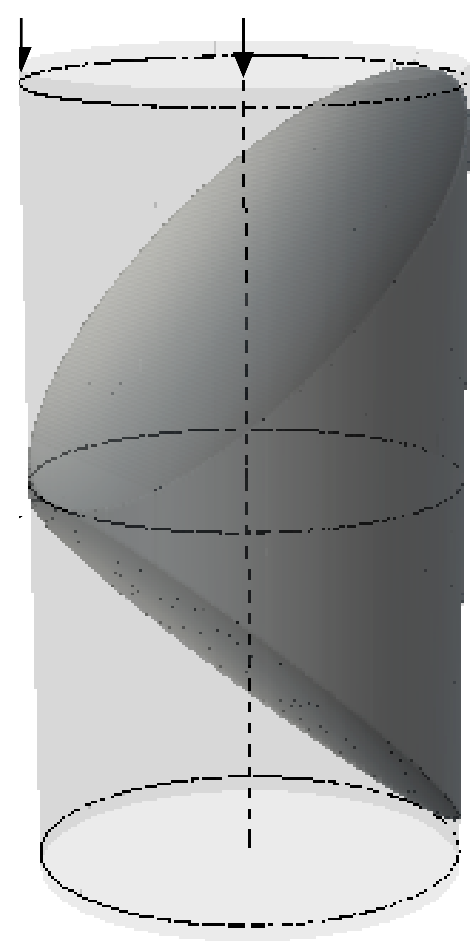
$$ds^2 = \frac{1}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

$$\phi(x,z) = z^{\Delta_-}\left[\phi_0(x) + \mathcal{O}(z^2)\right] + z^{\Delta_+}\left[\psi_0(x) + \mathcal{O}(z^2)\right], \quad \Delta_\pm = 2 \pm \sqrt{4+m^2}$$

source for $\mathcal{O}$ in the dual theory

expectation value in the dual theory: $\langle\mathcal{O}\rangle \sim \psi_0$

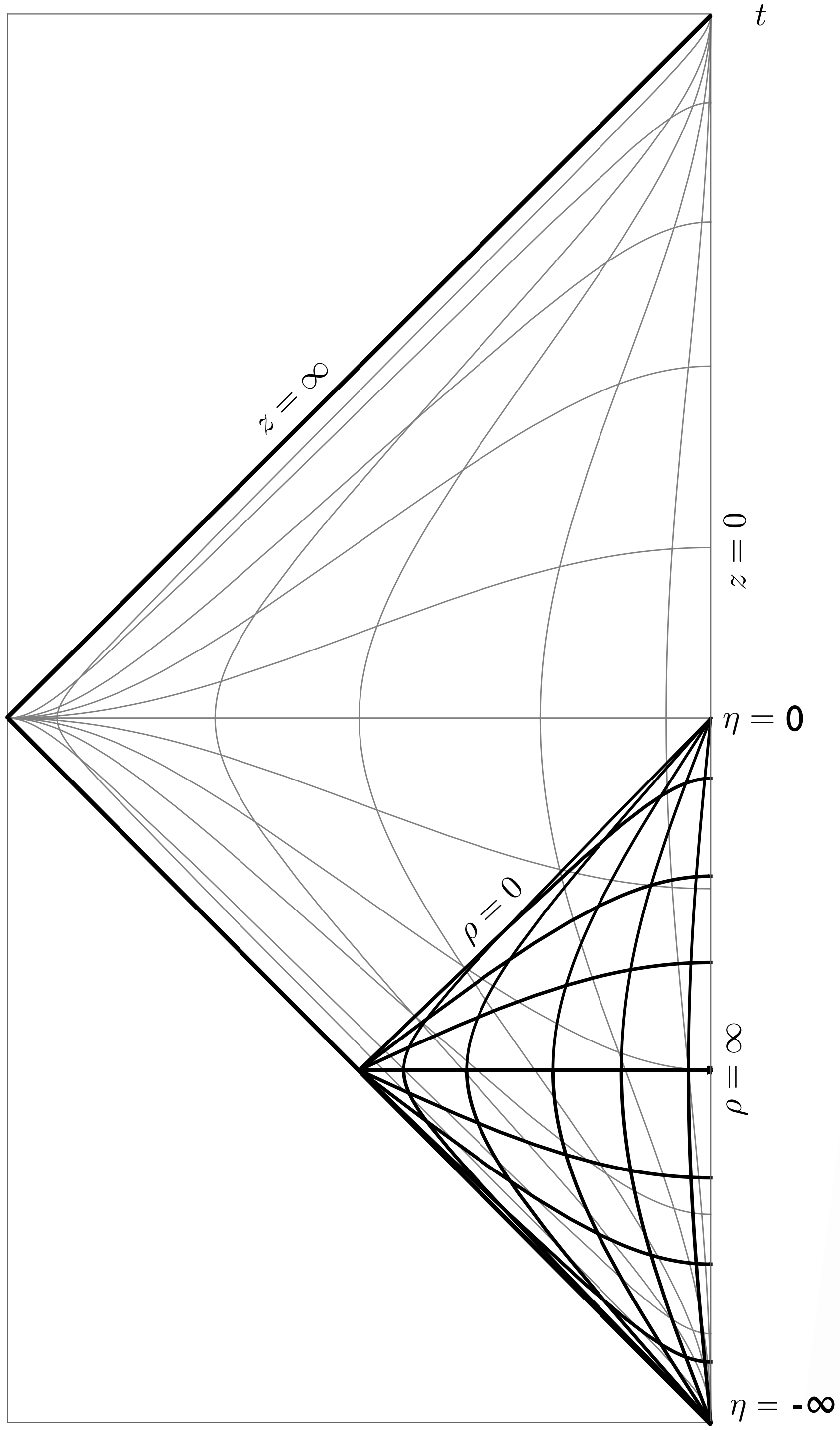


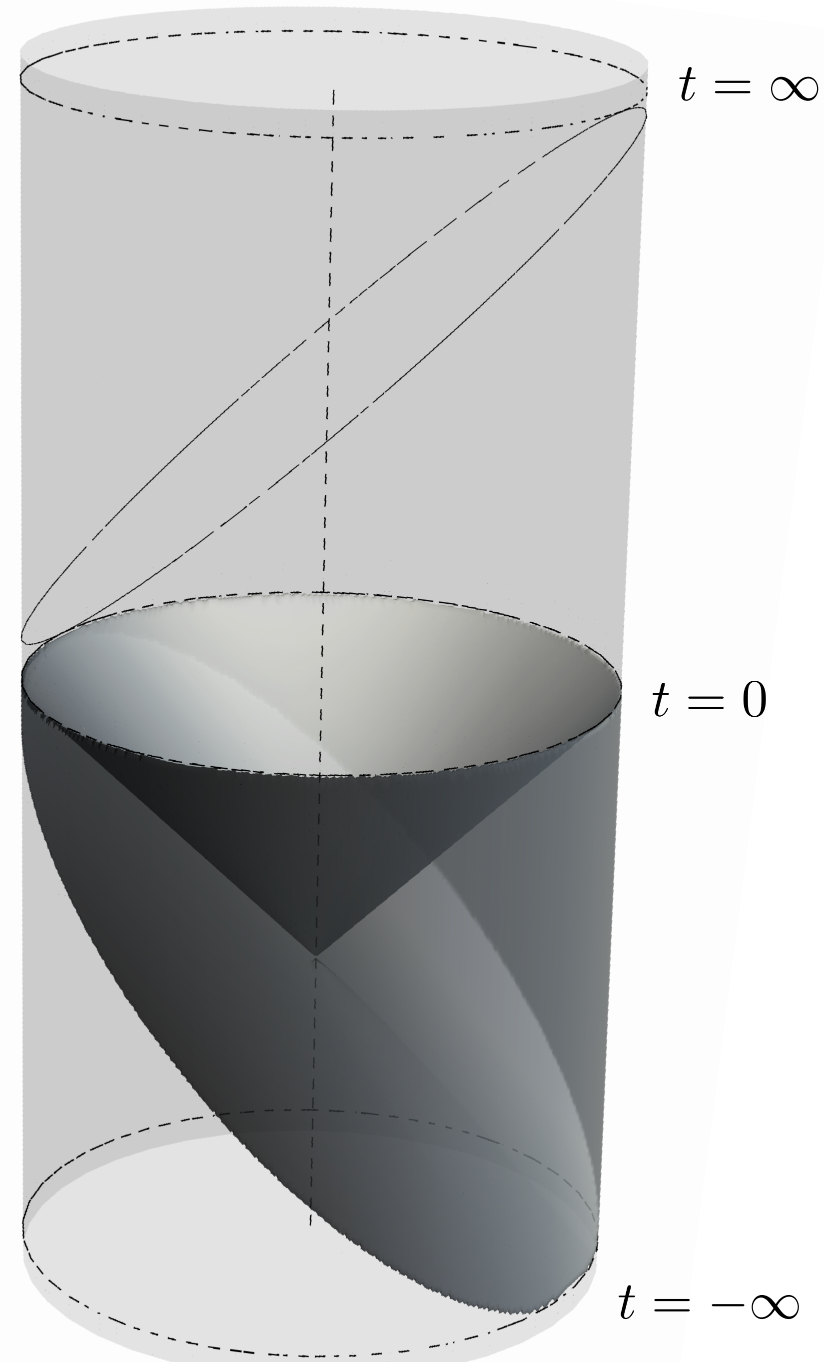


The dual region

KH, Stokes, Trodden (2014)

- Dual AdS state should be a configuration which is constant on a foliation of AdS_5 by dS_4 leaves:





Bulk solutions

KH, Stokes, Trodden (2014)

dS-slice coordinate

bulk scalar field mass

$$\phi(\rho) \;=\; C_+ \frac{e^{-\sqrt{4+m^2}\rho}\left(\sqrt{4+m^2}+\coth\rho\right)\operatorname{csch}^2\rho}{\sqrt{4+m^2}} + C_- \frac{e^{\sqrt{4+m^2}\rho}\left(\sqrt{4+m^2}-\coth\rho\right)\operatorname{csch}^2\rho}{3+m^2}$$

2 integration constants (2nd order equation)

Near-boundary asymptotics In Poincare coords:

$$\phi(\rho) \simeq C_+ z^{\Delta_+}\left[\boxed{\frac{1}{(-t)^{\Delta_+}}} + \mathcal{O}(z^2)\right] + C_- z^{\Delta_-}\left[\boxed{\frac{1}{(-t)^{\Delta_-}}} + \mathcal{O}(z^2)\right]$$

Expectation value

source

Spontaneous symmetry breaking ➔ no source ➔ set $C_- = 0$

$$\text{VEV:} \qquad \langle \mathcal{O} \rangle \propto \frac{C_+}{(-t)^{\Delta_+}}$$

Challenges and future work

- Calculate higher point functions holographically (consistency relations) KH, James Stokes, Mark Trodden (arXiv:1505.05513)

- Requires matching onto a standard radiation dominated cosmology (may require NEC violation at some stage)

Brandenberger, Davis, Perreault 1105.5649

- Can reheating/matching be described holographically?

- How are the scale invariant perturbations of the CFT fields imprinted onto the adiabatic mode?

Brandenberger, Wang 1206.4309