Non-gaussian imprints of primordial magnetic fields from inflation

Rajeev Kumar Jain

CP3 Origins
Cosmology & Particle Physics

University of Southern Denmark

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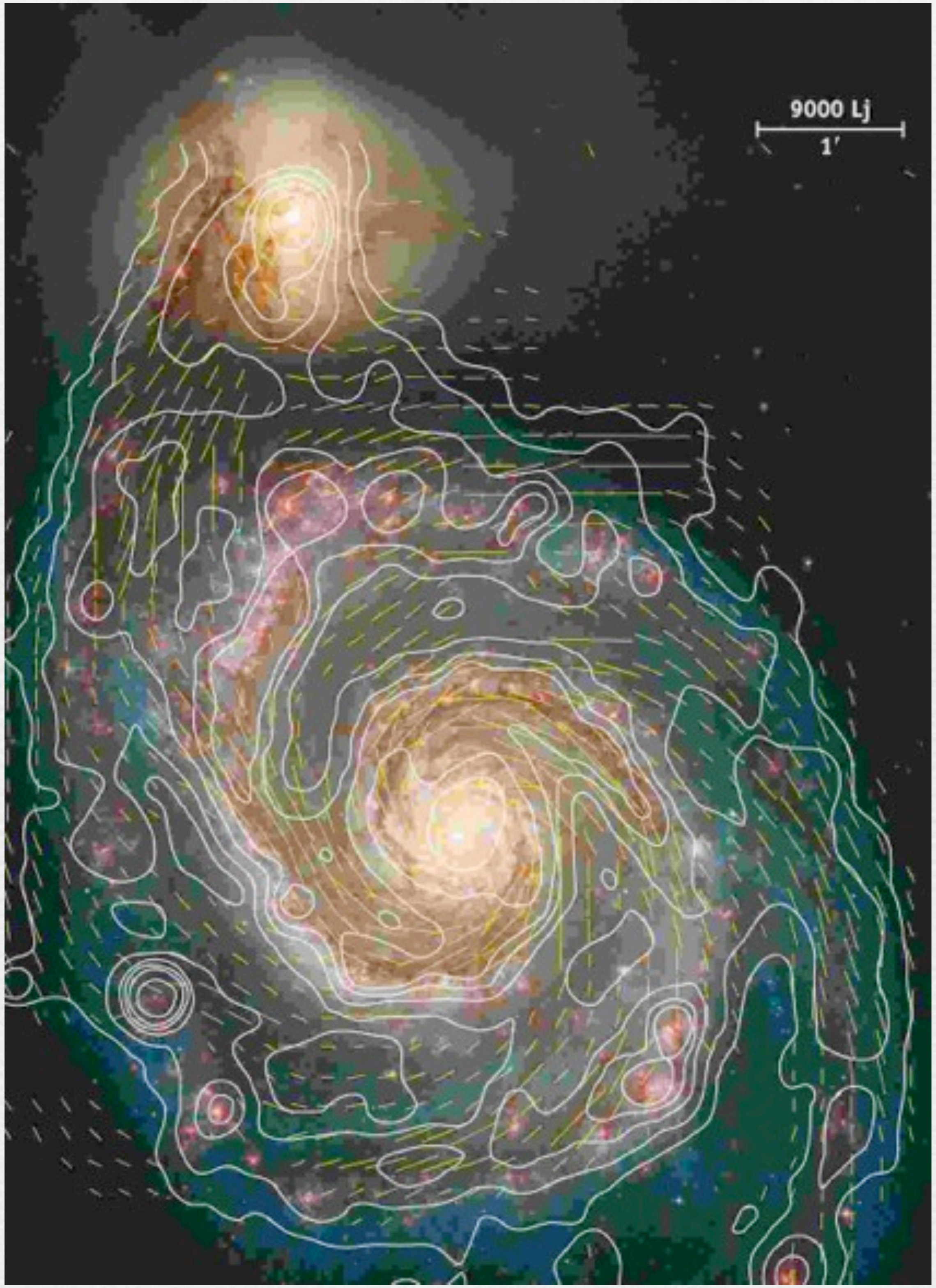
Outline of the talk

- Cosmic magnetic fields: Brief introduction and generation from inflation
- Deflationary magnetogenesis
- Non-gaussian imprints: Cross-correlation with primordial curvature perturbations
  - A *new* magnetic consistency relation
  - The squeezed limit and flattened shape
- Conclusions

Cosmic magnetic fields

- Our observed universe is magnetized on all scales.
- All the bound structures -- stars, galaxies and clusters carry magnetic fields, also present in the intergalactic medium.
- Stars: B ~ 0.1 – few G.
- Galaxies: B ~ 1 – 10 μG with coherence length as large as 10 kpc.
- Clusters: B ~ 0.1 – 1 μG, coherent on scales up to 100 kpc.
- Intergalactic medium: B > 10^{-16} G, coherent on Mpc scales, the lower bound arises due to the absence of extended secondary GeV emission around TeV blazars. (Neronov & Vovk, Science 328, 73, 2010)



Origin -- various mechanisms

- Primordial (early time)
  - Inflation
  - Phase transitions (QCD, EW)
  - Second order perturbation theory
- Astrophysical (late time)
  - Structure formation
  - Dynamo mechanism
  - Biermann battery

10^{-20}-10^{-30} G as seed field

Primordial magnetic fields from inflation

- Standard EM action is conformally invariant - the EM fluctuations do not grow in any conformally flat background like FRW - need to break it to generate magnetic fields. (Turner & Widrow, 1988)

- Various possible couplings:
  - Kinetic coupling: $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$
  - Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$

Primordial magnetic fields from inflation...

- Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - strong constraints from backreaction, final field strength not enough (Durrer, Hollenstein, **RKJ**, 2011; Byrnes, Hollenstein, **RKJ**, Urban, 2012)
- Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$
  - negative mass-squared needed for generating relevant magnetic fields, breaks gauge invariance

Primordial magnetic fields from inflation...

- Gauge-invariant coupling: $\lambda(\eta)F_{\mu\nu}F^{\mu\nu}$
  - For $\lambda(\eta) \propto a^{2\alpha} \propto \eta^{2\gamma}$, the magnetic field spectrum is

$$\frac{d\rho_B}{d\,\mathrm{lnk}}(\eta, k) \propto \left(\frac{k}{aH}\right)^{4+2\delta}$$

where $\delta = \gamma$ if $\gamma \leq 1/2$ and $\delta = 1 - \gamma$ if $\gamma \geq 1/2$.

The tilt of the spectrum is $n_B = 4 + 2\delta$ and $n_B = 0$ for $\alpha = 2$ or $\gamma = -2$. However, $n_B = 0$ also for $\gamma = 3$ but then the electric field vary strongly and so not interesting.

Various constraints

- Background
    - Strong coupling
    - Backreaction
- Perturbations
    - Power spectrum
    - Induced bispectrum
- Energy scale of inflation (from B-modes)

Constraint from strong coupling

- Adding the EM coupling to the SM fermions

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}\lambda(\phi)F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi\right]$$

- The physical EM coupling now is

$$e_{phys} = e/\sqrt{\lambda(\phi)}$$

- Since $\sqrt{\lambda} \propto a^{\alpha}$ then for $\alpha > 0$, the physical coupling decreases by a large factor during inflation, and must have been very large at the beginning of inflation.

- QFT out of control initially. (Demozzi et.al, 2009)

- Solutions ?? Speculations...(Caldwell & Motta, 2012, Ferreira, **RKJ** & Sloth, 2013)

Constraint from backreaction

- The produced magnetic fields should not backreact on the background dynamics of the universe i.e.

$$\rho_{\rm em} < \rho_{\rm inf}$$

- Backreaction + strong coupling constraints at most lead to B ~10^{-32} G today. (Demozzi & Mukhanov, 2009)

- Very weak strength -- not even enough as seed field for dynamo to work!

Is it possible to overcome this result?

Deflationary magnetogenesis

- Flux conservation leads to adiabatic decay of magnetic fields after inflation.
- Problem with modifying the inflationary part to generate even larger field strength during inflation.
- Rather, modify the post-inflationary evolution of magnetic fields until today.
- Consider prolonged reheating rather than instantaneous reheating.
- Deflation after inflation.

Ferreira, **RKJ** & Sloth, 2013

Deflationary magnetogenesis

- For radiation dominated universe immediately after inflation: $\rho_I/\rho_r = (a_0/a_f)^4$
- If the universe is instead dominated by a fluid with equation of state ω until the end of reheating:

$$\rho_I/\rho_r = (a_{reh}/a_f)^{3(1+\omega)}(a_0/a_{reh})^4$$

or $$\frac{a_0}{a_f} = \frac{1}{R}\left(\frac{\rho_I}{\rho_r}\right)^{\frac{1}{4}}$$

- Define the reheating parameter R as

$$\log(R) = \frac{-1+3\omega}{4}\log\left(\frac{a_{reh}}{a_f}\right)$$

Deflationary magnetogenesis

- The magnetic field spectrum today is

$$\left.\frac{d\rho_B}{d\log k}\right|_{a_0} = \left.\frac{d\rho_B}{d\log k}\right|_{a_f} \left(\frac{a_f}{a_0}\right)^4$$

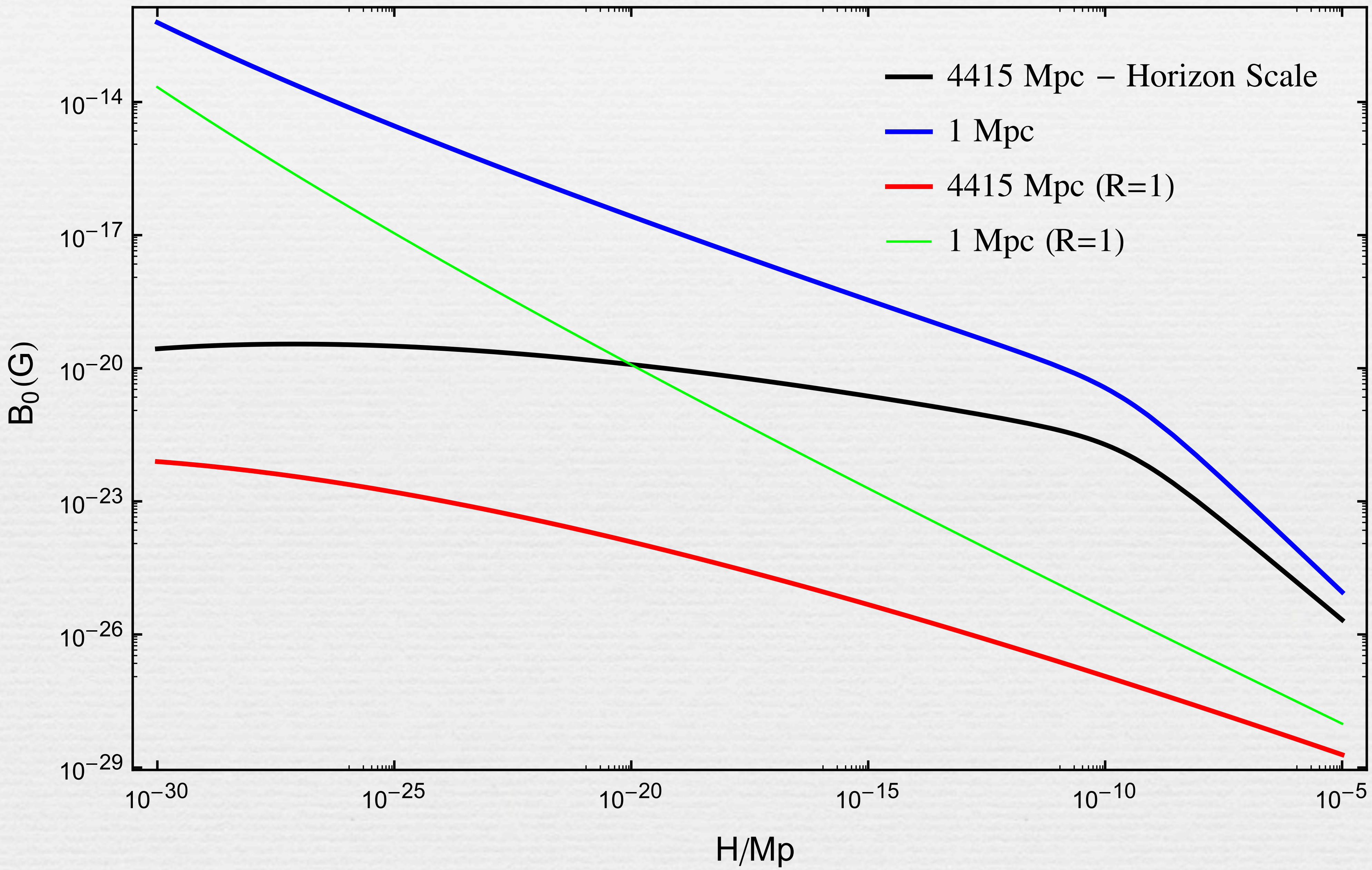
- In terms of R, we get

$$B_k(\alpha, H) = \frac{\Gamma(-\alpha - 1/2)}{2^{3/2+\alpha}\pi^{3/2}}\, H^2 \left(R\,\Omega_r^{1/4}\right)^{-(1+\alpha)} \left(\frac{H_0}{H}\right)^{\frac{1}{2}(5+\alpha)} \left(\frac{k}{a_0 H_0}\right)^{3+\alpha}$$

- To get optimal values of the magnetic fields today, maximize in α and R.

Ferreira, **RKJ** & Sloth, 2013

Final magnetic field strength



Ferreira, **RKJ** & Sloth, 2013

Magnetic non-Gaussianity

- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial curvature perturbations.
- Such cross-correlations are non-Gaussian in nature and it is very interesting to compute them in different models of inflationary magnetogenesis.
- We consider the following correlation here:

$$\langle \zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(k_3)\rangle$$

(Ordinary) non-Gaussianity

- The primordial perturbations are encoded in the two-point function or the power spectrum

$$\langle\zeta_k\zeta_{k'}\rangle = (2\pi)^3\delta(\vec{k}+\vec{k'})P_\zeta(k)$$

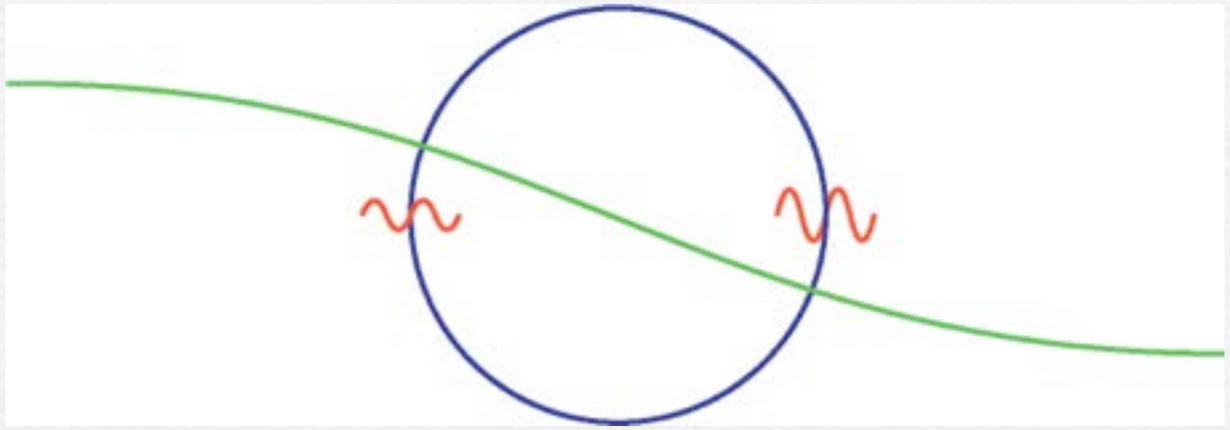
- A non-vanishing three-point function $\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$ is a signal of NG.

- Introduce f_{NL} as a measure of NG.

$$f_{NL} \sim \langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle / P_\zeta(k_1)P_\zeta(k_2) \ + \ perm.$$

(semi)Classical estimate

(for squeezed limit)



- Consider $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ in the squeezed limit i.e.
  - The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})} d\mathbf{x}^2$$

  - Taylor expand in the rescaled background

$$\langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} = \langle \zeta_{k_2} \zeta_{k_3} \rangle + \zeta_1 \frac{\partial}{\partial \zeta_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle + \ldots$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \approx \left\langle \zeta_{k_1} \langle \zeta_{k_2} \zeta_{k_3} \rangle_{\zeta_1} \right\rangle \sim \langle \zeta_{k_1} \zeta_{k_1} \rangle\, k \frac{d}{dk} \langle \zeta_{k_2} \zeta_{k_3} \rangle$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \sim -(n_s - 1) \langle \zeta_{k_1} \zeta_{k_1} \rangle \langle \zeta_{k_2} \zeta_{k_3} \rangle$$ (Maldacena, 2002)

Non-gaussian cross-correlation

- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- Introduce the magnetic non-linearity parameter

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

- *Local* resemblance between f_{NL} and b_{NL}

$$\zeta = \zeta^{(G)} + \frac{3}{5}f_{NL}^{local}\left(\zeta^{(G)}\right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2}b_{NL}^{local}\zeta^{(G)}\mathbf{B}^{(G)}$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- Use the same semi-classical argument to derive the consistency relation.

- Consider $\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$ in the squeezed limit.

- The effect of the long wavelength mode is to shift the background of the short wavelength mode.

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle = \Big\langle\zeta(\tau_I,\mathbf{k}_1)\left\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\right\rangle_B\Big\rangle$$

- Since the gauge field only feels the background through the coupling, all the effects of the long wavelength mode is indeed captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots\ .$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- Compute the two point function of the vector field in the modified background

$$\langle A_i(\tau,\mathbf{x}_2)A_j(\tau,\mathbf{x}_3)\rangle_B = \left\langle \frac{1}{\lambda_B} v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\right\rangle$$
$$\simeq \frac{1}{\lambda_0}\langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle - \frac{1}{\lambda_0^2}\frac{d\lambda}{d\ln a}\zeta_B \langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle$$

where $v_i = \sqrt{\lambda}\, A_i$ is the linear canonical vector field.

- One finally finds

$$\lim_{k_1\to 0}\langle \zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$
$$\simeq -\frac{1}{H}\frac{\dot\lambda}{\lambda}\langle\zeta(\tau_I,\mathbf{k}_1)\zeta(\tau_I,-\mathbf{k}_1)\rangle_0 \langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle_0$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In terms of magnetic fields, the correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle$$
$$= -\frac{1}{H} \frac{\dot{\lambda}}{\lambda} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

- With the coupling $\lambda(\phi(\tau)) = \lambda_I (\tau/\tau_I)^{-2n}$, we obtain

$$\boxed{b_{NL} = n_B - 4}$$

- For scale-invariant magnetic field spectrum, $n_B = 0$ and therefore, $b_{NL} = -4$

- Not so small......compared to $b_{NL} \sim \mathcal{O}(\epsilon, \eta)$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In the squeezed limit $k_1 \ll k_2, k_3 = k$, we obtain a new *magnetic consistency relation*

$$\boxed{\langle \zeta(k_1)\mathbf{B}(k_2)\cdot\mathbf{B}(\mathbf{k_3})\rangle = (n_B - 4)(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta(k_1)P_B(k)}$$

with $b_{NL}^{\rm local} = (n_B - 4)$

- Compare with Maldacena's consistency relation

$$\boxed{\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = -(n_s - 1)(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta(k_1)P_\zeta(k)}$$

with $f_{NL}^{\rm local} = -(n_s - 1)$

The full in-in calculation

- One has to cross-check the consistency relation by doing the full in-in calculation

$$\langle\Omega|\,\mathcal{O}(\tau_I)\,|\Omega\rangle = \langle 0|\,\bar{\mathrm{T}}\left(e^{i\int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}}\right)\mathcal{O}(\tau_I)\mathrm{T}\left(e^{-i\int_{-\infty}^{\tau_I} d\tau H_{\mathrm{int}}}\right)|0\rangle$$

- The result is

$$\begin{aligned}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle = {} & \frac{1}{H}\frac{\dot\lambda_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)| \\ & \times\left[\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)\left(\delta_{lj}-\frac{k_{3,l}k_{3,j}}{k_3^2}\right)\left(k_2k_3\,\tilde{\mathcal{I}}^{(1)}_n+\mathbf{k}_2\cdot\mathbf{k}_3\,\tilde{\mathcal{I}}^{(2)}_n\right)\right. \\ & \left.-\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)k_{3,l}\left(\delta_{jm}-\frac{k_{3,j}k_{3,m}}{k_3^2}\right)k_{2,m}\,\tilde{\mathcal{I}}^{(2)}_n\right]\end{aligned}$$

A generic result

RKJ & Sloth, 2013

Cross-correlation with magnetic fields

- Using this relation

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{a_0^4} \left(\delta_{ij} \mathbf{k}_2 \cdot \mathbf{k}_3 - \mathbf{k}_{2,i} \mathbf{k}_{3,j} \right) \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3) \rangle$$

- The cross-correlation with magnetic fields is

$$\begin{aligned} \langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = & -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) |\zeta_{k_1}^{(0)}(\tau_I)|^2 |A_{k_2}^{(0)}(\tau_I)| |A_{k_3}^{(0)}(\tau_I)| \\ & \times \left[\left(\mathbf{k}_2 \cdot \mathbf{k}_3 + \frac{(\mathbf{k}_2 \cdot \mathbf{k}_3)^3}{k_2^2 k_3^2} \right) k_2 k_3 \tilde{\mathcal{I}}_n^{(1)} + 2 (\mathbf{k}_2 \cdot \mathbf{k}_3)^2 \tilde{\mathcal{I}}_n^{(2)} \right] . \end{aligned}$$

- The two integrals can be solved exactly for different values of n.

RKJ & Sloth, 2013

The integrals...

- The two integrals are

$$\tilde{\mathcal{I}}_n^{(1)} = \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2}$$
$$\times\ \mathrm{Im}\Big[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I)$$
$$\times \int^{\tau_I} d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n-1/2}(-k_2\tau)H^{(2)}_{n-1/2}(-k_3\tau)\Big]$$

$$\tilde{\mathcal{I}}_n^{(2)} = \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2}$$
$$\times\ \mathrm{Im}\Big[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I)$$
$$\times \int^{\tau_I} d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n+1/2}(-k_2\tau)H^{(2)}_{n+1/2}(-k_3\tau)\Big]$$

The flattened shape

- In this limit, $k_1 = 2k_2 = 2k_3$, the second integral dominates

$$\tilde{\mathcal{I}}_2^{(2)} \simeq -\frac{3k_1^3}{(k_2k_3)^{5/2}}\ln(-k_t\tau_I)$$

- The cross-correlation thus becomes

$$\langle\zeta(\tau_I,\mathbf{k}_1)\mathbf{B}(\tau_I,\mathbf{k}_2)\cdot\mathbf{B}(\tau_I,\mathbf{k}_3)\rangle \simeq 96\ln(-k_t\tau_I)P_\zeta(k_1)P_B(k_2)$$

- For the largest observable scale today, $\ln(-k_t\tau_I) \sim -60$,

$$\left|b_{NL}^{flat}\right| \sim 5760$$

RKJ & Sloth, 2013

The squeezed limit

- In this limit, the integrals are

$$\tilde{\mathcal{I}}_n^{(1)} = \pi \int^{\tau_I} d\tau \tau J_{n-1/2}(-k\tau) Y_{n-1/2}(-k\tau)$$

$$\tilde{\mathcal{I}}_n^{(2)} = \tilde{\mathcal{I}}_{n+1}^{(1)} \ .$$

- The cross-correlation now becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

with $b_{NL} = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$ in agreement with the magnetic consistency relation.

RKJ & Sloth, 2013

Conclusions

- Inflationary + deflationary magnetogenesis can produce strong enough fields on large scales without the backreaction and strong coupling problem.

- Low scale inflationary magnetogenesis is still a viable possibility.

- The consistency relation is an important theoretical tool to cross-check the full in-in calculations, it's violation will rule out an interesting class of inflationary magnetogenesis models.

- The magnetic non-Gaussianity parameter is large in the flattened limit and can have non-trivial phenomenological consequences.

Thank you for your attention