Primordial magnetogenesis : role of nonlinear electromagnetism

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   - galaxies & clusters can host $\sim \mu$G fields up to ten kpc coherence length
   - IGM in voids can host $\sim 10^{-16} G$ field of Mpc order coherence scale

2. Magnetic fields in the astronomical structures need Dynamos to explain observed fields and maintain against decay

   But dynamo can act only on "seed" fields !!

3. Magnetc fields in the void regions would be difficult to explain purely by astrophysical processes in the late universe. Cosmic magnetic fields $\rightarrow$ may be relic from the early universe

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Interesting to ask -

What is the origin of cosmic magnetic field ?

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  BBN constraint : $B \lesssim 10^{-7}$ G at galaxy formation

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What are the mechanisms for magnetogenesis ?

Inflationary magnetogenesis [1]

- During inflation $\rightarrow$ tiny quantum fluctuations get stretched to long wavelength classical fluctuations $\Rightarrow$ matter structure formation

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- Whether same mechanism applies to magnetogenesis ?
  - Maxwell ED in FRW spacetime is not sufficient

$$S = -\int \sqrt{-g}\ d^4x\ \frac{F_{\mu\nu}F^{\mu\nu}}{16\pi} \tag{1}$$

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    conformal flatness of FRW spacetime $\longrightarrow$ leads to conformal invariance of Maxwell ED action
  - $B \propto 1/a^2$ (no amplification) due to conformal invariance of Maxwell ED
  - To achieve sufficient strength one needs to break conformal invariance to fulfill the requirement

$$B \sim 1/a^{\varepsilon} \text{ with typically } \varepsilon \ll 1$$

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Breaking of conformal invariance .... multitude of ways

$$S = \int \sqrt{-g}\ d^4x\ b(t) \left[\frac{-f^2}{16\pi} F_{\mu\nu} F^{\mu\nu} - \alpha R A^2 + \beta \theta F_{\mu\nu} \tilde{F}^{\mu\nu} - D_\mu \psi (D^\mu \psi)^* \right]$$

where $f = f(\phi, R)$.

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- Coupling of e.m. field with scalar field (ϕ) such as inflaton, dilaton; extra dimensional factor $b(t)$; curvature scalar (R); axion (θ); complex scalar (ψ) and so on.
- Breaking of conformal invariance can lead to amplification in e.m. wave from vacuum fluctuations
- At the end of inflation universe reheats leading to production of charged particles $\Rightarrow$ increase in plasma conductivity
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$\Rightarrow$ general mechanism for cosmic magnetogenesis during inflation

Two major issues

Back reaction & Strong coupling

Test whether NLED can give interesting magnetogenesis ?

- NLED provides yet another way of breaking conformal invariance
- On large scales present observation $\Rightarrow$ ED linear
- At smaller scale deviation from linearity may be there

$L_{em} = L(S, P)$ non linear function of S & P

where two Lorentz invariants are $S = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ and $P = \frac{1}{4} F_{\mu\nu} {}^{*}F^{\mu\nu}$

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- During inflation quantum fluctuations are excited within the horizon. Upon leaving the causal domain they become classical perturbations
- Assume ED is nonlinear during inflation
- Becomes linear at reheating and subsequently at radiation dominated stage

Consider minimal coupling of NLED with gravity

$$S_{total} = S_{grav} + \frac{1}{4\pi}\int d^4x\sqrt{-g}L(S,P)$$

- Treat electromagnetic field as test field $\longrightarrow$ in background FRW geometry

The equations of motion for electromagnetic field :

$$\nabla_\mu \mathscr{F}^{\mu\nu} = 0 \quad \text{and} \quad \nabla_\mu {}^*F^{\mu\nu} = 0$$

where $\mathscr{F}_{\mu\nu} = -\left(L_S F_{\mu\nu} + L_P {}^*F_{\mu\nu}\right)$ and $L_{S,P} = \partial L/\partial S, P$

- The electromagnetic wave equation $\longrightarrow$ coupled & nonlinear
- To a fundamental observer having 4-velocity $u_\mu = (1/a, 0, 0, 0)$

$$F_{\mu\nu} = 2E_\mu u_\nu + 2E_\nu u_\mu - \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$

NLED wave equation [1]

Wave equation for magnetic field becomes

$$
\begin{aligned}
& \frac{1}{a^2}\frac{\partial^2}{\partial\eta^2}(a^2\vec{B}) + \frac{1}{a^2}\frac{\partial_\eta L_S}{L_S}\partial_\eta(a^2\vec{B}) + \frac{1}{a^2}\frac{\partial_\eta L_P}{L_S}\partial_\eta(a^2\vec{E}) - \nabla^2\vec{B} \\
+ \quad & \vec{E}\times\vec{\nabla}\left(\frac{\partial_\eta L_S}{L_S}\right) + \frac{\partial_\eta L_P}{L_S}\left(\frac{\partial_\eta L_S}{L_S}\vec{E} - \frac{\partial_\eta L_P}{L_S}\vec{B}\right) - \vec{B}\times\vec{\nabla}\left(\frac{\partial_\eta L_P}{L_S}\right) \\
- \quad & \frac{\partial_\eta L_P}{L_S}\left[\frac{(\vec{\nabla}L_S)\times\vec{B}}{L_S} + \frac{(\vec{\nabla}L_P)\times\vec{E}}{L_S}\right] + \vec{\nabla}\times\left[\frac{(\vec{\nabla}L_S)\times\vec{B}}{L_S}\right] \\
+ \quad & \vec{\nabla}\times\left[\frac{(\vec{\nabla}L_P)\times\vec{E}}{L_S}\right] = 0
\end{aligned}
$$

Define $\mathscr{B} = a^2 B$ and $\mathscr{E} = a^2 E$

[1] Kunze, 2008

Consider the model [4] for NLED

$$L_{em} = -\left(S^2/\Lambda^8\right)^{\frac{\delta-1}{2}} S$$

Λ is constant, δ is a parameter

- As $L = L(S)$ only, wave equation for magnetic field reduces to:

$$\frac{\partial^2 \mathscr{B}}{\partial \eta^2} + \frac{\partial_\eta L_S}{L_S} \partial_\eta \mathscr{B} - \nabla^2 \mathscr{B} + \mathscr{E} \times \nabla \left(\frac{\partial_\eta L_S}{L_S}\right) + \nabla \times \left[\frac{(\nabla L_S) \times \mathscr{B}}{L_S}\right] = 0$$

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- Considering
  - Long wavelength limit leading to simplifications
  - Mode expansion

$$\vec{\mathscr{E}}(\vec{x}, \eta) = \int d^3 k e^{i\vec{k}\cdot\vec{x}} \vec{\mathscr{E}}_k(\eta) \qquad \vec{\mathscr{B}}(\vec{x}, \eta) = \int d^3 k e^{i\vec{k}\cdot\vec{x}} \vec{\mathscr{B}}_k(\eta).$$

- Equation for magnetic mode functions reduces to

$$\vec{\mathscr{B}}''_k + \frac{{L_S}'}{L_S} \vec{\mathscr{B}}'_k = 0$$

[4] Arodz et.al. 2001

Inflation as a general power law : $a = a_f \left[\frac{\eta}{\eta_f}\right]^{\frac{p}{1-p}}$

The magnetic field can be achieved from :

$$S = \Lambda^4 \left[\frac{4\beta(\delta-1)+1}{(2\delta-1)\alpha {M_p}^2 \eta_f^2}\right]^{\frac{1}{1-2\delta}} \left(\frac{\eta}{\eta_f}\right)^{\frac{2-4\beta}{2\delta-1}}$$

where $\beta = \frac{p}{1-p}$ and α is dimensionless quantity

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The ratio of magnetic energy density between horizon exit and end of inflation : [5]

$$\frac{\rho_B(a_e)}{\rho_B(a_f)} = \frac{B_k^2(a_e)}{B_k^2(a_f)} = \exp\left[N \frac{2-4\beta}{\beta(2\delta-1)}\right]$$

The ratio of magnetic to background radiation energy density at end of inflation

$$r = \frac{\rho_B}{\rho_\gamma} = \exp\left[-N \frac{2-4\beta}{\beta(2\delta-1)}\right] \left[\frac{M^3}{M_p^2}\right]^2 \frac{1}{T_{RH}^2}$$

[5] Turner, Widrow, 1988

- In order to seed the galactic dynamo one needs : $r \gtrsim 10^{-37}$

$$\Rightarrow \boxed{\exp\left[N \frac{2-4\beta}{\beta(2\delta-1)}\right] \lesssim \frac{1}{10^{-37}\, T_{RH}^2}\left[\frac{M^3}{M_p^2}\right]^2}$$

- The quantities above depend on inflation model parameters & T_{RH}
- For a choice of $T_{RH} \simeq 10^9$ GeV and $M \simeq 10^{14}$ GeV the following constraint [6] on δ :

$$\delta \geqslant \frac{\frac{1}{p} - 2.21}{1.58}$$

- A seed field generation for galactic dynamo may be possible!

[6] RK and Kundu, 2015

Born Infeld Lagrangian

$$L_{BI} = \frac{b^2}{4\pi}\left(1 - \sqrt{1 + \frac{2S}{b^2} - \frac{P^2}{b^4}}\right)$$

In **Minkowski spacetime** the electromagnetic field equations are

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \qquad \vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

where

$$\vec{D} = \frac{\vec{E} + \frac{1}{b^2}(\vec{E}\cdot\vec{B})\vec{B}}{\sqrt{1 + \frac{2S}{b^2} - \frac{P^2}{b^4}}} \quad \text{and} \quad \vec{H} = \frac{\vec{B} - \frac{1}{b^2}(\vec{E}\cdot\vec{B})\vec{E}}{\sqrt{1 + \frac{2S}{b^2} - \frac{P^2}{b^4}}}$$

Born-Infeld physical plane waves (for $P = 0$) in Minkowski spacetime :

$$B^i = \frac{f(\phi)}{\sqrt{I}} e^i$$

where $f(\phi)$ is arbitrary function.

In FRW

Solutions may be plane waves with variable frequency

- The energy-momentum conservation laws, $T^{\mu\nu}{}_{;\nu} = 0$ in curved space :

$$\left[\sqrt{-g}\, T^{\mu\nu}\right]_{,\nu} + \Gamma^{\mu}_{\nu\rho} \sqrt{-g}\, T^{\rho\nu} = 0$$

- For FRW geometry the energy balance becomes

$$a^{-2}\left[a^4\, T^0_0\right]_{,0} + a^2\, T^{\alpha}_{0}{}_{,\alpha} - a_{,0}\, a\ T = 0$$

- T is null in Maxwell ED $\Rightarrow \{T^{\nu}_{0}\}^{FRW} = a(\eta)^{-4}\{T^{\nu}_{0}\}^{M}$ will solve energy conservation in the spatially flat FRW universe.

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- But T is non null in BI electromagnetism, for energy balance a different scaling is required
- Ansatz for scaling

$$\{T^{\nu}_{0}\}^{FRW} = (1 - \varepsilon\, a^{-4}\, b^{-2})\, a^{-4}\{T^{\nu}_{0}\}^{M}$$

where ε is function of trace.

$$\rho_B \simeq \frac{1}{a^4}\left[\frac{b^2}{4\pi} + \frac{B^2}{8\pi} - \frac{B^4}{8\pi b^2}\right]$$

$\Rightarrow$ Born Infeld type of electrodynamics in the early universe may not responsible for giving rise to seeds for primordial magnetic fields.

- Theoretical predictions are highly model parameter dependent
- Look for implications of primordial NLED other than magnetogenesis
- To explore what signals it could leave behind in the early universe

THANK YOU for YOUR ATTENTION