On the phenomenology of extended Brans-Dicke gravity

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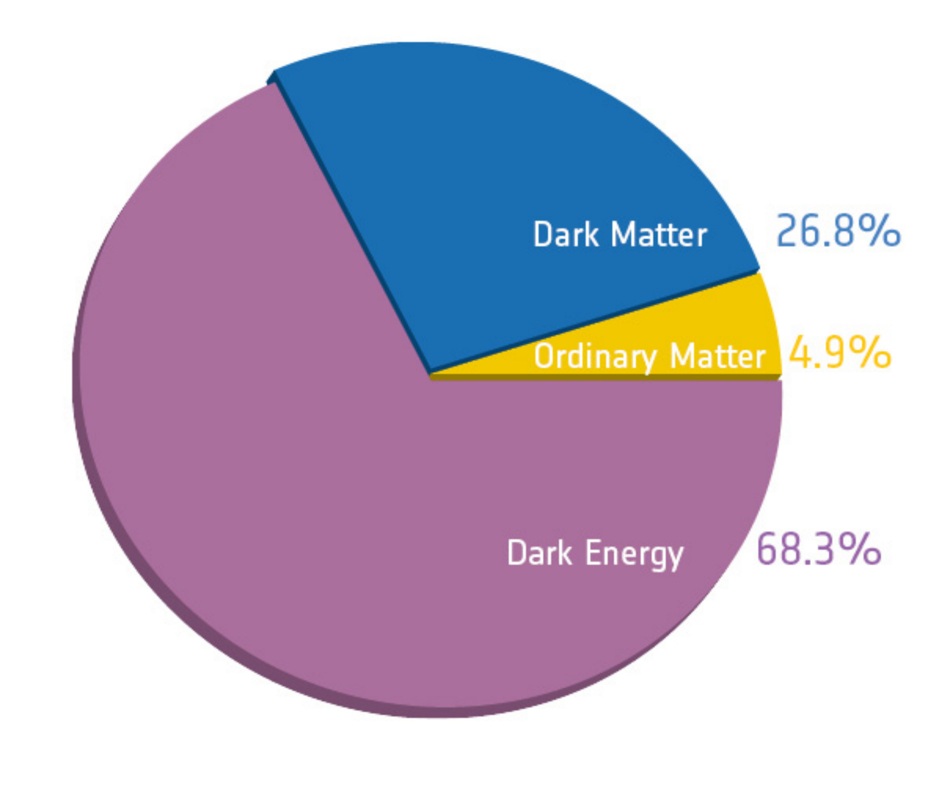
OUTLINE

- Why should we consider modifying gravity?
- Extended Brans-Dicke gravity with a constant potential
- Extended Designer Brans-Dicke gravity
- The phenomenological parameters
- Conclusions and outlook

Why Modify Gravity?



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- Our Universe is undergoing an accelerated expansion
- Approx. 70 % is an exotic negative pressure component, Dark Energy
- Our best guess is ΛCDM. However, it is not definitive
- Does G.R. need to be corrected/modified?

Why Modify Gravity?

- MG theories evade Solar-system tests through screening mechanisms
- At the background level, modified gravity can be indistinguishable from ΛCDM with fine-tuning
- Next generation of surveys may place tight constraints on the largest scales
- Dynamics of linear perturbations of MG theories can be significantly different
- Essential to understand how the observables are interrelated to the fundamental parameters of the theories

Why Modify Gravity?

- Evolution of the perturbed potentials can be parameterized in the sub-horizon ($k^2/a^2H^2>>1$) regime [De Felice, 2011]

$$\frac{k^2}{a^2}\Psi \approx -4\pi G_{eff}(k,a)\rho_m\delta \qquad \eta(k,a) \equiv -\frac{\Phi}{\Psi}$$

- G_{eff} and η are scale and time dependent functions that are also related to the fundamental parameters of the theory one considers

Extended Brans-Dicke gravity

- We consider an action of the form

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R - \frac{w_{BD}}{\phi}(\partial\phi)^2 - 2V(\phi)\right] + S_m$$

- Scalar field equation of motion

$$\nabla^\alpha\nabla_\alpha\phi = \frac{\kappa^2 T}{2w_{BD}+3} + \frac{4V(\phi) - 2\phi V_\phi}{2w_{BD}+3}$$

$$2w_{BD}+3 \equiv d$$

- Effective eq. of state: $w_{eff} = \frac{\dot{\phi}^2 w(\phi) + 4H\dot{\phi} + 2\ddot{\phi} - 2V(\phi)}{\dot{\phi}^2 w(\phi) - 6H\dot{\phi} + 2V(\phi)}$

Extended Brans-Dicke gravity

- The GR limit is recovered when w_{BD} -> ∞
- Solar-system tests place lower bound on the Brans-Dicke parameter of 40.000 [B. Bertotti, 2003]
- Cosmological constraints place the lower bound at around 1.000 [A. Avilez, 2014]
- Accelerating solutions without $V(\phi)$ exist for negative (order unity) Brans-Dicke parameter, or considering w_{bd} as a function of the scalar field

Extended Brans-Dicke gravity with a constant potential

- Constant potential $V(\phi) = 3H_0^2(1-\Omega_m)$

$$\phi_0 = \frac{2w_{BD}+4}{2w_{BD}+3}$$

- Matter dominated regime: $\phi = \phi_0 a^{1/(1+w_{BD})}$

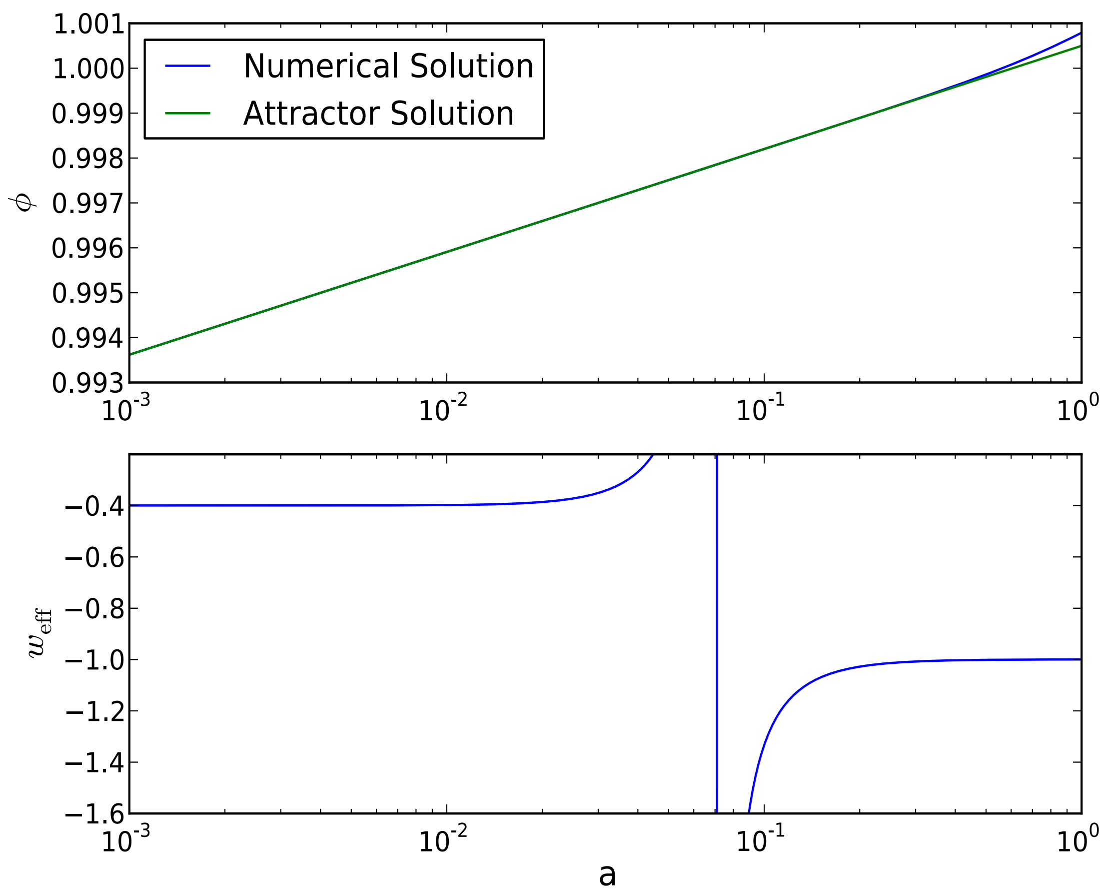
[H. Nariai, 1968]

- Inserting in the eq. of state

$$w_{eff} \approx \frac{4-4w_{BD}V(\phi)a^3/H_0^2}{-10+4w_{BD}V(\phi)a^3/H_0^2}$$

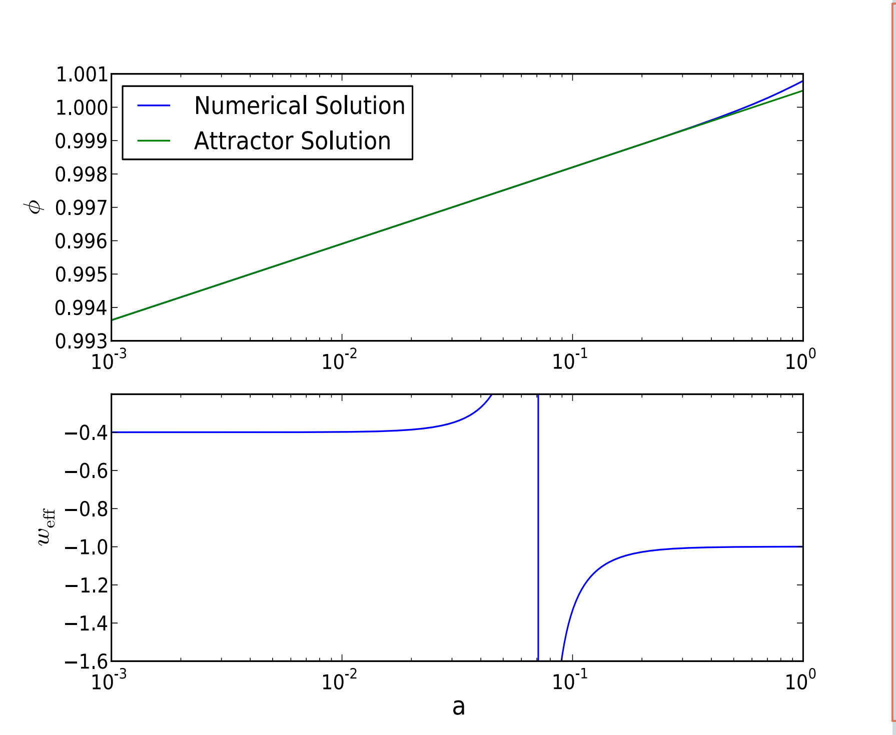
- Fractional energy density $\Omega_\phi \approx \frac{1}{3}\left[-\frac{5}{2w_{BD}} + a^3\frac{V(\phi)}{H_0^2}\right]$

Extended Brans-Dicke gravity with a constant potential



Extended Brans-Dicke gravity with a constant potential

- Departure from attractor solution at late-times



- Numerical solution matches attractor solution at early-times -> presence of potential negligible
- Sharp transition in effective eq. of state due to effective dark energy density crossing zero
- Departure at late-times, with the scalar field moving away from attractor solution and yielding a value today larger than ϕ_0

Extended designer Brans-Dicke gravity

- Designer Brans-Dicke approach

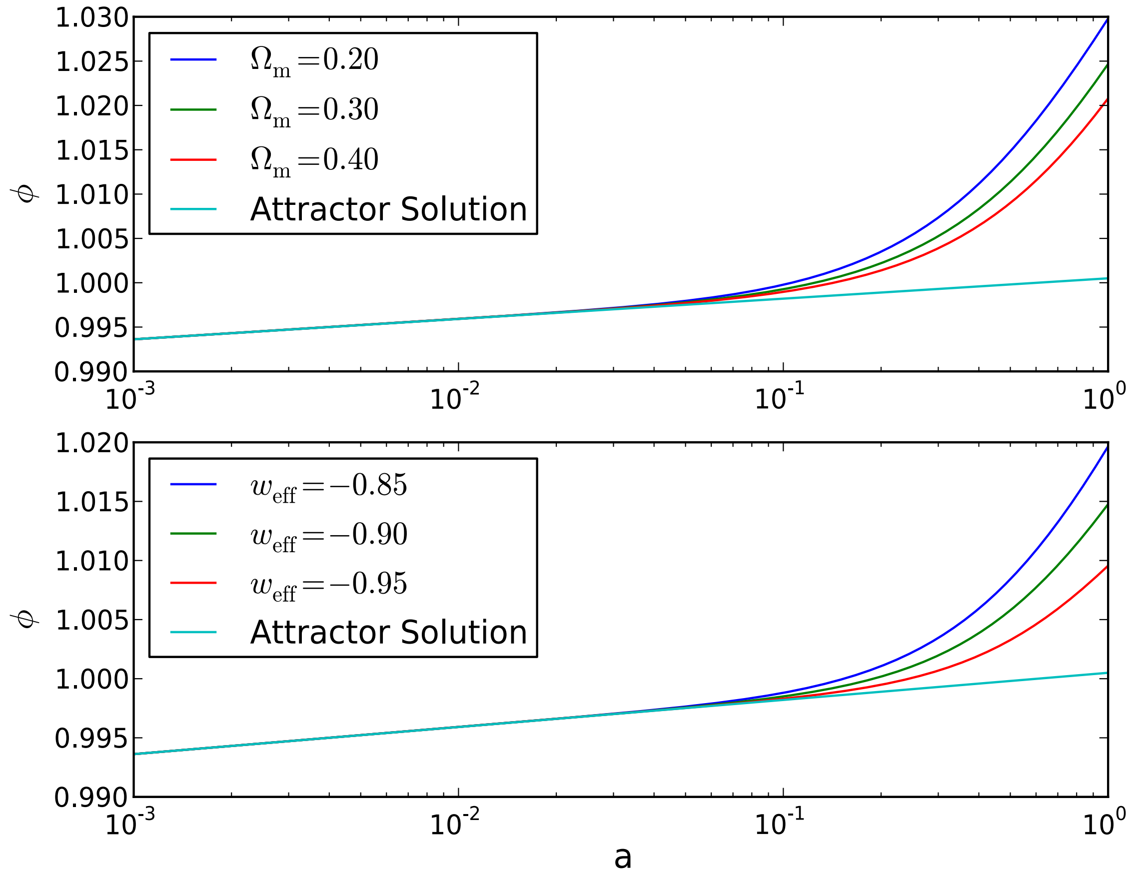
$$H^2 \equiv \frac{E(a)}{\phi} = \frac{H_0^2}{\phi}\left[\Omega_m a^{-3} + (1-\Omega_m)\, a^{-3(1+w_{eff})}\right]$$

- Potential given by

$$V(\phi) = 3H_0^2 (1-\Omega_m)\, a^{-3(1+w_{eff})}$$

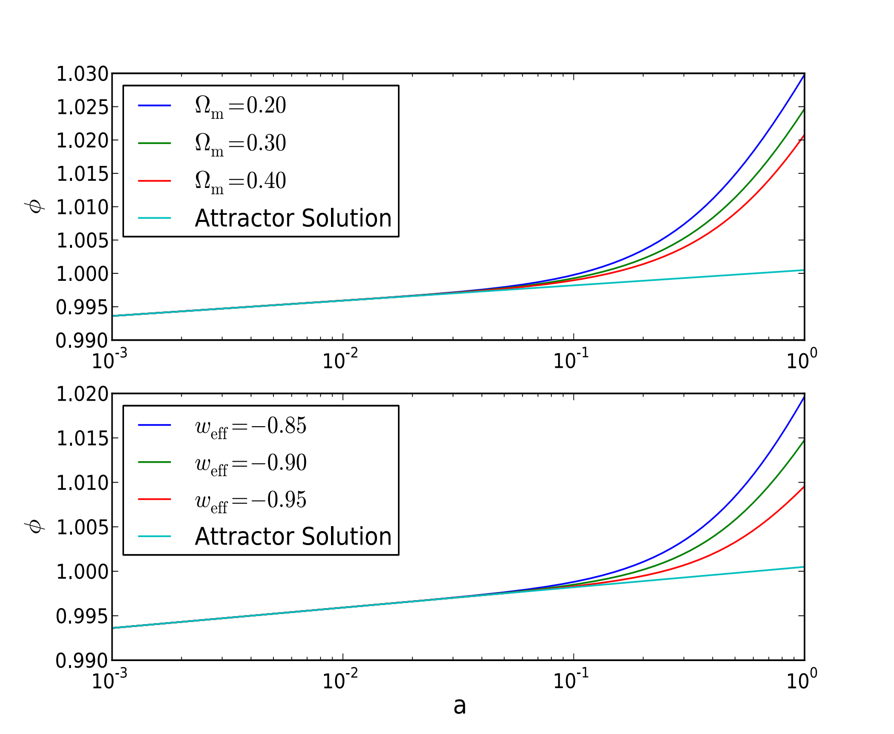
- Assuming main contribution to effective dark energy density comes from potential, hence, sub-dominant scalar field dynamics

Extended designer Brans-Dicke gravity



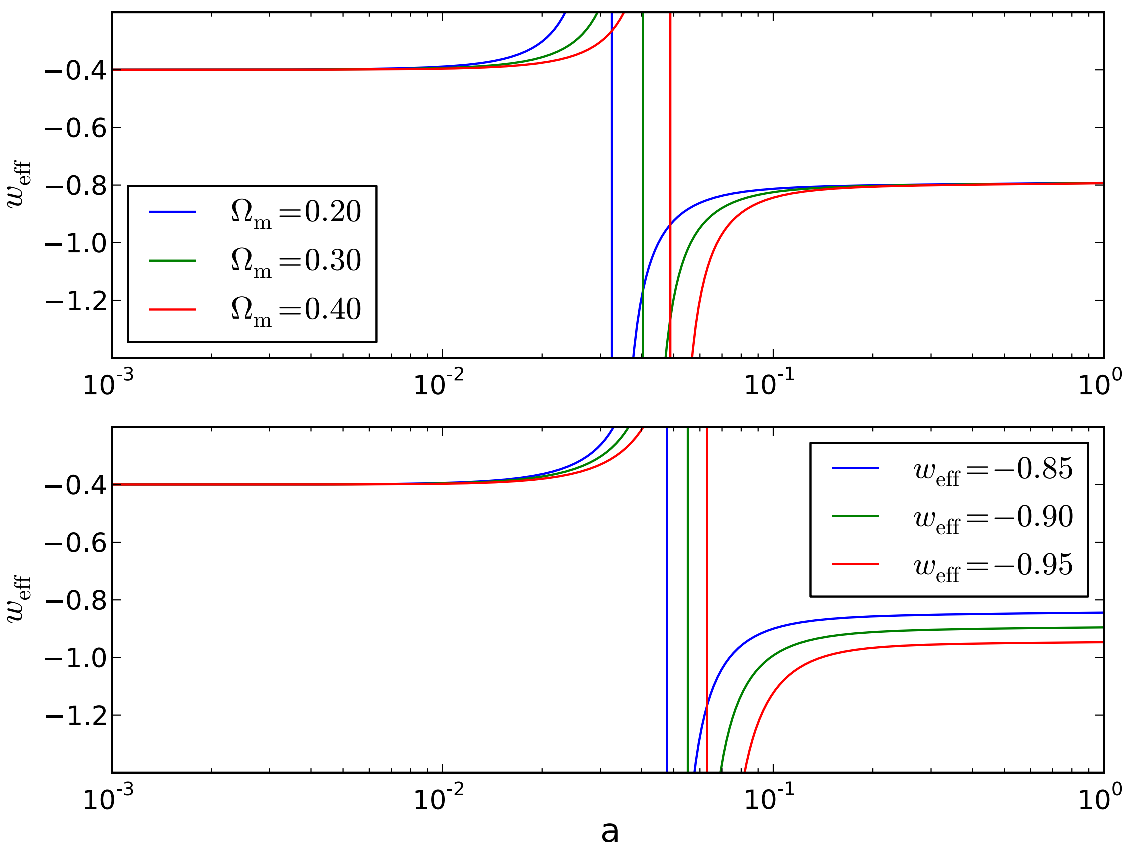
Extended designer Brans-Dicke gravity

- Earlier departures from attractor solution



- Designer approach recovers attractor solution -> expected since potential is suppressed at early-times
- Significant departures from attractor solution at late-times
- Departures happen earlier the more significant we set the dark energy component to be

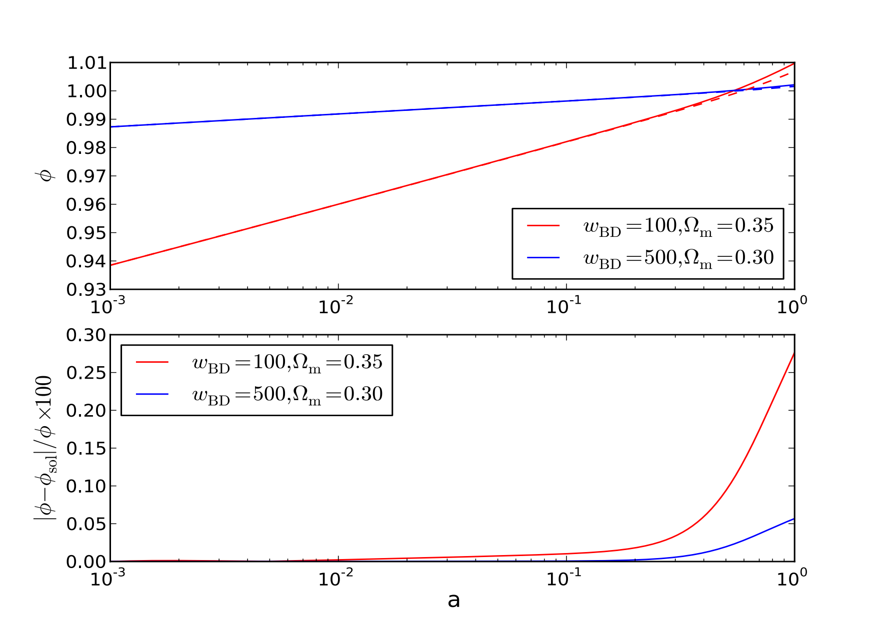
Extended designer Brans-Dicke gravity



Analytical solutions

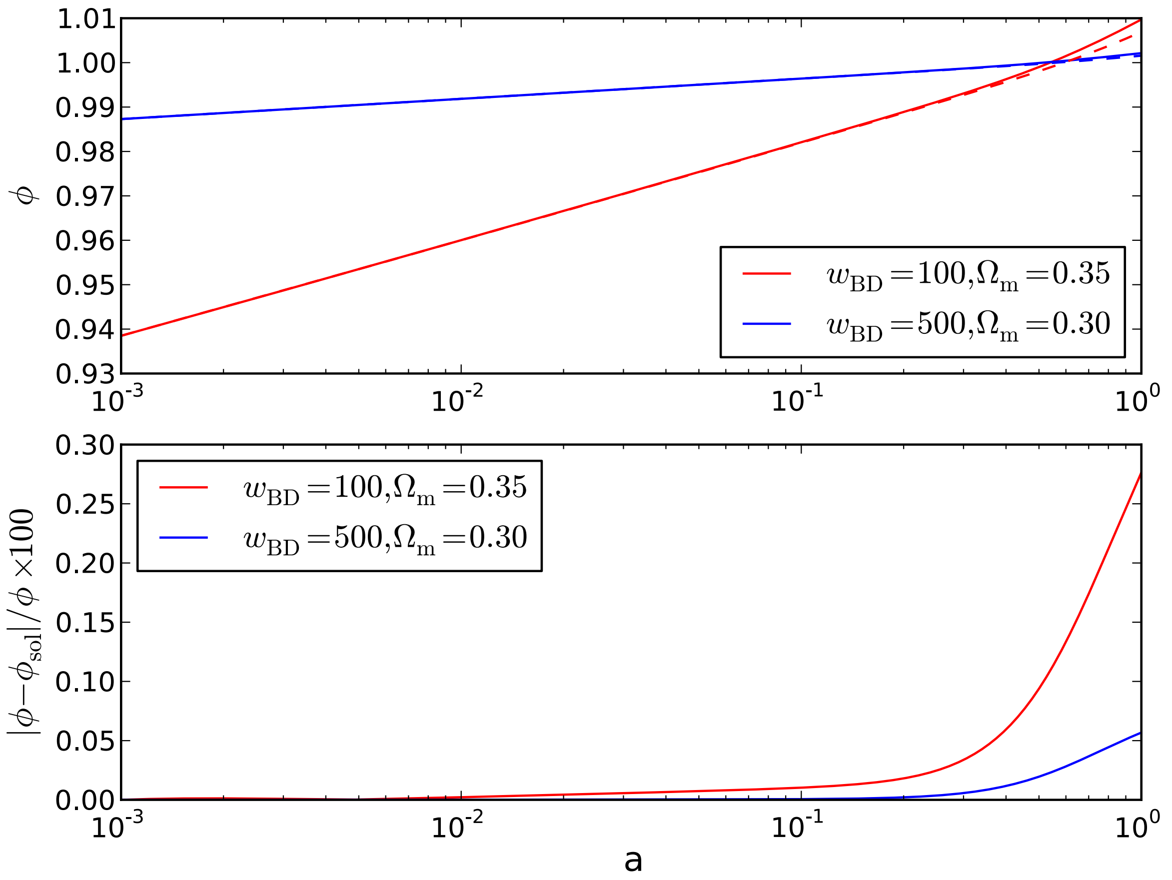
- Late-time analytical solutions for ϕ
  - $w_{eff} = -1$

$$\phi(a) = \phi(a_i) g^{-1}(a_i) \underbrace{a^{\frac{2}{d}} \left(2a^3(1-\Omega_m)+\Omega_m\right)^{\frac{2}{3d}}}_{g(a)}$$



- Analytical solution tends to overestimate departure from attractor solution
- Nevertheless, sub-percent order errors throughout

Analytical solutions

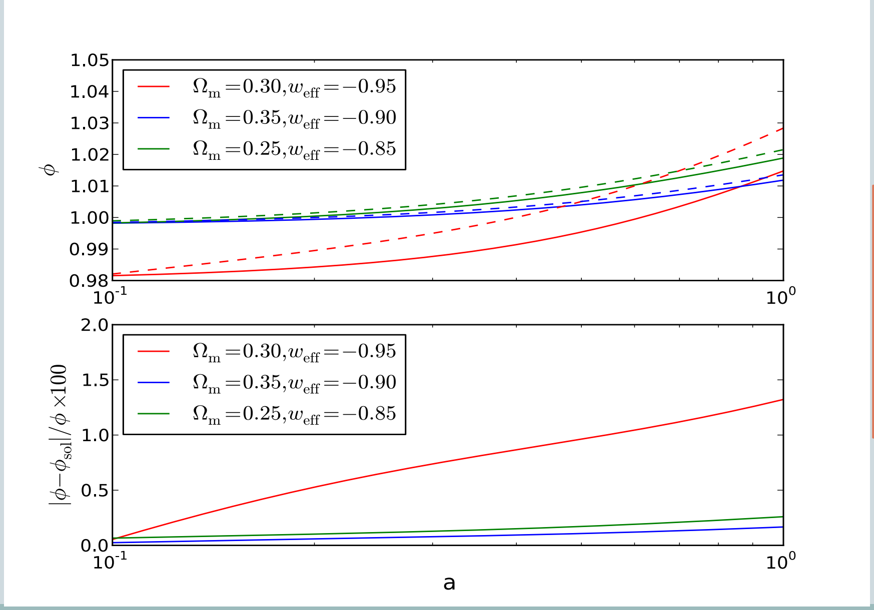


Analytical solutions

- $w_{eff} > -1$

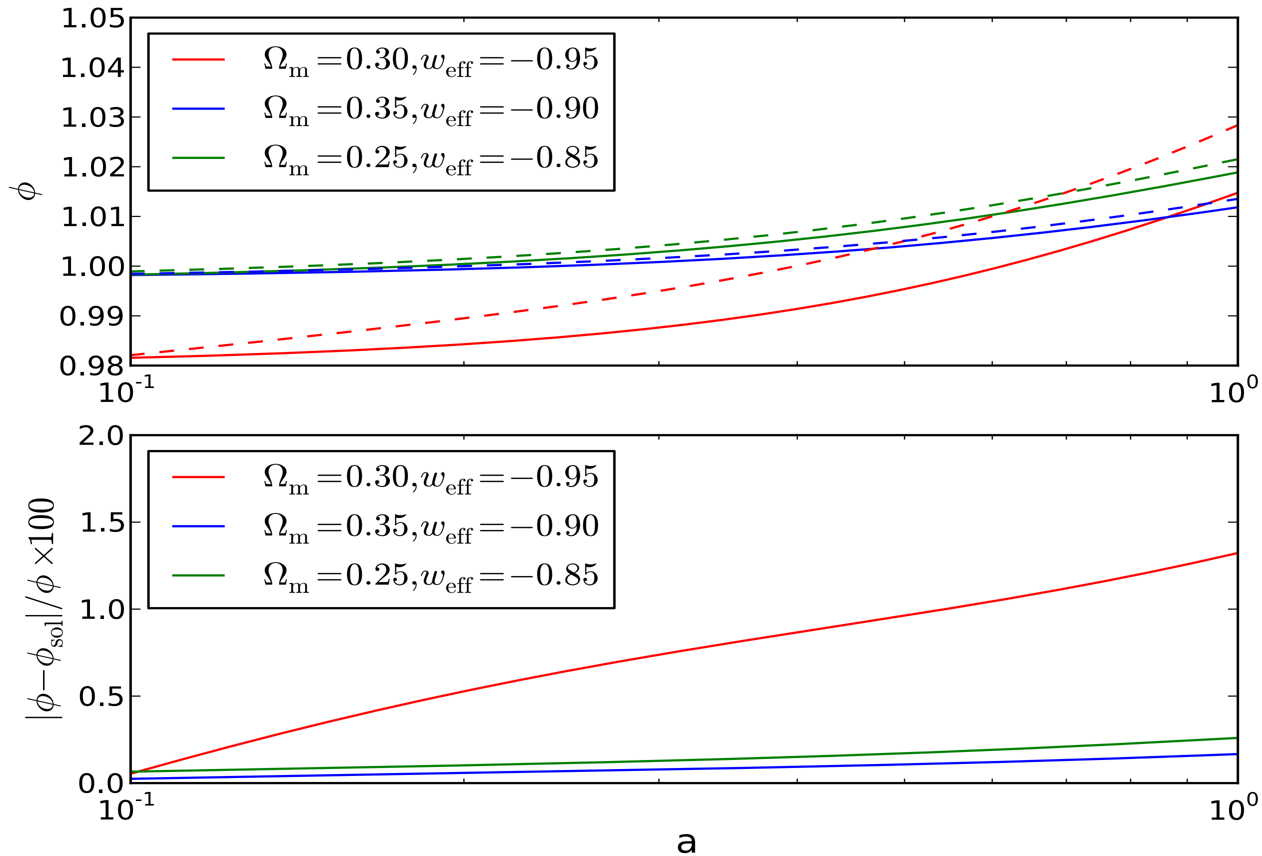
$$\phi(a) = \phi(a_i) f^{-1}(a_i) \underbrace{\left(\frac{1 + x(a)}{x(a) - 1} \right)^{-\frac{\sqrt{6d}\,(1+w_{eff})^{3/2}}{w_{eff}\,(-2+3d\,(1+w_{eff}))}}}_{f(a)}$$

$$x(a) = \sqrt{1 + \Omega_m \frac{a^{3w_{eff}}}{1 - \Omega_m}}$$



- Our solution now underestimates the departure from the attractor solution
- Sub-percent order errors for large w_{BD}

Analytical solutions



Analytical solutions

- Global solution, $w_{eff} \geq -1$

$$\phi_{global} = \phi(a_i) f(a) g(a) f^{-1}(a_i) g^{-1}(a_i)$$

- Simply the product of the particular solutions

- Will be an exact solution of the eq. of motion when $w_{eff} = -1$, and an approximate solution when $w_{eff} > -1$

- We will see it works great to describe the phenomenological parameters

Phenomenological Parameters

- Sub-horizon ($k^2/a^2H^2>>1$) parameters in quasi-static regime [De Felice, 2011]

$$\frac{G_{eff}}{G}=\frac{1}{\phi}\frac{4+2w_{BD}+2\phi\left(Ma/k\right)^2}{3+2w_{BD}+2\phi\left(Ma/k\right)^2}$$

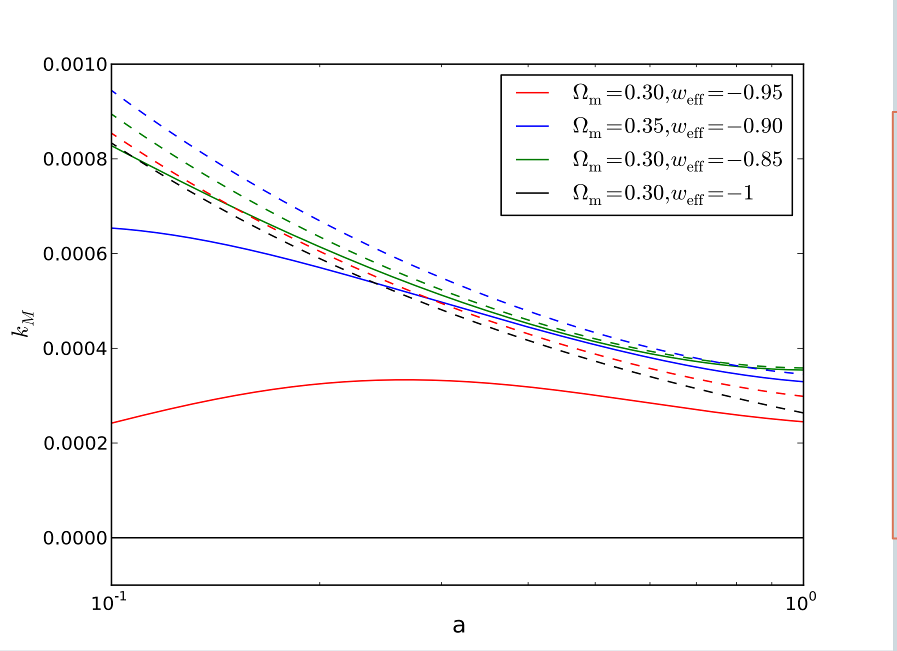
$$-\frac{\Phi}{\Psi}\equiv\eta=\frac{1+w_{BD}+\phi\left(Ma/k\right)^2}{2+w_{BD}+\phi\left(Ma/k\right)^2}$$

$$k_M=\sqrt{\frac{\phi}{1+w_{BD}}}Ma$$

Phenomenological Parameters

- For large w_{BD} and $w_{eff} > -1$ we find

$$k_M \approx \frac{3H_0}{\sqrt{2}}\sqrt{a^{-1}\Omega_m(2+w_{eff})+2(1-\Omega_m)(1+w_{eff})a^{-(1+3w_{eff})}}$$



- k_M is a fairly negligible quantity
- Furthermore, k_M will be very similar to the comoving horizon, aH
- Hence, assuming $k/k_M >> 1$ is a great approximation at sub-horizon scales!

Phenomenological Parameters

- Sub-horizon ($k^2 >> a^2 H^2$) parameters in quasi-static regime

$$\frac{G_{eff}}{G} = \frac{1}{\phi}\frac{4+2w_{BD}+2\phi\left(Ma/k\right)^2}{3+2w_{BD}+2\phi\left(Ma/k\right)^2} \approx \frac{1}{\phi}\frac{4+2w_{BD}}{3+2w_{BD}} \equiv \xi_{QS}$$

$$-\frac{\Phi}{\Psi} \equiv \eta = \frac{1+w_{BD}+\phi\left(Ma/k\right)^2}{2+w_{BD}+\phi\left(Ma/k\right)^2} \approx \frac{1+w_{BD}}{2+w_{BD}} \equiv \eta_{QS}$$

Constant throughout cosmological evolution

Phenomenological Parameters

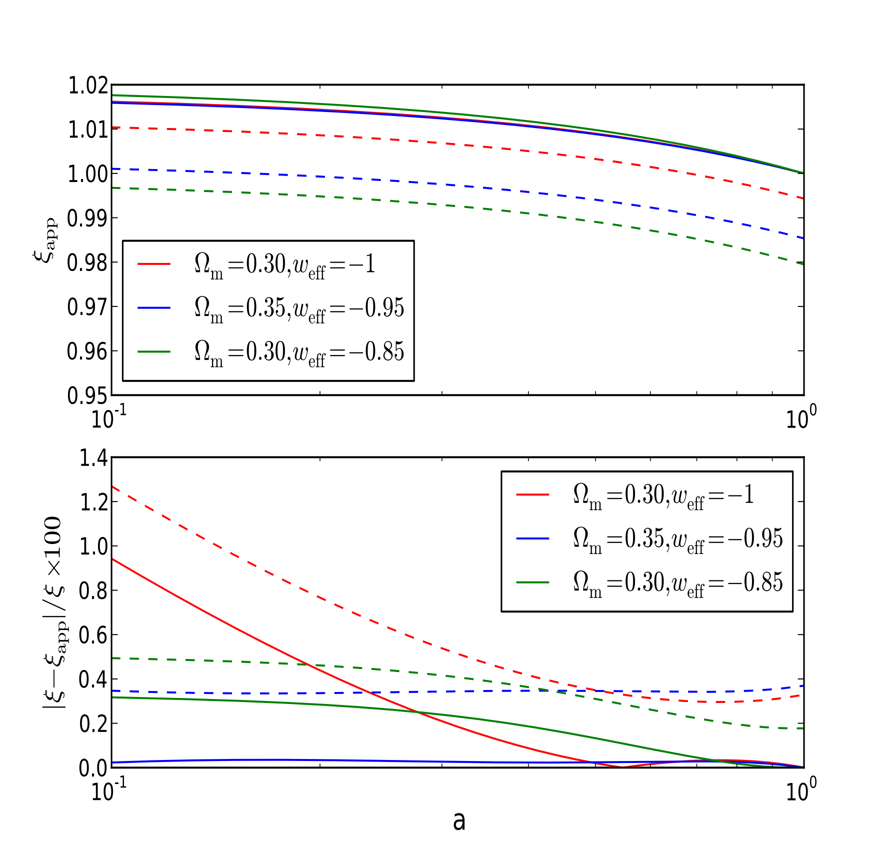
- Taylor expansion of ξ_{QS} at a = 1

$$\xi_{QS1} \approx 1 + (1-a)\left[\frac{8-6\Omega_m}{d(2-\Omega_m)} + \frac{3\sqrt{6d(1-\Omega_m)}(1+w_{eff})^{3/2}}{3d(1+w_{eff})-2}\right]$$

$$\xi_{QS2} \approx \left(\frac{\Omega_m}{2-\Omega_m}\right)^{\frac{2}{3d}} \left(\frac{1-\sqrt{1-\Omega_m}}{1+\sqrt{1-\Omega_m}}\right)^{-\frac{\sqrt{6d}\,(1+w_{eff})^{3/2}}{w_{eff}(-2+3d(1+w_{eff}))}} \xi_{QS1}$$

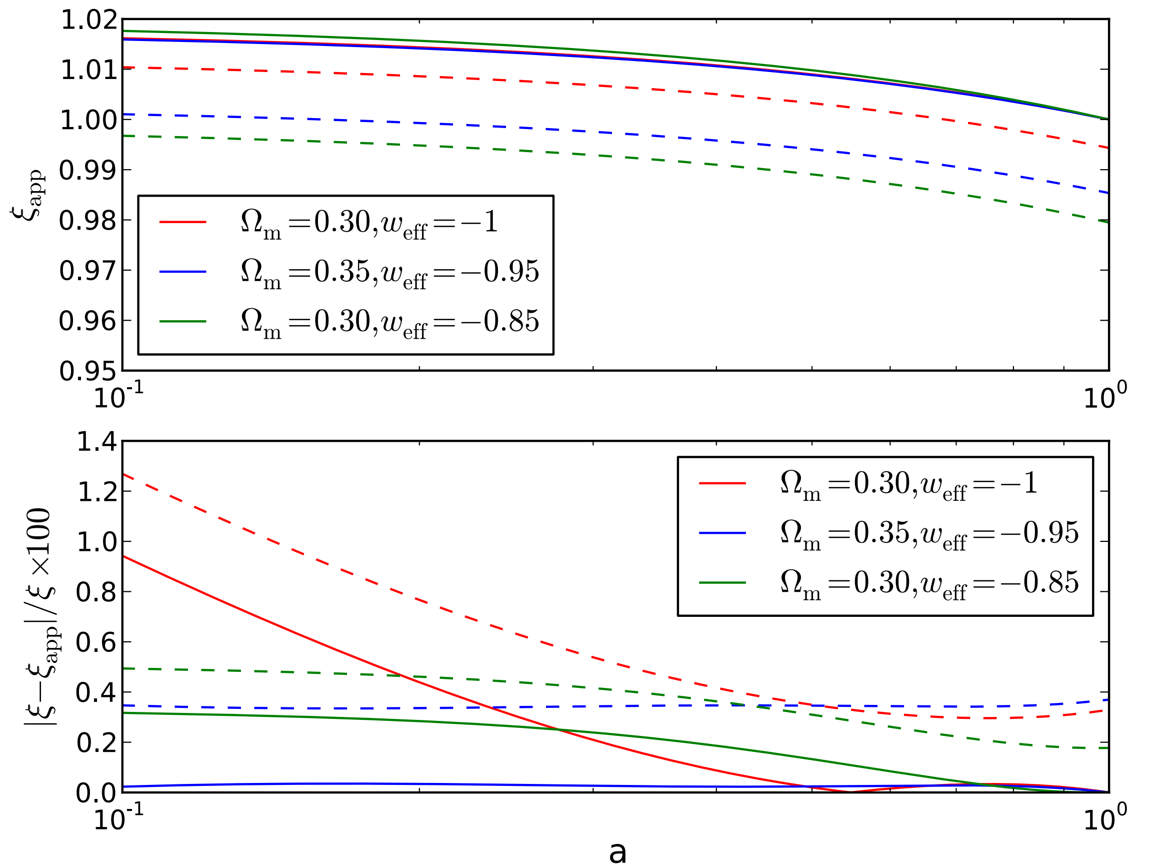
Phenomenological Parameters

- Analytical approximation works very well!



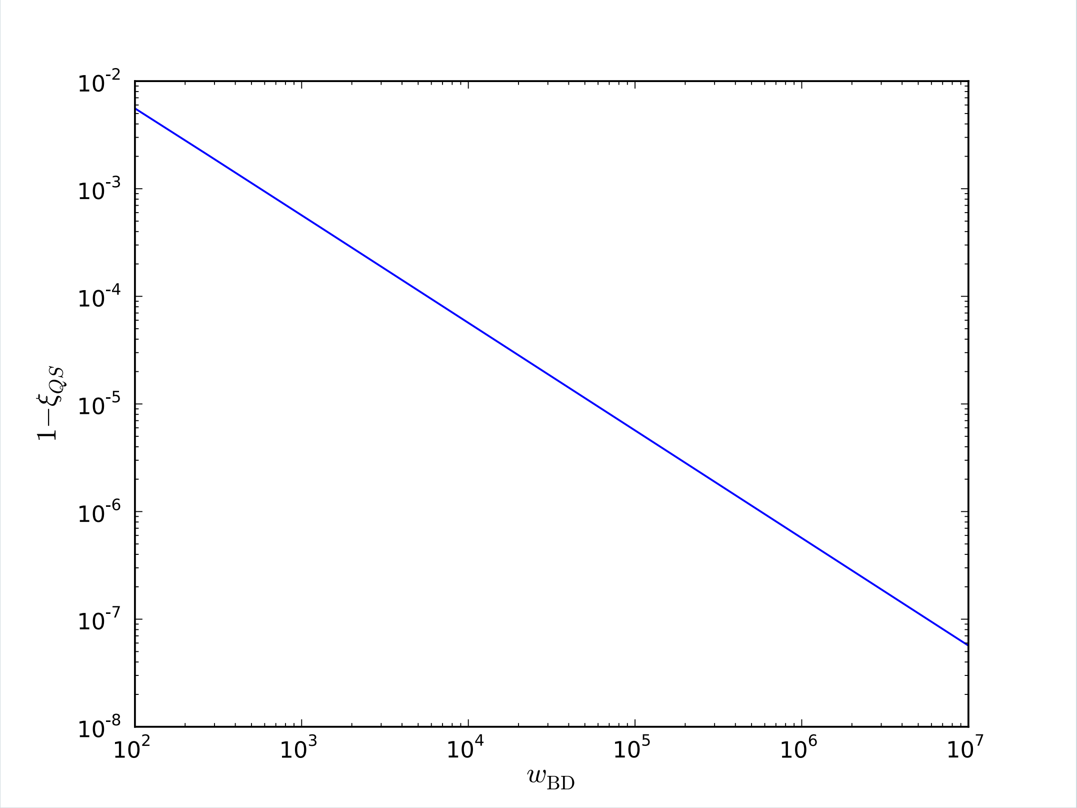
- Two possible regimes. Either $G_{eff}/G = 1$ or smaller than 1 today, depending on $\phi = \phi_0$ or $\phi > \phi_0$ today
- Approximation works better for larger w_{BD}
- Considerable deviation from 1 for larger w_{eff}

Phenomenological Parameters



Phenomenological Parameters

- G_{eff}/G evaluated today for $w_{eff} = -1$



Conclusions

- Novel designer extended Brans-Dicke model
- Analytical solutions for the Brans-Dicke scalar field provide good agreement with numerical predictions
- Explicit understanding of behavior of phenomenological QS parameters on the model parameters
- Clear departure from standard GR that can be constrained