On the phenomenology of extended Brans-Dicke gravity

NELSON A. LIMA
INSTITUTE FOR ASTRONOMY
UNIVERSITY OF EDINBURGH

COSMOCRUISE 2015

COLLABORATOR: PEDRO G. FERREIRA (U. OXFORD)
SUPERVISOR: PROF. ANDREW LIDDLE
Why should we consider modifying gravity?

Extended Brans-Dicke gravity with a constant potential

Extended Designer Brans-Dicke gravity

The phenomenological parameters

Conclusions and outlook
Why Modify Gravity?

- Our Universe is undergoing an accelerated expansion
- Approx. 70% is an exotic negative pressure component, Dark Energy
- Our best guess is $\Lambda$CDM. However, it is not definitive
- Does G.R. need to be corrected/modified?
Why Modify Gravity?

- MG theories evade Solar-system tests through screening mechanisms
- At the background level, modified gravity can be indistinguishable from ΛCDM with fine-tuning
- Next generation of surveys may place tight constraints on the largest scales
- Dynamics of linear perturbations of MG theories can be significantly different
- Essential to understand how the observables are interrelated to the fundamental parameters of the theories
• Evolution of the perturbed potentials can be parameterized in the sub-horizon \((k^2/a^2H^2 >> 1)\) regime [De Felice, 2011]

\[
\frac{k^2}{a^2} \Psi \approx -4\pi G_{\text{eff}} (k, a) \rho_m \delta \quad \eta(k, a) \equiv -\frac{\Phi}{\Psi}
\]

• \(G_{\text{eff}}\) and \(\eta\) are scale and time dependent functions that are also related to the fundamental parameters of the theory one considers
Extended Brans-Dicke gravity

- We consider an action of the form

\[ S = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} \left[ \phi R - \frac{w_{BD}}{\phi} (\partial \phi)^2 - 2V(\phi) \right] + S_m \]

- Scalar field equation of motion

\[ \nabla^\alpha \nabla_\alpha \phi = \frac{\kappa^2 T}{2w_{BD} + 3} + \frac{4V(\phi) - 2\phi V_\phi}{2w_{BD} + 3} \]

\[ 2w_{BD} + 3 \equiv d \]

- Effective eq. of state: 

\[ w_{\text{eff}} = \frac{\dot{\phi}^2 w(\phi) + 4H \dot{\phi} + 2\ddot{\phi} - 2V(\phi)}{\dot{\phi}^2 w(\phi) - 6H \dot{\phi} + 2V(\phi)} \]
The GR limit is recovered when $w_{BD} \to \infty$

Solar-system tests place lower bound on the Brans-Dicke parameter of 40.000 [B. Bertotti, 2003]

Cosmological constraints place the lower bound at around 1.000 [A. Avilez, 2014]

Accelerating solutions without $V(\phi)$ exist for negative (order unity) Brans-Dicke parameter, or considering $w_{bd}$ as a function of the scalar field
Extended Brans-Dicke gravity with a constant potential

- Constant potential $V(\phi) = 3H_0^2(1 - \Omega_m)$

$$\phi_0 = \frac{2w_{BD} + 4}{2w_{BD} + 3}$$

- Matter dominated regime: $\phi = \phi_0 a^{1/(1 + w_{BD})}$

[H. Nariai, 1968]

- Inserting in the eq. of state

$$w_{eff} \approx \frac{4 - 4w_{BD}V(\phi)a^3 / H_0^2}{-10 + 4w_{BD}V(\phi)a^3 / H_0^2}$$

- Fractional energy density

$$\Omega_\phi \approx \frac{1}{3} \left[ -\frac{5}{2w_{BD}} + a^3 \frac{V(\phi)}{H_0^2} \right]$$
Extended Brans-Dicke gravity with a constant potential
Extended Brans-Dicke gravity with a constant potential

- Departure from attractor solution at late-times

- Numerical solution matches attractor solution at early-times $\rightarrow$ presence of potential negligible

- Sharp transition in effective eq. of state due to effective dark energy density crossing zero

- Departure at late-times, with the scalar field moving away from attractor solution and yielding a value today larger than $\phi_0$
Extended designer Brans-Dicke gravity

- Designer Brans-Dicke approach

\[ H^2 \equiv \frac{E(a)}{\phi} = \frac{H_0^2}{\phi} \left[ \Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w_{\text{eff}})} \right] \]

- Potential given by

\[ V(\phi) = 3H_0^2 (1 - \Omega_m) a^{-3(1+w_{\text{eff}})} \]

- Assuming main contribution to effective dark energy density comes from potential, hence, sub-dominant scalar field dynamics
Extended designer Brans-Dicke gravity

\[ \phi = \begin{cases} \Omega_m = 0.20 \\ \Omega_m = 0.30 \\ \Omega_m = 0.40 \\ \text{Attractor Solution} \end{cases} \]

\[ w_{\text{eff}} = \begin{cases} -0.85 \\ -0.90 \\ -0.95 \\ \text{Attractor Solution} \end{cases} \]
Extended designer Brans-Dicke gravity

- Earlier departures from attractor solution

- Designer approach recovers attractor solution -> expected since potential is suppressed at early-times

- Significant departures from attractor solution at late-times

- Departures happen earlier the more significant we set the dark energy component to be
Extended designer Brans-Dicke gravity
Analytical solutions

- Late-time analytical solutions for $\phi$
  - $w_{\text{eff}} = -1$
    
    $$\phi(a) = \phi(a_i) g^{-1}(a_i) a^d \left( 2 a^3 \left( 1 - \Omega_m \right) + \Omega_m \right)^{\frac{2}{3d}}$$

- Analytical solution tends to overestimate departure from attractor solution
- Nevertheless, sub-percent order errors throughout
Analytical solutions
\( \omega_{\text{eff}} > -1 \)

\[
\phi(a) = \phi(a_i) f^{-1}(a_i) \left( \frac{1 + x(a)}{x(a) - 1} \right) - \frac{\sqrt{6d (1 + \omega_{\text{eff}})^{3/2}}}{\omega_{\text{eff}} (-2 + 3d (1 + \omega_{\text{eff}}))}
\]

\[
x(a) = \sqrt{1 + \Omega_m} \frac{a^{3\omega_{\text{eff}}}}{1 - \Omega_m}
\]

- Our solution now underestimates the departure from the attractor solution
- Sub-percent order errors for large \( \omega_{BD} \)
Analytical solutions

\[ \Omega_m = 0.30, w_{\text{eff}} = -0.95 \]
\[ \Omega_m = 0.35, w_{\text{eff}} = -0.90 \]
\[ \Omega_m = 0.25, w_{\text{eff}} = -0.85 \]
Analytical solutions

- Global solution, $w_{\text{eff}} \geq -1$

  $$\phi_{\text{global}} = \phi(a_i) f(a) g(a) f^{-1}(a_i) g^{-1}(a_i)$$

- Simply the product of the particular solutions

- Will be an exact solution of the eq. of motion when $w_{\text{eff}} = -1$, and an approximate solution when $w_{\text{eff}} > -1$

- We will see it works great to describe the phenomenological parameters
Phenomenological Parameters

- Sub-horizon \((k^2/a^2H^2>>1)\) parameters in quasi-static regime [De Felice, 2011]

\[
\frac{G_{\text{eff}}}{G} = \frac{1 + 4w_{BD} + 2\phi(Ma/k)^2}{\phi 3 + 2w_{BD} + 2\phi(Ma/k)^2}
\]

\[
\frac{\Phi}{\Psi} \equiv \eta = \frac{1 + w_{BD} + \phi(Ma/k)^2}{2 + w_{BD} + \phi(Ma/k)^2}
\]

\[
k_M = \sqrt{\frac{\phi}{1 + w_{BD}}} Ma
\]
For large $w_{BD}$ and $w_{eff} > -1$ we find

$$k_M \approx \frac{3H_0}{\sqrt{2}} \sqrt{a^{-1} \Omega_m (2 + w_{eff}) + 2(1 - \Omega_m)(1 + w_{eff})a^{-(1+3w_{eff})}}$$

- $k_M$ is a fairly negligible quantity
- Furthermore, $k_M$ will be very similar to the comoving horizon, $aH$
- Hence, assuming $k/k_M >> 1$ is a great approximation at sub-horizon scales!
Sub-horizon \((k^2 > a^2 H^2)\) parameters in quasi-static regime

\[
\frac{G_{\text{eff}}}{G} = \frac{1}{\phi} \frac{4 + 2 w_{BD} + 2 \phi (Ma / k)^2}{3 + 2 w_{BD} + 2 \phi (Ma / k)^2} \approx \frac{1}{\phi} \frac{4 + 2 w_{BD}}{3 + 2 w_{BD}} \equiv \xi_{QS}
\]

\[
-\frac{\Phi}{\Psi} \equiv \eta = \frac{1 + w_{BD} + \phi (Ma / k)^2}{2 + w_{BD} + \phi (Ma / k)^2} \approx \frac{1 + w_{BD}}{2 + w_{BD}} \equiv \eta_{QS}
\]

Constant throughout cosmological evolution
Phenomenological Parameters

- Taylor expansion of $\xi_{QS}$ at $a = 1$

\[
\xi_{QS1} \approx 1 + (1 - a) \left[ \frac{8 - 6\Omega_m}{d(2 - \Omega_m)} + \frac{3\sqrt{6d(1 - \Omega_m)(1 + w_{\text{eff}})^{3/2}}}{3d(1 + w_{\text{eff}}) - 2} \right]
\]

\[
\xi_{QS2} \approx \left( \frac{\Omega_m}{2 - \Omega_m} \right)^{\frac{2}{3d}} \left( \frac{1 - \sqrt{1 - \Omega_m}}{1 + \sqrt{1 - \Omega_m}} \right)^{-\frac{\sqrt{6d(1+w_{\text{eff}})^{3/2}}}{w_{\text{eff}}(-2 + 3d(1+w_{\text{eff}}))}} \xi_{QS1}
\]
• Analytical approximation works very well!

- Two possible regimes. Either $G_{\text{eff}}/G = 1$ or smaller than 1 today, depending on $\phi = \phi_0$ or $\phi > \phi_0$ today

- Approximation works better for larger $w_{BD}$

- Considerable deviation from 1 for larger $w_{\text{eff}}$
Phenomenological Parameters

- $\Omega_m = 0.30, w_{\text{eff}} = -1$
- $\Omega_m = 0.35, w_{\text{eff}} = -0.95$
- $\Omega_m = 0.30, w_{\text{eff}} = -0.85$
Phenomenological Parameters

- $G_{\text{eff}}/G$ evaluated today for $w_{\text{eff}} = -1$
Conclusions

- Novel designer extended Brans-Dicke model
- Analytical solutions for the Brans-Dicke scalar field provide good agreement with numerical predictions
- Explicit understanding of behavior of phenomenological QS parameters on the model parameters
- Clear departure from standard GR that can be constrained