Emptiness of the Universe as a Cause of Inflation and Dark Energy

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Both inflation and dark energy are tied by the fact that in both cases we are dealing with an exponentially accelerated expansion of the Universe

Apart from this fact, there is one more fact that unites them, and this is the emptiness of the Universe

- By the end of its evolution, the Universe is going to become empty

$$\varepsilon_m \sim a(t)^{-3} \to 0 \qquad t \to \infty$$

- The present Universe is already ~70% empty, so the effects associated with the emptiness of space must already be very noticeable, and they are (dark energy).

- At the beginning of cosmological evolution before matter was born, the Universe is also assumed to be empty

- Both inflation and dark energy are associated with the emptiness of the Universe.

The empty space-time is not really empty because of natural quantum metric fluctuations (gravitons)

How evolve such empty space filled with gravitons?

To solve the problem, we have to have the theory of quantum gravity that does not exist for now

Fortunately, for the empty space-time (with no matter fields), it is finite in one-loop approximation (t'Hooft and Veltman, 1974)

We have managed to get the rigorous set of finite one-loop quantum gravity equations for the empty FLRW metric (with no mathematical contradictions)

Marochnik, Usikov & Vereshkov (2008 a, b; 2013); Vereshkov & Marochnik (2011)

One-loop approximation

Quantum state of gravitons is determined by their interaction with a classical macroscopic field, and the macroscopic (background) geometry, in turn, depends on the quantum state of gravitons

Graviton-graviton interactions are not taken into account

Self-Consistent Equations of One-Loop Quantum Gravity that are Finite off the Graviton Mass Shell

Faddeev – Popov gauged path integral

↓

Factorization of classical and quantum variables

(allowing the existence of a self–consistent system of equations for gravitons, ghosts and macroscopic geometry)

↓

Exponential parameterization

↓

Transition to the one–loop approximation

(taking into account the fact that contributions of ghost fields to observables cannot be eliminated in any way)

↓

Choice of ghost sector, satisfying the condition of one–loop finiteness of the theory off the mass shell.

$$3\frac{a'^2}{a^4} = 8\pi G\varepsilon_g = \frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk\Big(\sum_\sigma <\Psi_g|\hat{\psi}'^{+}_{\mathbf{k}\sigma}\hat{\psi}'_{\mathbf{k}\sigma} + k^2\hat{\psi}^{+}_{\mathbf{k}\sigma}\hat{\psi}_{\mathbf{k}\sigma}|\Psi_g>$$

$$-2<\Psi_{gh}|\bar{\theta}'_{\mathbf{k}}\theta'_{\mathbf{k}} + k^2\bar{\theta}_{\mathbf{k}}\theta_{\mathbf{k}}|\Psi_{gh}>\Big)$$

$$\hat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a}\hat{\phi}_{\mathbf{k}\sigma}$$

Quantum state vectors of gravitons and ghosts, Ψ_g Ψ_{gh}

Shrödinger-like graviton equation

conformal time $\eta = \int dt/a$

Heisenberg's graviton operator $\hat{\phi}_{k,\sigma}$

$$\hat{\phi}''_{k,\sigma} + (k^2 - \frac{a''}{a})\hat{\phi}_{k,\sigma} = 0$$

a''/a Plays the role of "one-dimensional potential"

$$k = 2\pi/\lambda$$

$$x = k\eta$$

de Sitter background $a_S = \exp(Ht) = -(H\eta)^{-1}$

$$\hat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a_S}\sqrt{\frac{16\pi G\hbar}{k}}\left[\hat{c}_{\mathbf{k}\sigma}f(x) + \hat{c}^{+}_{-\mathbf{k}-\sigma}f^{+}(x)\right]$$

De Sitter mode function

$$f(x) = (1+\frac{i}{x})e^{ix}$$

divergent integrals

$x^2 = 0$ is the barrier; behind this barrier $x^2 < 0$ $\quad x = i\xi$

$$f(x) = (1+\frac{i}{x})e^{ix} \Rightarrow f(\xi) = (1+\frac{1}{\xi})e^{-\xi}$$

Wick rotation $t \to i\tau,\ x \to i\xi$ $\Rightarrow$ convergent integrals

Example: Milne background

$$a(t) = q \times t \quad q = const \quad a = \exp(q\eta) \quad \eta = \int dt / a$$

η plays the role of the spatial coordinate of Schrödinger equation

The role of "one-dimensional potential" plays a'' / a

$$f'' + (k^2 - \frac{a''}{a}) f = 0 \qquad a'' / a = q^2 \quad \Rightarrow \quad f'' + (k^2 - q^2) f = 0$$

Incident wave

Reflected wave

$$f = C_1 \exp(i\sqrt{k^2 - q^2}\eta) + C_2 \exp(-i\sqrt{k^2 - q^2}\eta) \qquad x^2 = (k\eta)^2 \geq (q\eta)^2$$

$$f = C_3 \exp(-\sqrt{q^2 - k^2}\eta) + C_4 \exp(\sqrt{q^2 - k^2}\eta) \qquad x^2 = (k\eta)^2 \leq (q\eta)^2$$

$C_4 = 0$

Transmitted wave is damped

In terms of quantum tunneling, topologically impenetrable barrier is $x^2{}_{barrier} = (q\eta)^2$

For the de Sitter background topologically impenetrable barrier is $x^2{}_{barrier} = 0$

De Sitter transmitted waves are behind this barrier, i.e. in the region $x^2 < 0$

Transmitted wave formally corresponds to transition to imaginary time $k\eta = x \rightarrow i\xi$

Wick rotation $t \to i\tau, \eta \to i\varsigma$ and imaginary time formalism is used in the following fields

- Non–relativistic Quantum Mechanics
- Quantum Chromodynamics (QCD)
- Axiomatic quantum field theory (AQFT)
- Euclidian Quantum Gravity
- Theory of Josephson Effect

Calculations using imaginary time agree with experimental data

de Sitter state is exact solution to self-consistent finite eqs. of one-loop quantum gravity in imaginary time

$$a_S = a_0 \exp(H_\tau \tau)$$

$$H_\tau = \frac{1}{a}\frac{da}{d\tau} = i\left(\frac{\pi}{G\hbar N}\right)^{1/2} = iH$$

N is the number of gravitons in the system

Equation of state in imaginary time

$$-p_\tau = \varepsilon_\tau = \frac{3\hbar N}{8\pi^2} H_\tau^{\ 4}$$

Marochnik, Usikov & Vereshkov, Found. Phys. **38**, 546, 2008

- de Sitter state is invariant with respect to Wick rotation (!)

$$a = \exp(H_\tau \times \tau) = \exp(iH \times it) = \exp(H \times t)$$

$$H = \left(\frac{\pi}{G\hbar N}\right)^{1/2}$$

- The equation of state is also invariant with respect to Wick rotation (!)

$$-p_\tau = \varepsilon_\tau = \frac{3\hbar N}{8\pi^2} H_\tau^{\,4} \Rightarrow -p = \varepsilon = \frac{3\hbar N}{8\pi^2} H^4$$

It is because of the remarkable fact

$$H_\tau^{\,4} = (iH)^4 = H^4$$

It suggests that de Sitter takes place in real time too

The price that we pay for the emergence of de Sitter expansion in the real time empty Universe is an assumption that time is a complex variable, and this fact has a deep but still not understood meaning

CONSISTENCY WITH OBSERVATIONS

Inflation

CMB anisotropy

In units $\hbar = c = 1$ this de Sitter solution reads

$$H^2 = 8\pi^2 \frac{M_{pl}^{\ 2}}{N} \qquad M_{pl} = (\hbar c / 8\pi G)^{1/2} = 2.4 \times 10^{18} GeV / c^2$$

Fluctuations of the number of gravitons that are presumably described by Gaussian distribution are

$$<(\Delta N)^2> / <N>^2 = <N>^{-1} \qquad \frac{<(\Delta N)^2>}{<N>^2} = \frac{<(\Delta \varepsilon)^2>}{<\varepsilon>^2} = \frac{1}{8\pi^2} \times \frac{H^2}{M^2_{pl}}$$

Fluctuations of the number of gravitons in the Universe produce CMB anisotropy. Typical energy scale of inflation $H \sim 10^{15} GeV$

$$\Delta T / T \sim 10^{-5}$$

Observed CMB anisotropy is consistent with inflation from gravitons

Equation-of-state parameter

The theory predicts that the equation-of-state parameter should be $w > -1$ for inflation and for dark energy $w < -1$

This is consistent with observational data

Combination of Planck+WP+BAO (Abe et al., 2013)

$$w = -1.13_{-0.25}^{+0.24} < -1 \quad \text{(dark energy)}$$

The observed tilt $n_s - 1$ of power spectrum $k^{n_s - 1}$ deviates slightly from the scale-invariant form corresponding to $n_s = 1$ .The observed value is $n_s \approx 0.96$ (Komatsu et al., 2008). This means that in reality we deal with a quasi-de Sitter expansion.

Assuming that quantum metric fluctuations are of power spectrum

$$N_k = N_0 (k / k_0)^{\beta}$$ k_0 is a pivot scale,

we get relations

$$n_s - 1 = -\beta \qquad \beta = 3(1 + w)$$

$$w \approx -0.987 > -1 \qquad \text{(inflation)}$$

Consistency with Observations

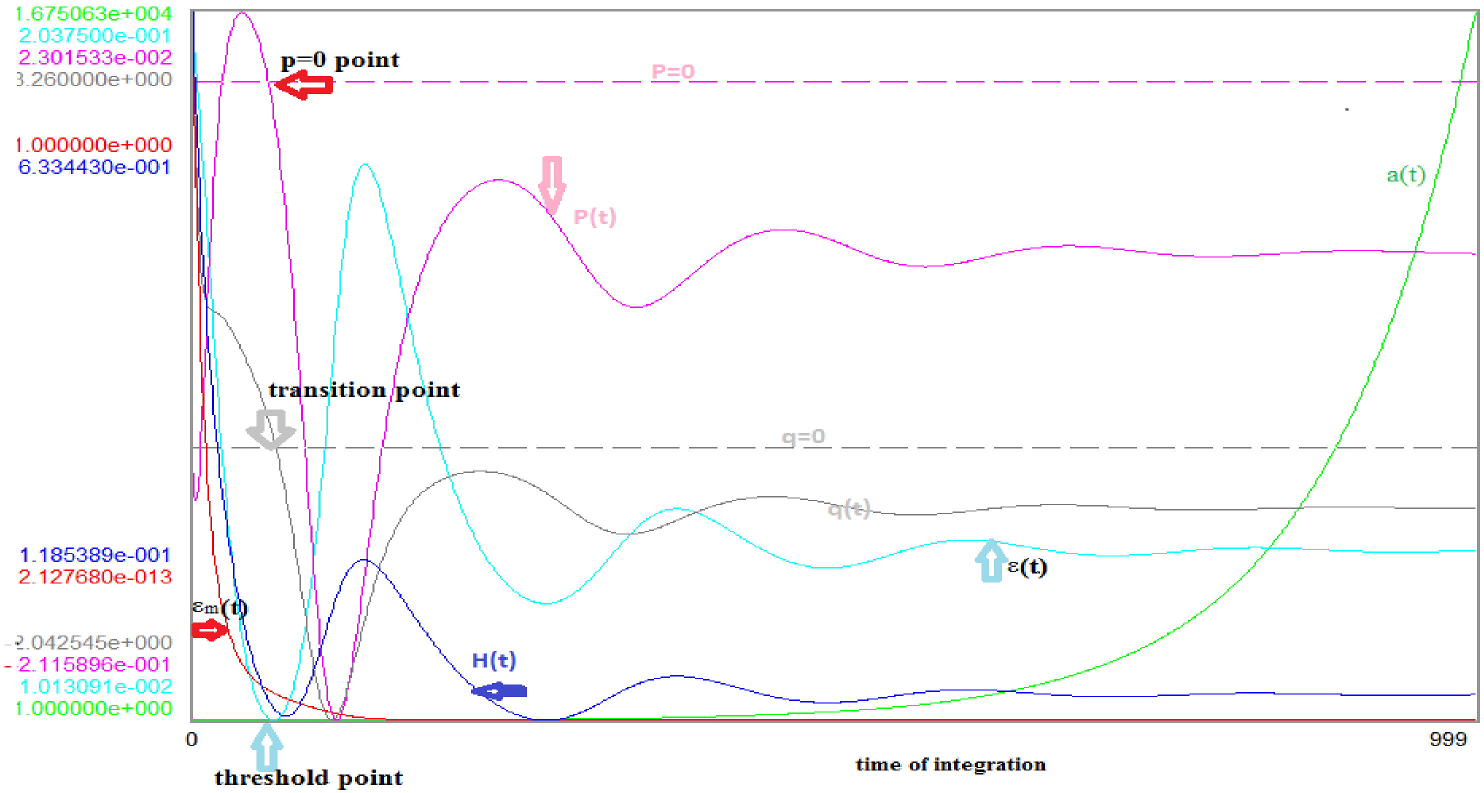
Dark Energy

Numerical solutions

There is no explicit solutions to the system of one-loop equations that include non-relativistic matter

Numerical solutions asymptotically come to the Sitter

A typical real-time numerical solution for non-relativistic matter + dark energy of instanton origin



Such solutions oscillate from the start, and then they asymptotically come to the de Sitter mode

$$H \to const;\ \varepsilon \to 3H^2;\ p \to -3H^2$$

The fact is independent of initial conditions

“Old cosmological constant problem”

Emptying the Universe is expanding at the rate of

$$H = \left(\frac{\pi}{G\hbar N}\right)^{1/2}$$

Observational value of the Hubble constant (Riess et al 2011)

$$H = 73.8 \pm 2.4 km \times \sec^{-1} \times Mpc^{-1}$$

Number of gravitons in the contemporary Universe should be

$$N \sim 10^{122}$$

This “mysterious” number has nothing to do with the vacuum energy which could be a solution to the “old cosmological constant problem”

Marochnik, Usikov & Vereshkov, Found. Phys. **38**, 546, 2008

“Coincidence problem”

Why is the acceleration happening during the contemporary epoch of matter domination?

Combination of two facts which are the existence of a threshold and coincidence of “one-dimensional potentials” for the de Sitter and matter dominated backgrounds

Threshold

Theoretical prediction

The dark energy takes over after the energy density of non-relativistic matter drops below the threshold which is the energy density of instantons

$$1 + z_{threshold} \leq\approx (\frac{\varepsilon_{de}}{\varepsilon_m})^{1/3} = (\frac{\Omega_{de}}{\Omega_m})^{1/3}$$

$$1 + z_{threshold} \approx 1.3$$

Coincidence of “one-dimensional potentials”

$$f'' + (k^2 - \frac{a''}{a}) f = 0$$

Shrödinger-like graviton equation for mode function f

Matter dominated background

$$a_{p=0} = \eta^2$$

de Sitter background

$$a_{p=-\varepsilon} = -(H\eta)^{-1}$$

“One-dimensional potentials” are identically the same

$$(a''/a)_{p=0} = (a''/a)_{p==-\varepsilon} = 2/\eta^2$$

Provides smooth transition from matter dominated epoch to de Sitter epoch of the Universe evolution

Conclusion

At the beginning and by the end of its evolution the Universe is expected to be *empty*

The natural state of the empty homogenous isotropic space is de Sitter expansion

This fact naturally explains the origin of both inflation and dark energy

This theory is consistent with observational data

Marochnik, Usikov & Vereshkov, arXiv: 0811.4484 v2, 2008

Vereshkov & Marochnik, arXiv: 1108.4256, 2011;
J. Mod. Phys., 4, 285, 2013;

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