CONSTRAINTS ON PRIMORDIAL MAGNETIC FIELDS WITH CMB ANISOTROPIES

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SCIENTIFIC CASE



LARGE SCALE STRUCTURE MAGNETIC FIELDS

In 1949 the first observation of a diffuse magnetic field in our galaxy!

Evidence of the presence of large scale magnetic fields in large scale structure especially in galaxies and galaxy clusters.

Amplitudes-- few to several µGauss

> Evidence for their presence at high redshifts (Bernet et al. 2008, Wolfe et al. 2008)



M51 polarization, credits: MPIfR Bonn

Coherence Lenght-- depending on the host size, up to Mpc scale for clusters

Magnetic fields generated in the early universe may represent initial seeds which -amplified by structure formation- may contribute to the generation of the observed large scale magnetic fields

See Ryu 2011 for a review

CLUES FROM HIGH ENERGY?

High energy TeV photons from blazar interact with the background photons producing pair cascades. If there is the presence of diffuse magnetic fields on cosmological scales the charged component of the cascade interacts with the magnetic fields reducing the flux of secondary photons in the GeV range from the blazar (*Dolag et al. 2000*).

FERMI observations of the Blazar 1ES 0229+200 presents a lack of flux of GeV photons with respect to the predictions.

One of the possible interpretations of this lack of photons is the presence of a diffuse magnetic field not associated with existing structures.

Assuming a model for the background light and the Blazar emission it is possible to derive hypothetical *lower limits* on the field amplitude.

Current limits are of the order of: $10^{-18} - 10^{-15}$ Gauss. Dedicated observations will come in the next years . (*Neronov & Vovk 1010, Tavecchio et al. 2010, Taylor et al. 2011, Vovk et al. 2012*).

EARLY UNIVERSE

- PMF may be generated in the early universe!
- PMF might have been created by causal mechanisms such as phase transitions *Vachaspati 1991, Joyce & Shaposhnikov 1997*
- But may be created also during inflation...PMF cannot be generated by the expanding universe as for nearly massless minimally coupled scalar fields or gravitons because of conformal invariance within Einstein gravity. *Giovannini & Shaposhnikov 2000, Martin & Yokoama 2007, Demozzi et al. 2009*
- Conformal invariance has to be broken during inflation ... *Turner & Widrow 1988, Garretson, Field & Carroll 1992*
- or afterwards during (p)reheating through the coupling with the inflaton or other charged fields ...
 Finelli & Gruppuso 1999, Calzetta & Kandus 2002, Garcia Bellido et al. 2008, Byrnes et al. 2012
- or it can be broken in scenarios alternative to Einstein gravity (as by the coupling with the dilaton).
 Ratra 1988, Gasperini, Giovannini & Veneziano 1995....



PRIMORDIAL MAGNETIC FIELDS

DIRECTLY FROM THE EARLY UNIVERSE HILLS GROWN AT THE LIGHT OF THE BIG BANG AND AGED UNDER THE SOFT LIGHT OF THE CMB Magnetic fields can be generated in the early universe through many mechanisms....

PMF represent a new observational window on the early universe

Magnetic fields in the early universe may contribute to the generation of large scale magnetic fields observed in galaxy and galaxy clusters ...

Cosmological magnetic fields not associated with existing structures may provide an interpretation to the FERMI data on Blazar 1ES0229+200....

PRIMORDIAL MAGNETIC FIELDS (PMF) LIKE A GOOD WINE ARE BECOMING MORE AND MORE INTERESTING WITH PASSING TIME

THE CMB AND THE PMF

VERY SMALL DEDICATED LITERATURE... THIS IS ONLY A SMALL SUBSET...

SCALAR

- Adams et al. 1996
- Koh & Lee 2000
- Grasso & Rubinstein 2001
- Bonvin & Caprini 2010
- Kahniashvili & Ratra 2007
- Giovannini 2004,2006,2006/2
- Giovannini & Kunze 2008
- Yamazaki et al. 2005,2006
- Finelli et al. 2008

REVIEWS

- Subramanian 2010
- Durrer 2007
- Giovannini 2004
- Caprini 2011

VECTOR

- Caprini & Durrer 2003
- Subramanian et al 2003
- Kojima et al. 2008
- Lewis 2004
- Subramanian & Barrow 2002

TENSOR

- Durrer et al. 2000
- Lewis 2000
- Mack et al. 2002
- Paoletti et al. 2009
- Kojima & Ichiki 2009

ALL

- Shaw & Lewis (2010,2012)
- Giovannini 2006/3,2006/4,2007,2009,2009/2
- Giovannini & Kunze 2008/2,2008/3
- Kahniashvili et al.2010

- Yamazaki et al. 2005,2010,2011,2012
- Paoletti & Finelli 2011,2012
- Ballardini, Finelli, Paoletti 2015

PMF affect the evolution of cosmological perturbations and therefore have a direct impact on CMB anisotropies.

In addition PMF presence may have also an indirect effect on the CMB polarization

PMF MODELLED AS A STOCHASTIC BACKGROUND PMF may affect CMB anisotropies on three level

CMB ANGULAR POWER SPECTRA IN TEMPERATURE AND POLARIZATION

FARADAY ROTATION

NON-GAUSSIANITIES, CMB BISPECTRA AND TRISPECTRA

The CMB, with its different probes combined in a single observable, represents one of the best laboratories to investigate PMF nature and constrain their characteristics. A **stochastic background** of PMF represent and extra component to the cosmological fluid which adds to matter, neutrinos and radiation. Although PMF are a radiation-like component, their behaviour is completely different.

We can neglect all the contributions to the background.



MAGNETICALLY INDUCED PERTURBATIONS

Cosmological perturbations are described by the coupled system of Einstein equations for metric perturbations and the Boltzmann equations for the fluid

perturbations

 $\delta G_{\mu\nu} = 8\pi (\delta T_{\mu\nu} + \tau_{\mu\nu}^{PMF})$

MAGNETIC ENERGY MOMENTUM TENSOR

$$\begin{aligned} \tau_0^0 &= -\rho_B = -\frac{|\vec{B}|^2}{8\pi G} \\ \tau_i^0 &= \frac{\vec{E} \times \vec{B}}{8\pi G} = 0 \\ \tau_j^i &= \frac{1}{4\pi G} \left(\frac{|\vec{B}|^2}{2} \delta_j^i - \vec{B^i} \vec{B_j} \right) \end{aligned}$$

PERTURBED METRIC TENSOR FLUID PERTURBED ENERGY MOMENTUM TENSOR

Lorentz force term in baryons equations

 $F^{\mu
u}J_{\mu}$ $\nabla_{\mu}\delta T^{\mu\nu}$

The tight coupling of baryons and photons induce and indirect contribution of the Lorentz force also on photon velocity

PMF generates independent modes which are completely sourced by the PMF energy momentum tensor components.

> THE PMF EMT IS THE KEY TO MAGNETIC PERTURBATIONS

PRIMORDIAL MAGNETIC FIELDS ENERGY MOMENTUM TENSOR



STOCHASTIC BACKGROUND OF PMF

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{k'}) \rangle = \frac{(2\pi)^3}{2} \delta^{(3)}(\mathbf{k} - \mathbf{k'}) \left[(\delta_{ij} - \hat{k}_i \hat{k}_j) P_B(\mathbf{k}) + i \epsilon_{ijl} \hat{k}_l P_H(\mathbf{k}) \right]$$

NON-HELICAL PART

HELICAL PART

Power-law power spectrum $P_B(k) = Ak^{n_B} P_H(k) = A_H k^{n_H}$.

Magnetized perturbations survive silk damping but are suppressed on smaller scales. The damping scale can be model as dependent on the field amplitude.

Subramanian and Barrow 1997, Jedamzik et al 1997

$$k_D = (2.9 \times 10^4)^{\frac{2}{n_{B+5}}} \left(\frac{B_{\lambda}}{nGauss}\right)^{\frac{-2}{n_{B+5}}} \left(\frac{k_{\lambda}}{1Mpc^{-1}}\right)^{\frac{n_{B+3}}{n_{B+5}}} \Omega_b h^2 Mpc$$

 $\langle \vec{B}_i^*(\vec{k})\vec{B}_j(\vec{k'})\rangle = \begin{cases} \delta^3(\vec{k}-\vec{k'})(\delta_{ij}-\hat{k}_i\hat{k}_j)\frac{P_B(k)}{2} & \text{for } k < k_D\\ \text{for } k > k_D \end{cases}$

PARAMETRIZATIONS AND MHD

RMS OF THE FIELDS

SMOOTHED FIELDS

Used to have a reference scale, usually 1 Mpc

HELICAL COMPONENT

$$\langle B^{2}(x) \rangle = \int_{k < k_{D}} d^{3}k P_{B}(k) = \frac{4\pi A}{n_{B} + 3} \frac{k_{D}^{n_{B} + 3}}{k_{*}^{n_{B}}}$$
$$\langle B^{2}_{\lambda}(x) \rangle = \int d^{3}k e^{-\lambda^{2}k^{2}} P_{B}(k) = 2\pi A \frac{\Gamma\left(\frac{n_{B} + 3}{2}\right)}{\lambda^{n_{B} + 3}}$$

$$\langle \mathcal{B}_{\lambda}^2 \rangle = \lambda \int_0^\infty \frac{\mathrm{d}k \, k^3}{2\pi^2} \mathrm{e}^{-k^2 \lambda^2} |P_H(k)| = \frac{|A_H|}{4\pi^2 \lambda^{n_H+3}} \, \Gamma\left(\frac{n_H+4}{2}\right)$$

 $n_B > -3$ to avoid divergences

MHD approximation can be considered good in the cosmological fluid

 $\rho_B(x,\tau) = \frac{\rho_B(x)}{a^4(\tau)} \to B(x,\tau) = \frac{B(x)}{a^2(\tau)}$

Conservation equations of the fields give: Analogous relation for vectors between anisotropic stress and lorentz force

What we need to predict the CMB angular power spectrum are the scalar, vector and tensor components of the EMT, plus the Lorentz force in Fourier space PMF anisotropic stress

$$\sigma_B = \frac{\rho_B}{3} + L$$

$$\begin{aligned} \tau_0^0 &= -\rho_B = -\frac{|\vec{B}|^2}{8\pi G} \\ \tau_i^0 &= \frac{\vec{E} \times \vec{B}}{8\pi G} = 0 \\ \tau_j^i &= \frac{1}{4\pi G} \left(\frac{|\vec{B}|^2}{2} \delta_j^i - \vec{B^i} \vec{B_j} \right) \end{aligned}$$

PMF EMT FOURIER SPECTRA

Are given by complex convolutions of the fields. With the development of a dedicated integration technique we have derived the analitical solutions

Magnetized CMB angular power spectrum strongly depends on the behavior of the PMF EMT components in the infrared limits k->0





 n_B $(n_H) = -3/2, -1, 0, 1, 2, 3, 4$ ranging from the solid to the longest dashed

$$\begin{split} n_{B} &> -3/2 \rightarrow white \quad noise \\ n_{B} &< -3/2 \rightarrow k^{(2n+3)} \\ |\rho_{B}(k)|_{n_{B}=2}^{2} &= \frac{A^{2}k_{D}^{7}}{512\pi^{4}k_{*}^{4}} \left[\frac{4}{7} - \tilde{k} + \frac{8\tilde{k}^{2}}{15} - \frac{\tilde{k}^{5}}{24} + \frac{11\tilde{k}^{7}}{1240} \right], \\ |\Pi_{B}^{(V)}(k)|_{n_{B}=2}^{2} &= \frac{A^{2}k_{D}^{7}}{256\pi^{4}k_{*}^{4}} \left[\frac{4}{15} - \frac{5\tilde{k}}{12} + \frac{4\tilde{k}^{2}}{15} - \frac{\tilde{k}^{3}}{12} + \frac{7\tilde{k}^{5}}{960} - \frac{\tilde{k}^{7}}{1920} \right], \\ |\Pi_{B}^{(T)}(k)|_{n_{B}=2}^{2} &= \frac{A^{2}k_{D}^{7}}{256\pi^{4}k_{*}^{4}} \left[\frac{8}{15} - \frac{7\tilde{k}}{6} + \frac{16\tilde{k}^{2}}{15} - \frac{7\tilde{k}^{3}}{24} - \frac{13\tilde{k}^{5}}{480} + \frac{11\tilde{k}^{7}}{1920} \right]. \end{split}$$

Paoletti et Al. 2009

Non-Helical vs Maximum Helica

Ballardini, Finelli and Paoletti 2015



MAGNETICALLY INDUCED ANGULAR POWER SPECTRA



INITIAL CONDITIONS

Magnetically induced perturbations are divided into different kinds depending on their initial conditions. Different types of initial conditions source modes with completely different nature.

- **Compensated:** Magnetized modes which are the solutions to the Einstein-Boltzmann equations (in the radiation era for large wavelengths) sourced by PMF energy mometum tensor after neutrino decoupling. The «compensated» definition comes from the compensation of magnetic terms by the fluid perturbations in the solutions of the equation. *(Giovannini 2004, Lewis 2004, Finelli et al. 2008, Paoletti et al. 2009)*
- **Passive:** This mode is generated prior to the neutrino decoupling when the anisotropic stress of PMF has no counterpart on the fluid. Therefore we have a not-compensated source in the metric perturbation equations which have an extra logarithmic growing mode solution. After neutrino decoupling their anisotropic stress turns on and we fall back in the compensated case. But a footprint of the logarithmic pre-decopling mode remains in the form of an offset in the amplitude of the inflationary mode. It affects only scalar and tensor perturbations. It depends on the ratio: $\ln(\tau_B/\tau_v)$ (*Shaw and Lewis 2010*)
- Inflationary: This mode is strictly related to inflationary generated fields and is dependent on the coupling f F² and on the angle between the hypermagnetic field and the electromagnetic field (*Bonvin et al. 2011,2013*)

Primary modes



NON-HELICAL COMPENSATED MAGNETICALLY INDUCED MODES





Planck 2015 Results XIX

NON-HELICAL MAGNETIZED CMB ANGULAR POWER SPECTRA



Planck 2015 Results XIX

HELICAL MAGNETIZED CMB ANGULAR POWER SPECTRA MAXIM





$$A_B = A_H$$

 $n_B = n_H$

The presence of an extra term in the energy momentum tensor diminishes the PMF contribution for the helical case

The antisymmetric part of the helical component generates non-zero ODD CMB cross correlator TB and EB



PLANCK 2015 CONSTRAINTS ON PMF

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Planck 2015 results. XIX. Constraints on primordial magnetic fields

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ABSTRACT

We predict and investigate four types of imprint of a stochastic background of primordial magnetic fields (PMFs) on the counic microwave background (CMB) anisotopies: the impact of PMFs on the CMB temperature and polarization spectra, related to their contribution to cosmo-bical aperturbations, the effect on CMB polarization induced by Brander trattion, magnetically-induced nor 2 mo-bical aperturbations, the effect on CMB polarization induced by Brander trattion, magnetically-induced nor and the second stratting of statistical isotropy. We present constraints on the amplitude of PMFs derived from different bioscentra, and the magnetically-induced breaking of statistical isotropy. We present constraints on the amplitude of PMFs derived from different bioscentra isotropy of the statistical isotropy of the statistical isotropy of the statistical isotropy of the constraints of the amplitude of PMFs derived from different bioscentra isotropy of the statistical isotropy of the impact of PMFs on the ionization history of the Universe is included in the statistical isotropy of the s these waves. Together, these results comprise a comprehensive set of constraints on possible PMFs with Planck data.

Key words. magnetic fields - cosmology: cosmic background radiation - early Universe

1. Introduction

* Corresponding author: D. Paoletti paoletti@lasfbo.inaf. f.f. Cosmic magnetism

Magnetic fields are one of the fundamental and ubiquitous components of our Universe. They are a common feature of many



CONSTRAINTS WITH PLANCK LIKELIHOOD I

1.0 Planck TT,TE,EE+lowP:C+P Planck TT+lowP:C+P Planck TT,TE,EE+lowP:C Planck TT,TE,EE+lowP:C			
		$B_{1\mathrm{Mpc}}/\mathrm{nG}$	
	TT, TE, EE+lowP: C	< 4.4	
B/H	TT+lowP: C	< 4.4	
0.4	TT, TE, EE+lowP: C+P	< 4.5	
	TT+lowP: C+P	< 4.5	
0.2	$TT + \tau_{reion}$ prior: C+P	< 4.4	
$B_{1 \mathrm{Mpc}} \left[nG \right]$			
SPECTRAL INDEX	nG	al-	

SPECTRAL INDEX	nG
n _B >0	$B_{1 Mpc} < 0.55$
n _B =2	$B_{1 Mpc} < 0.01$
n _B =-2.9	$B_{1 Mpc} < 2.1$

CONSTRAINTS WITH PLANCK LIKELIHOOD II



Strong degeneracy between the amplitude and the spectral index



Degeneracy between the amplitude and the foreground residual parameters for the Poissonian terms

CONSTRAINTS FOR HELICAL FIELDS

MAXIMALLY HELICAL

The constraint on PMF amplitude with an helical component is

 $B_{1 Mpc} < 5.6 nG$

Which can be translated into a contraint on the amplitude of the helical component

 $\mathcal{B}_{1 \,\mathrm{Mpc}} < 4.6 \,\mathrm{nG}$

The constrains are derived with the Planck TT and lowP likelihood and they include only the even-power spectra



JOINT PLANCK+BICEP 2/KECK Array





 $B_{1\,\mathrm{Mpc}} < 4.7\,\mathrm{nG}$

IMPACT OF THE IONIZATION HISTORY

The presence of PMF modifies the ionization history. This is due to the injection of energy into the plasma caused by the dissipation of the PMF. In particular we have two main mechanisms (*Sethi & Subramanian 2005, Chluba et al. 2015, Kunze & Komatsu 2015*):

AMBIPOLAR DIFFUSION <

→ MHD DECAYING TURBULENCE



Chluba, Paoletti, Finelli ad Rubino-Martin 2015



Chluba, Paoletti, Finelli & Rubino-Martin 2015

Very large effect for blue spectral indices

For blue indices the ambipolar diffusion term dominates whereas red indices are dominated by MHD decaying turbulence

Using this effect the Planck TT+lowP constraints the smoothed amplitude (1 Mpc) of scale invariant PMF ($n_B = -2.9$) are less than 1 nG

CONCLUSIONS

- Ever increasing accuracy of data allows to strongly constraints PMF amplitude.
- In particular, the CMB, carrying different probes within the same observable, is one of the best laboratories to investigate and constrain PMF characteristics.
- A stochastic background of PMF leaves a predictable imprint on CMB anisotropies through scalar vector and tensor constributions both in temperature and polarization.
- Vector contribution is the dominant one for regular compensated modes whereas the passive tensor is the dominant one on large angular scale but only for almost scale invariant PMF.
- It is possible to consider also an helical component in the fields which generates two contributions: the simmetric part lowers the amplitude of magnetically induced power spectra whereas the antisimmetric part generates non-zero odd TB and EB spectra.
- PMF have also a significant impact on the ionization history. Their dissipation injects energy into the plasma raising the ioniation fraction and electron temperature. This effect gives strong contraints on PMF with the CMB polarization.

Planck 2015 likelihood constraints on the amplitude of PMF are of the order of few nG

Model	nG
Planck TT+lowP	$B_{1 Mpc} < 4.4$
Planck TT,TE,EE+lowP	$B_{1 Mpc} < 4.4$
n _B >0	$B_{1 Mpc} < 0.55$
n _B =2	$B_{1 Mpc} < 0.01$
n _B =-2.9	$B_{1 Mpc} < 2.1$
Helical PMF	$B_{1 Mpc} < 5.6$
Planck+BICEP 2/KECK ARRAY	B _{1 Mpc} <4.7
Impact on the ionization history	$B_{1 Mpc} < 1$

PART OF THE WORK PRESENTED HAVE BEEN DONE IN THE FRAMEWORK OF THE PLANCK COLLABORATION



Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.

SUPPORT SLIDES

$$\begin{array}{lll} \langle \Pi^{*(S)}(k)\Pi^{(S)}(k')\rangle &=& |\Pi^{(S)}(k)|^{2}\delta(k-k') \\ \langle \Pi^{*(V)}_{i}(k)\Pi^{(V)}_{j}(k')\rangle &=& \frac{1}{2}|\Pi^{(V)}(k)|^{2}P_{ij}(k)\delta(k-k') \\ \langle \Pi^{*(T)}_{ij}(k)\Pi^{(T)}_{tl}(k')\rangle &=& \frac{1}{4}|\Pi^{(T)}(k)|^{2}\mathcal{M}_{ijtl}(k)\delta(k-k') \end{array} \end{array}$$

SCALAR, VECTOR AND TENSOR MAGNETIZED PERTURBATIONS SOURCE TERMS

 $\begin{aligned} |\rho_B(k)|^2 &= \frac{1}{128\pi^2} \int d^3 p P_B(p) P_B(k-p) (1+\mu^2) & \stackrel{Durrer, Ferreira \& Kahniashvili 2000}{Mack, Kahniashvili \& Kosowsky 2002}\\ |\Pi_i^{(V)}(k)|^2 &= \frac{1}{64\pi^2} \int d^3 p P_B(p) P_B(k-p) [(1+\beta^2)(1-\gamma^2)+\gamma\beta(\mu-\gamma\beta)]\\ |\Pi_{ij}^{(T)}(k)|^2 &= \frac{1}{64\pi^2} \int d^3 p P_B(p) P_B(k-p)(1+2\gamma^2+\gamma^2\beta^2) \end{aligned}$

IN ADDITION THE LORENTZ FORCE SCALAR COMPONENT

$$|L(k)|^{2} = \frac{1}{128\pi^{2}a^{8}} \int d^{3}p \quad P_{B}(p) P_{B}(|\mathbf{k} - \mathbf{p}|)[1 + \mu^{2} + 4\gamma\beta(\gamma\beta - \mu)]$$

$$\mu = \vec{p} \cdot (\vec{k} - \vec{p}) / (p|\vec{k} - \vec{p}|). \qquad \gamma = \hat{k} \cdot \hat{p}, \ \beta = \vec{k} \cdot (\vec{k} - \vec{p}) / (k|\vec{k} - \vec{p}|).$$

Scalar component of anisotropic stress is related to the energy density and lorenz force. Vector component of the lorentz force is related with the vector component of the anisotropic stress

COMPENSATION

A stochastic background of PMF does not have an homogeneous part. In order to solve the metric perturbation equations the magnetic source terms are compensated by magnetic matter perturbations

$$\Phi = -\frac{(-165L_B + (-55 + 28R_{\nu})\Omega_B)}{(28(15 + 4R))} + \frac{(-165L_B + (-55 + 28R_{\nu})\Omega_B)}{(252(15 + 4R))}k^2\tau^2$$
$$\Psi = -\frac{(-165L_B + (-55 + 28R_{\nu})\Omega_B)}{(14(15 + 4R))} + \frac{(-165L_B + (-55 + 28R_{\nu})\Omega_B)}{(126(15 + 4R))}k^2\tau^2$$

CURVATURE COMPENSATED AT FIRST ORDER $\zeta = \frac{(-165L_B + (-55 + 28R_{\nu})\Omega_B)}{(84(15 + 4R))}k^2\tau^2$

PLANCK 2013 VS 2015



MAGNETIZED NON-GAUSSIANITY

A stochastic background of PMF has a fully non-Gaussian contribution to CMB anisotropies. PMF source terms are quadratic in the fields, therefore have a nearly χ^2 distribution (*Brown & Crittenden 2005*).

Higher order statistical moments do not vanish and in particular PMF induce a non zero bispectrum

CMB non-Gaussianity measurements can be used as a probe to constrain PMF.

• Passive tensor mode (generated at the GUT era)

 $B_{1 \,\mathrm{Mpc}} < 2.8 \,\mathrm{nG} \ (95\% \mathrm{CL}, n_{\mathrm{B}} = -2.9)$

• Passive scalar mode (directional bispectrum, also generated at the GUT era)

 $B_{1\,\mathrm{Mpc}} < 4.5\,\mathrm{nG}~(95\%\mathrm{CL}, n_\mathrm{B} = -2.9)$

• Compensated scalar mode

 $B_{1\,\mathrm{Mpc}} < 3.0\,\mathrm{nG}~(95\%\mathrm{CL}, n_\mathrm{B} = -2.9)$

 $B_{1 \,\mathrm{Mpc}} < 0.04 \,\mathrm{nG} \ (95\% \mathrm{CL}, n_{\mathrm{B}} = 2)$

Different techniques used to derive the magnetic bispectrum both for passive and compensated modes.

TENSOR PASSIVE BISPECTRUM

The tensor passive mode is the dominant contribution to the large scale angular power spectrum for scale invariant PMF (n_B =-2.9).

We have considered the magnetized passive tensor bispectrum for l<500 and the squeezed limit configuration in which the passive bispectrum is amplified.

 $\ell_1 \ll \ell_2 \approx \ell_3$

The magnetized bispectrum depends on the sixth power of the fields

Optimal estimator in separable modal methodology (*Shiraishi et. Al 2014, Planck Coll. 2014, Fergusson 2014,Liguori et al. 2014*) The limits on the bispectrum amplitude can be translated into limits for the fields SMICA FG cleaned maps T and E for PMF generated at the Grand Unification scale with n_B =-2.9

 $B_{1 Mpc} < 2.8 nG$

 $A_{\rm bis} \equiv \left(\frac{B_{1\,\rm Mpc}}{3\,\rm nG}\right)^6 \left[\frac{\ln(\tau_{\nu}/\tau_B)}{\ln(10^{17})}\right]^3 ,$

5

DIRECTIONAL BISPECTRUM

Considering the curvature perturbations induced by passive modes

$$\zeta_{\boldsymbol{k}} \approx 0.9 \ln\left(\frac{\tau_{\nu}}{\tau_{B}}\right) \frac{1}{4\pi\rho_{\gamma,0}} \qquad \sum_{ij} \left(\hat{k}_{i}\hat{k}_{j} - \frac{1}{3}\delta_{ij}\right) \int \frac{\mathrm{d}^{3}\boldsymbol{k}'}{(2\pi)^{3}} B_{i}(\boldsymbol{k}') B_{j}(\boldsymbol{k} - \boldsymbol{k}') \,.$$

PMF produce non-vanishing bispectrum of direction-dependence

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Legendre Polynomial

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = \sum_L c_L \Big(P_L(\widehat{\mathbf{k}_1} \cdot \widehat{\mathbf{k}_2}) P_{\Phi}(k_1) P_{\Phi}(k_2) + 2 \operatorname{perm} \Big)$$

The zeroth and the second expansion coefficients are related to the amplitude of magnetic fields:

Constraints on the amplitude for B_{1MPC} [nG] with n_B=-2.9 generated at the GUT scale

	SMICA	NILC	SEVEM	Commander
$B_{1 \mathrm{Mpc}}/\mathrm{nG}$	< 4.5	< 4.9	< 5.0	< 5.0

$$C_0 \approx -2 \times 10^{-4} \left(\frac{D_{\rm IMpc}}{\rm nG} \right) ,$$
$$C_2 \approx -2.8 \times 10^{-3} \left(\frac{B_{\rm IMpc}}{\rm nG} \right)^6$$

SCALAR MAGNETIZED BISPECTRUM

We derived the analytical magnetized compensated scalar bispectrum on large angular scales. The temperature anisotropy for PMF can be written as

$$\frac{\Theta_{\ell}^{(0)}(\eta_0, \mathbf{k})}{2\ell + 1} = \frac{\alpha}{4} \Omega_{\mathrm{B}}(\mathbf{k}) j_{\ell}(\mathbf{k}(\eta_0 - \eta_{\mathrm{dec}})),$$

The magnetized bispectrum depends on the energy density bispectrum

 $\langle
ho_B(\mathbf{k})
ho_B(\mathbf{q})
ho_B(\mathbf{p})
angle = rac{1}{(8\pi)^3}\int rac{d^3 ilde{k}\,d^3 ilde{q}\,d^3 ilde{p}}{(2\pi)^9} \langle B_i(ilde{\mathbf{k}})B_i(\mathbf{k}- ilde{\mathbf{k}})B_j(ilde{\mathbf{q}})B_j(\mathbf{q}- ilde{\mathbf{q}})B_l(ilde{\mathbf{p}})B_l(\mathbf{p}- ilde{\mathbf{p}})
angle.$

Contrary to the passive case for compensated mode there is no a-priori dominant geometrical configuration

By the comparison of the bispectrum and the spectrum it is possible to derive an effective f_{nl} in the local configuration to be compared with the measured (SMICA KSW) one to constrain PMF

$$\begin{split} f_{\rm NL}^{\rm eff} &\simeq \; \frac{3\pi^9 \, \alpha^3}{2304 \, \mathcal{A}^2} \frac{n_{\rm B} (n_{\rm B}+3)^2}{2n_{\rm B}+3} \frac{\langle B^2 \rangle^3}{\rho_{\rm rel}^3} \simeq \\ & 1.2 \times 10^{-3} \, (n_{\rm B}+3)^2 \left(\frac{\langle B^2 \rangle}{(10^{-9} \, {\rm G})^2} \right)^3 \, . \end{split}$$

 $B_{1 \,\mathrm{Mpc}} < 3.0 \,\mathrm{nG} \ (95\% \mathrm{CL}, n_{\mathrm{B}} = -2.9)$

 $B_{1 \,\mathrm{Mpc}} < 0.04 \,\mathrm{nG} \, (95\% \mathrm{CL}, n_{\mathrm{B}} = 2)$

FARADAY ROTATION

The presence of PMF induces a rotation of the polarization plane of CMB anisotropies rotating *E*-mode polarization into *B*-mode and vice versa. The Faraday depth is given by

$$\Phi = K \int n_{\rm e}(x, \boldsymbol{n}) B_{\parallel}(x, \boldsymbol{n}) \, \mathrm{d}\boldsymbol{x}.$$

B and **E** mode polarization rotated spectra

$$\begin{split} C_{\ell}^{BB} &= N_{\ell}^{2} \sum_{\ell_{1}\ell_{2}} \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{4\pi(2\ell+1)} N_{\ell_{2}}^{2} K(\ell,\ell_{1},\ell_{2})^{2} C_{\ell_{2}}^{EE} C_{\ell_{1}}^{\alpha} \left(C_{\ell_{1}0\ell_{2}0}^{\ell 0}\right)^{2} \\ C_{\ell}^{EE} &= N_{\ell}^{2} \sum_{\ell_{1}\ell_{2}} \frac{(2\ell_{1}+1)(2\ell_{2}+1)}{4\pi(2\ell+1)} N_{\ell_{2}}^{2} K(\ell,\ell_{1},\ell_{2})^{2} C_{\ell_{2}}^{BB} C_{\ell_{1}}^{\alpha} \left(C_{\ell_{1}0\ell_{2}0}^{\ell 0}\right)^{2} \\ C_{\ell}^{\Phi} &\approx \frac{9\ell(\ell+1)}{(4\pi)^{3}e^{2}} \frac{B_{\lambda}^{2}}{\Gamma(n_{B}+3/2)} \left(\frac{\lambda}{\eta_{0}}\right)^{n_{B}+3} \int_{0}^{x_{D}} \mathrm{d}x \, x^{n_{B}} \, j_{\ell}^{2}(x). \end{split}$$

$$C_{\ell}^{\alpha} = \nu_0^{-4} C_{\ell}^{\Phi},$$

The *EE* mode from Planck 70 GHz (2<l<29) spectrum has been used to derive the expected *BB* rotated mode. Comparison with measured *B*-modes at 70 GHz computing the minimum χ^{2.}



B_{1 Mpc} < 1380 nG

Estimate of the Galactic contribution, subdominant for our data

