Topology of the Universe from Planck CMB

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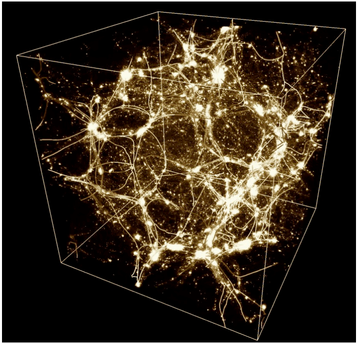
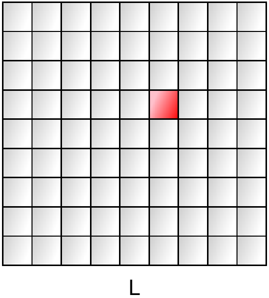
Familiar simulation of the multiconnected toroidal Universe

N-body with periodic boundary conditions



Can be viewed as tiling of infinite space with the copies of the box. This is flat 3-torus space. The box is a “fundamental domain”. Light travels in straight line



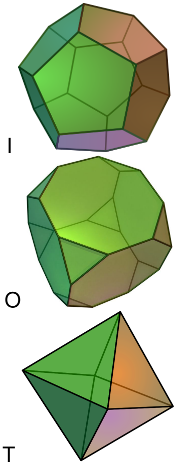
- “Artifacts of the box” at scales $\sim L$
  - Long-wave cutoff in power spectrum
  - Discrete and anisotropic set of modes
- No effects of the box at scales $\ll L$

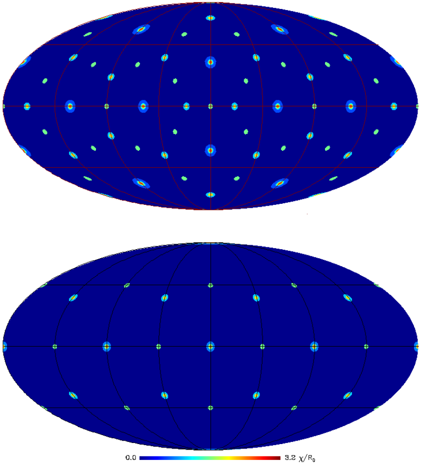
Artifacts of the periodic boundary conditions are observable features if our Universe is indeed 3-torus

CMB provide unique view to large scale organization of the Universe. Can our Universe be multiconnected ?

Tiled with copies of a fundamental domain?

Can we see in different directions repeated images of same objects ?

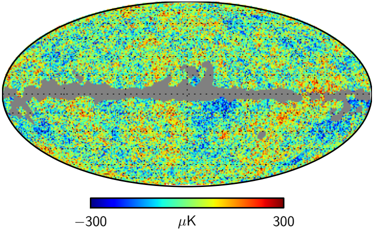




How far do we see ?

Measurements of Cosmic Microwave Background gives the answer –
up to $\approx 14,000$ Megaparsec (χ_{rec}).
Limited by transparency of our evolving Universe in the past

What we observe from these distances are temperature and polarization maps of CMB photons



Planck collaboration, 2013.
Map of CMB temperature fluctuations.

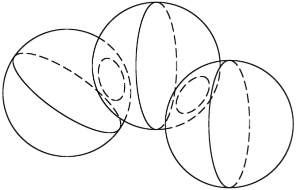
Crucial comparison is of the radius measure of the fundamental domain R_i to the distance travelled by photons χ_{rec}

- $R_i \ll \chi_{rec}$ - photons circumnavigated the Universe multiple times, same regions of space are seem from several distinct directions.
- $R_i \gg \chi_{rec}$ - observed volume is contained entirely within a single fundamental domain, effects of multiconnectivity disappear.

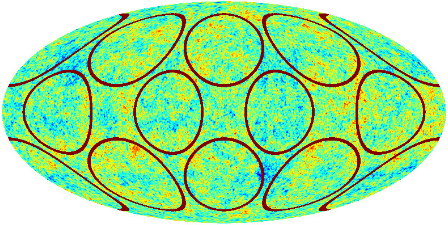
Search for topological signature in CMB maps

Direct search for images

"Circles on the sky"



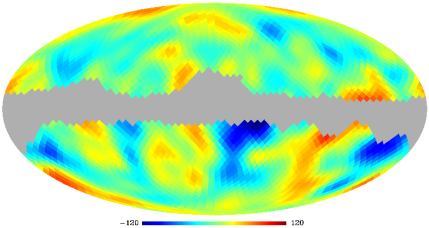
Example for Cubic Torus with $R_i/\chi_{rec} \approx 0.32$



Indirect search

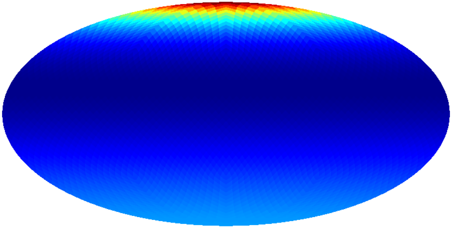
Likelihood analysis of the correlations between the pixels

Planck Smica FWHM=660 arcmin, fsky=0.79



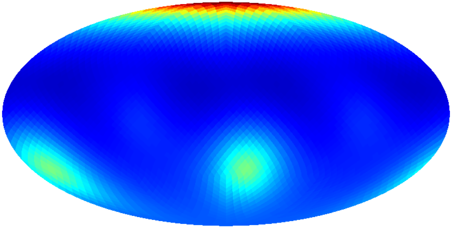
Pixel-pixel correlations and images: Fiducial infinite space

C(North Pole, θ)

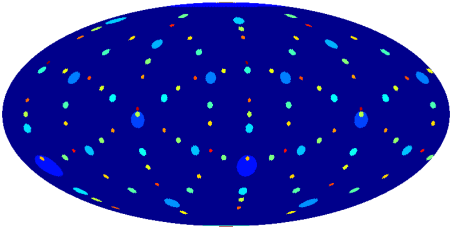


Pixel-pixel correlations and images: χ_{rec} just fits into the compact space

C(North Pole, θ)

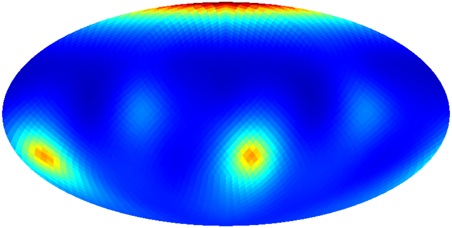


Images of North Pole



Pixel-pixel correlations and images: χ_{rec} just larger than the compact space

C(North Pole, θ)



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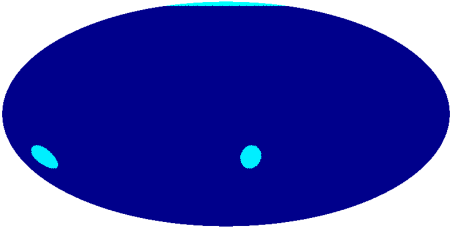

Pixel-pixel correlations and images: χ_{rec} much larger than the compact space

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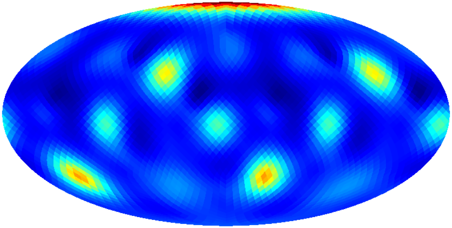

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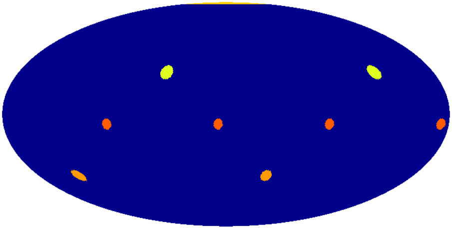


Formalism: Likelihood analysis

Log-likelihood

$$\ln(\mathscr{L}) = -\frac{1}{2}\left[n_p \ln(2\pi) + \ln(\det(\mathbf{C}_\mathrm{T} + \mathbf{N})) + \mathbf{x}^\dagger(\mathbf{C}_\mathrm{T} + \mathbf{N})^{-1}\mathbf{x}\right]$$

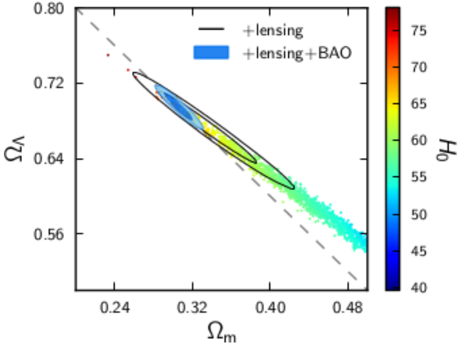
Technical ingredients:

- C_T – theoretical pixel correlation matrix, N – noise correlation function, x – experimental smoothed and masked data.
- Parameters: amplitude of the signal and relative orientation of the data map and theoretical model.
- judicial choice of fiducial modes to project data and model onto is a very important part of the procedure.

Flat and curved FRW models consistent with data still permit Universe to be multiconnected

Flat and closed Universes are allowed by CMB

- $\Omega_k = -0.01^{+0.018}_{-0.019}$ (2013 Planck+lensing+WP+highL) $\Omega_k = -0.01$ is a pretty large value - the volume of the sphere is only 100 times the observed volume to LSS. Or: the curvature radius $R_0 \approx 3.2\chi_{rec}$
- Small scale fluctuations of the CMB are virtually unchanged if Ω_k tracks degeneracy line.

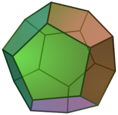
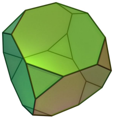
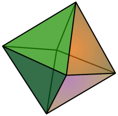


Models analyzed: constant curvature multiconnected spaces

Flat spaces. Size of fundamental domain is continous parameter

Equal and non-equal sides flat $T^3(L_x, L_y, L_z)$ tori, $R_i = L/2 = (0.32 - 1.1) \times \chi_{rec}$.

Spaces of positive curvature. Size of the fundamental domain is linked to the curvature radius

	Dodecahedral	Truncated Cube	Octahedral
			
N	120	48	24
$\mathscr{V}/R_0^3$	0.16	0.41	0.82
$\mathscr{R}_\mathrm{i}/R_0$	0.31 $(\pi/10)$	0.39 $(\pi/8)$	0.45
$\mathscr{R}_\mathrm{m}/R_0$	0.37	0.56	0.56
$\mathscr{R}_\mathrm{u}/R_0$	0.40	0.58	0.79 $(\pi/4)$

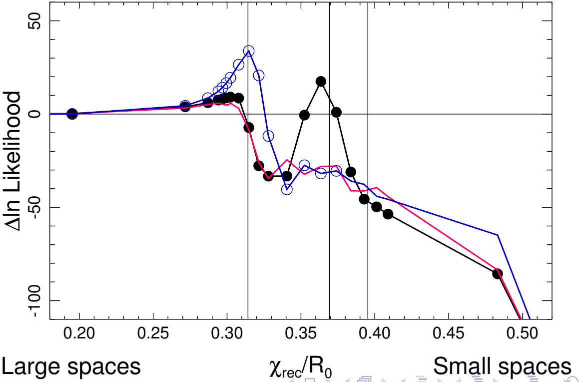
Likelihood detectability of the multiconnected topology

- we compare sequences of multiconnected models that have as its limiting point the fiducial Planck best-fit flat LCDM model
- LnLikelihood is given as the difference with this fixed fiducial model
- For curved spaces we vary the size of the domain by varying the curvature. In addition other parameters as modified to follow the degeneracy line

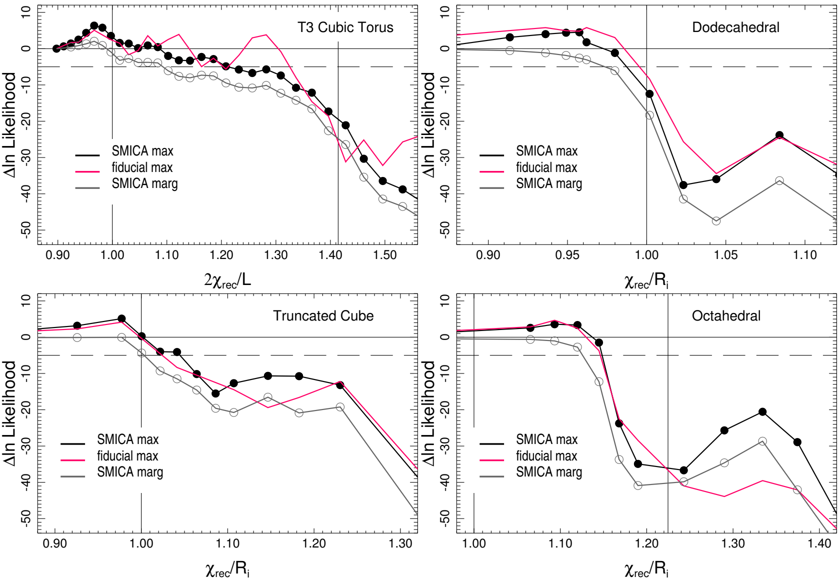
Detecting simulated dodecahedral space

Blue and Black – in two simulated maps indeed drawn from dodecahedral space with two different curvature radii.

Red – in a realization of the fiducial model.



2013 results - no evidence for compact topology

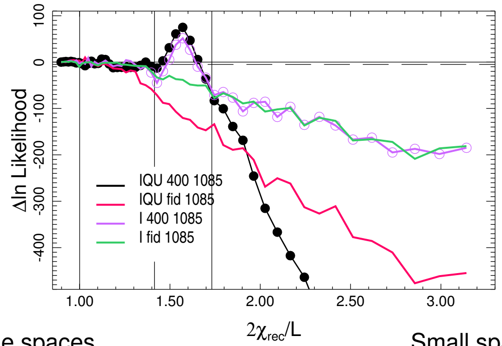


2015 results and future advancement – use of polarization

Detecting simulated toroidal space: strengthened detection and/or rejection of small spaces with polarization

- Black and Red – using polarization
- Blue and Green – temperature only

- Green and Red – rejection
- Blue and Black – detection

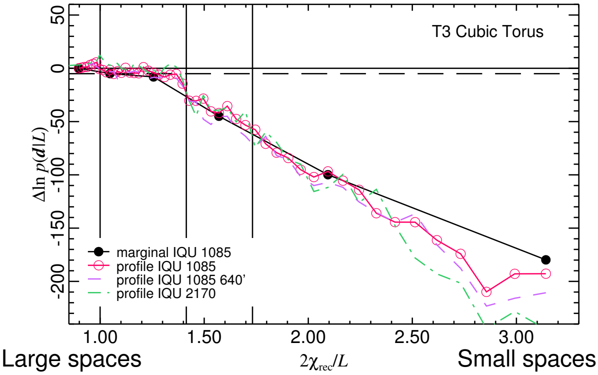


Large spaces

Small spaces

2015 results – use of polarization

2015 limited use of large-scale polarization data: strengthened rejection of small spaces



Lower limits on the size of the fundamental domain for different multiply-connected spaces

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R_i (or $L/2$) $> \chi_{rec}$ means last-scattering sphere fits completely into the fundamental domain of the multiconnected Universe.

Space	Quantity		$\Delta \ln \mathscr{L} < -5$		$\Delta \ln \mathscr{L} < -12.5$	
			max	marg	max	marg
T3 Cubic Torus	$L/(2\chi_{\rm rec})$	$>$	0.83	0.92	0.76	0.83
T2 Chimney	$L/(2\chi_{\rm rec})$	$>$	0.71	0.71	0.63	0.67
T1 Slab	$L/(2\chi_{\rm rec})$	$>$	0.50	0.50	–	–
Dodecahedron	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	1.01	1.03	1.00	1.01
Truncated Cube	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	0.95	1.00	0.81	0.97
Octahedron	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	0.87	0.89	0.87	0.88