

Cosmology using the combination of RSD power and bi spectra

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Cosmo Cruise 2015: The Edge of Universe



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Implication of cosmic acceleration

- Breaking down our knowledge of particle physics: we have limited knowledge of particle physics bounded by testable high energy, and our efforts to explain the cosmic acceleration turn out in vain.

 - Alternative mechanism to generate fine tuned vacuum energy

 - New unknown energy component

 - Unification or coupling between dark sectors

- Breaking down our knowledge of gravitational physics: gravitational physics has been tested in solar system scales, and it is yet confirmed at horizon size,

 - Presence of extra dimension

 - Non-linear interaction to Einstein equation

- Failure of standard cosmology model: our understanding of the universe is still standing on assumptions:

 - Inhomogeneous models: LTB, back reaction

Theoretical models to explain acceleration

• Breaking down our knowledge of particle physics: we have

Dynamical Dark Energy: modifying matter

$$G_{\mu\nu} = 4\pi G_N T_{\mu\nu} + \Delta T_{\mu\nu}$$

Alternative mechanism to generate fine tuned vacuum energy

New unknown energy component

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Geometrical Dark Energy: modifying gravity

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Key observables in cosmological science

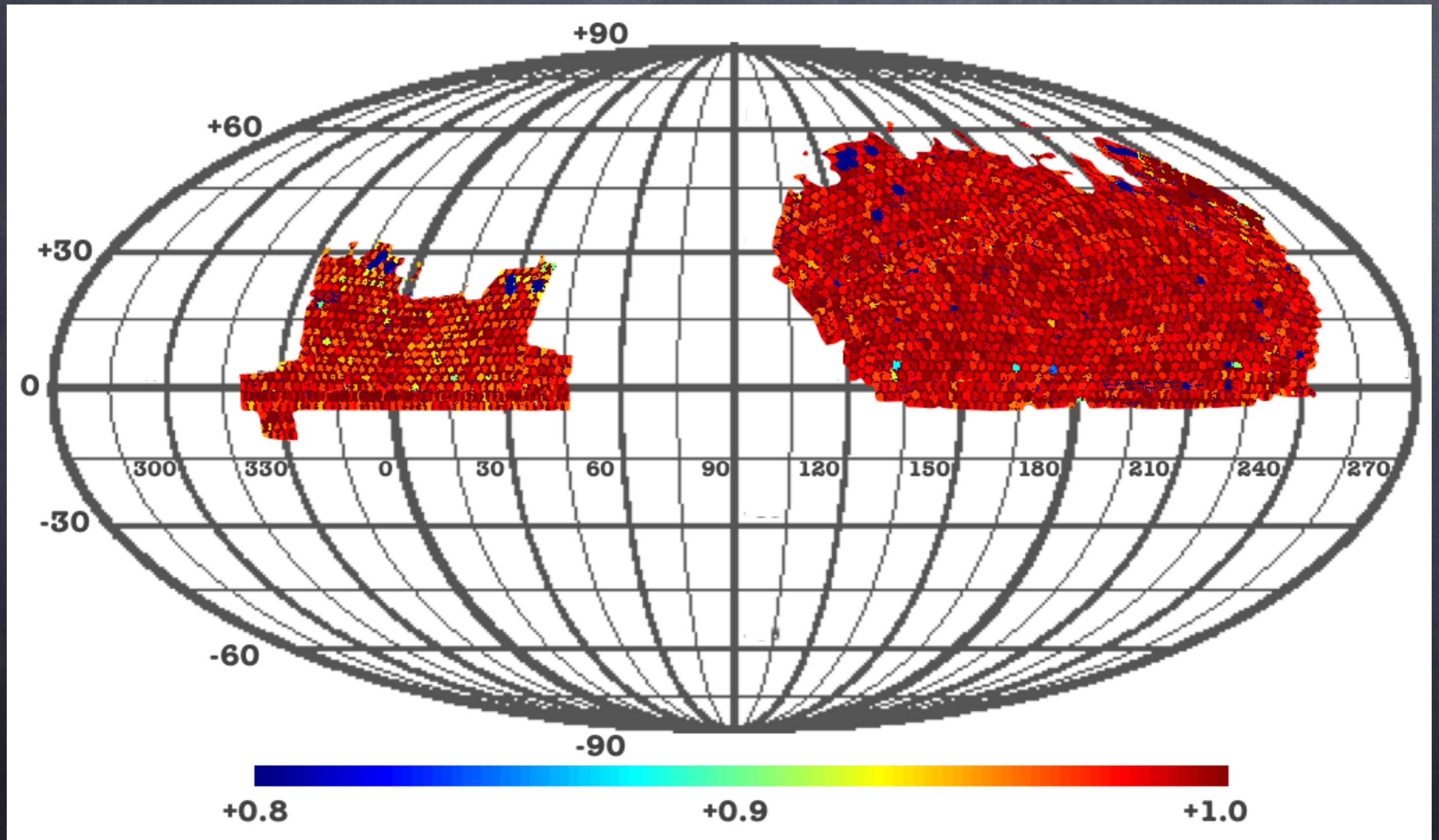
Angular diameter distance D_A : Exploiting BAO as standard rulers which measure the angular diameter distance and expansion rate as a function of redshift.

Radial distance H^{-1} : Exploiting redshift distortions as intrinsic anisotropy to decompose the radial distance represented by the inverse of Hubble rate as a function of redshift.

Coherent motion G_θ : The coherent motion, or flow, of galaxies can be statistically estimated from their effect on the clustering measurements of large redshift surveys, or through the measurement of redshift space distortions.

Spectroscopy wide deep field survey

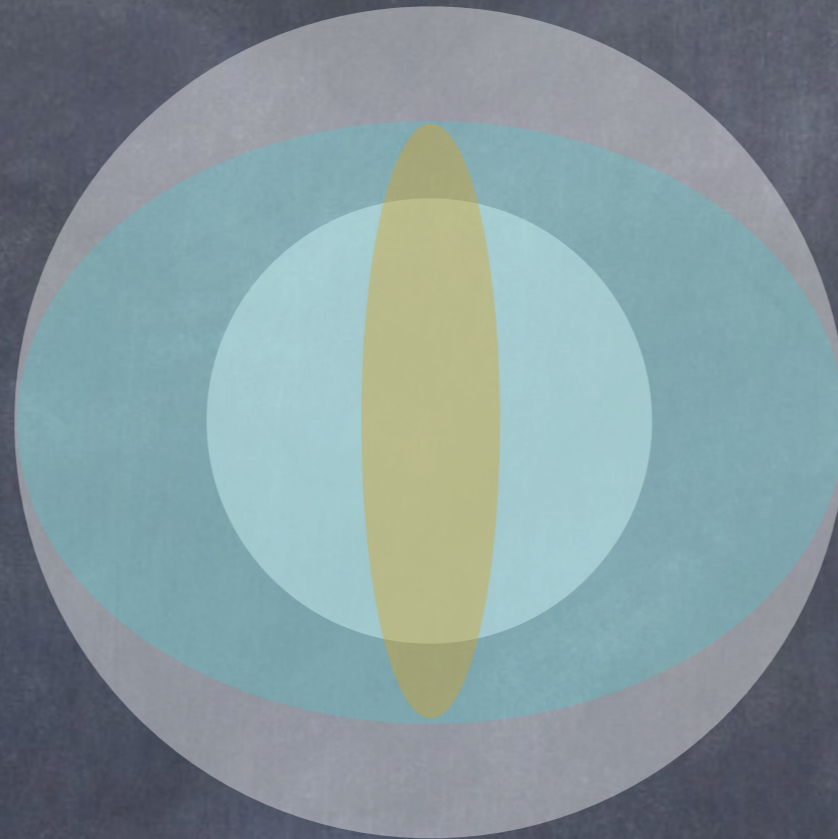
BOSS DR11 catalogue



Structure formation

Squeezing effect
at large scales

(Kaiser 1987)



Finger of God
effect at small
scales

(Jackson 1972)

$$P_s(k, \mu) = P_{gg}(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)$$



$$P_s(k, \mu) = [P_{gg}(k) + \Delta P_{gg} + 2\mu^2 P_{g\theta}(k) + \Delta P_{g\theta} + \mu^4 P_{\theta\theta}(k) + \Delta P_{\theta\theta} + \mu^2 A(k) + \mu^4 B(k) + \mu^6 C(k) + \dots] \exp[-(k\mu\sigma_p)^2]$$

Theoretical model in configuration space

$$P_s(k, \mu) = [Q_0(k) + \mu^2 Q_2(k) + \mu^4 Q_4(k) + \mu^6 Q_6(k)] \exp[-(k\mu\sigma_p)^2]$$

$$\xi(\sigma, \pi) = \int d^3k P(k, \mu) e^{ikx} = \sum \xi_\ell(s) \mathcal{P}_\ell(v)$$

$$\xi_\ell(s) = i^\ell \int k^2 dk P_\ell(k) j_\ell(ks)$$

$$P_0(k) = p_0(k)$$

$$P_2(k) = 5/2 [3p_1(k) - p_0(k)]$$

$$P_4(k) = 9/8 [35p_2(k) - 30p_1(k) + 3p_0(k)]$$

$$P_6(k) = 13/16 [231p_3(k) - 315p_2(k) - 105p_1(k) + 5p_0(k)]$$

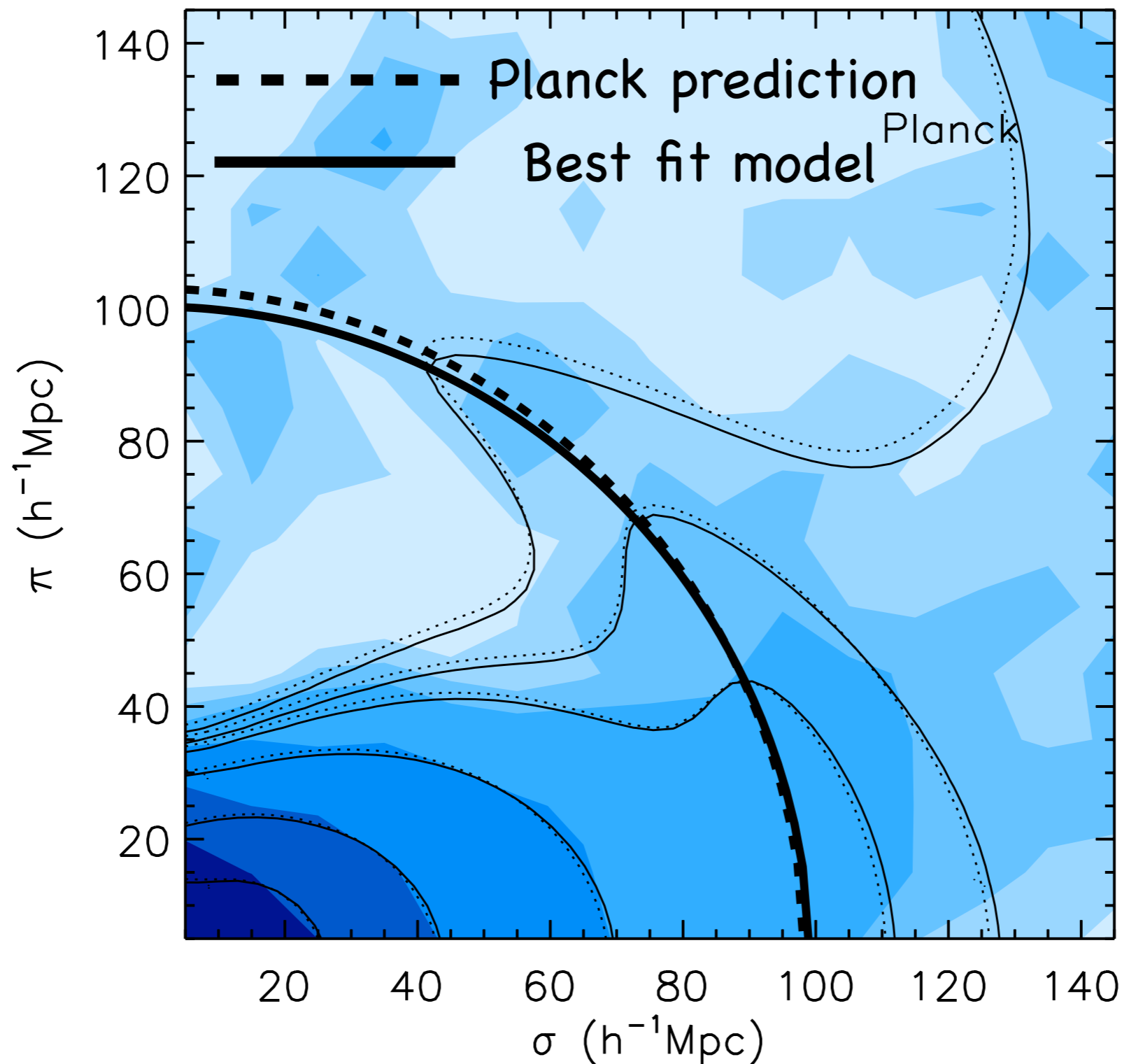
⋮

$$p_n(k) = 1/2 [\gamma(n+1/2, \kappa) / \kappa^{n+1/2} Q_0(k) + \gamma(n+3/2, \kappa) / \kappa^{n+3/2} Q_2(k) \\ + \gamma(n+5/2, \kappa) / \kappa^{n+5/2} Q_4(k) + \gamma(n+7/2, \kappa) / \kappa^{n+7/2} Q_6(k)]$$

$$\kappa = k^2 \sigma_p^2$$

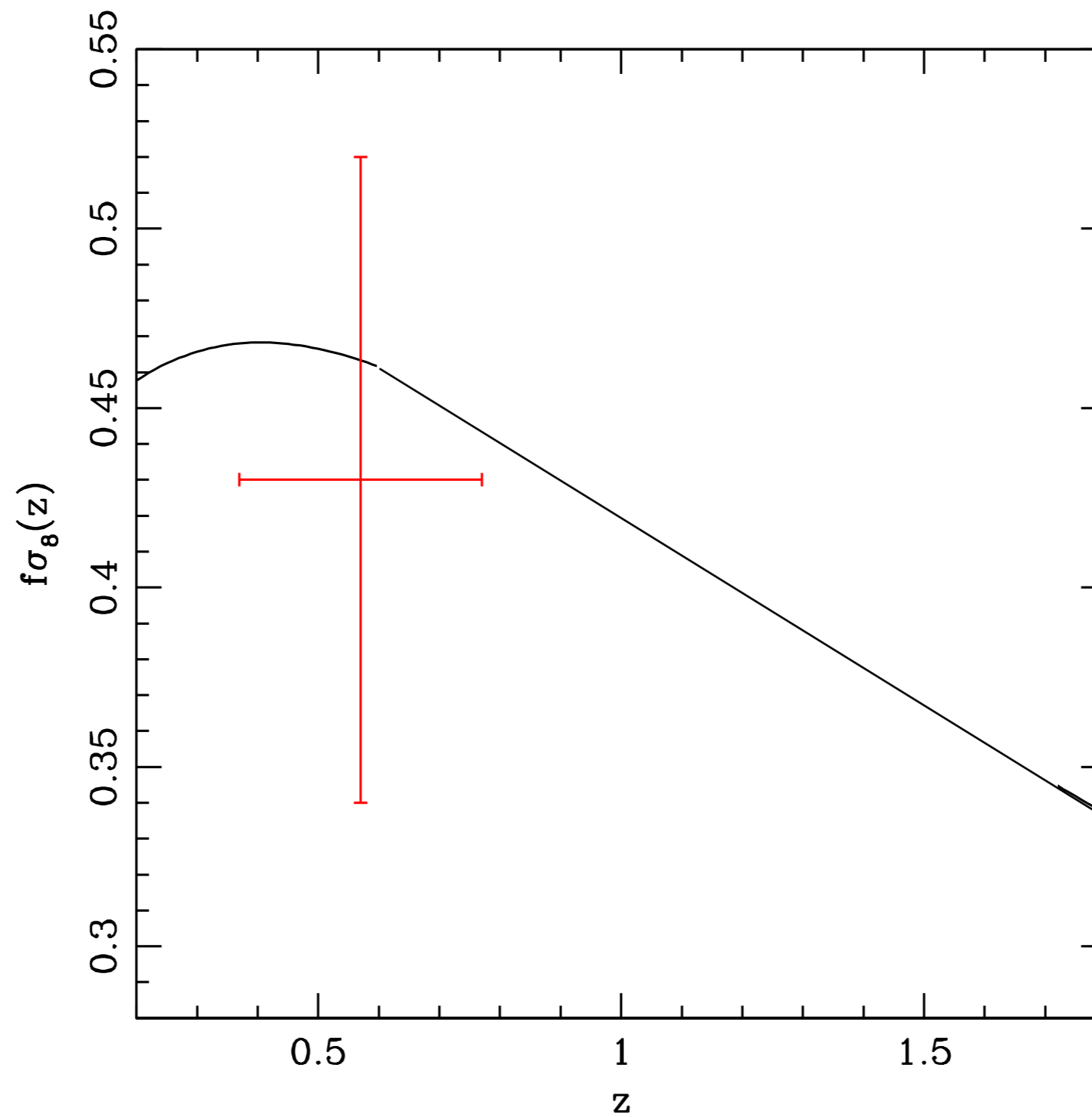
Measured correlation functions using DR11

Parameter space is $(D_A, H^{-1}, G_\delta, G_\theta, \text{FoG})$



Measured coherent motion

Results from BOSS maps



f(R) gravity

Corrections are introduced in the Einstein-Hilbert Lagrangian to modify the general relativity, which gets influential only low curvature, e.g. late time & not dense region. The corrections can be adjusted to generate the cosmic acceleration, Carroll, Duvvuri, Trodden, Turner (2004:CDTT)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2\mu^2} + \mathcal{L}_m \right]$$

cosmic acceleration was discovered with $f(R) = -a/R$. **Ruled out**

Two distinct branches of f(R) gravity was found depending on the sign of second order derivative of f(R) in terms of R,

$$f_{RR} = d^2f/dR^2 < 0 \quad \text{Unstable}$$

$$f_{RR} = d^2f/dR^2 > 0 \quad \text{Stable}$$

The original proposal of CDTT is ruled out due to instability.

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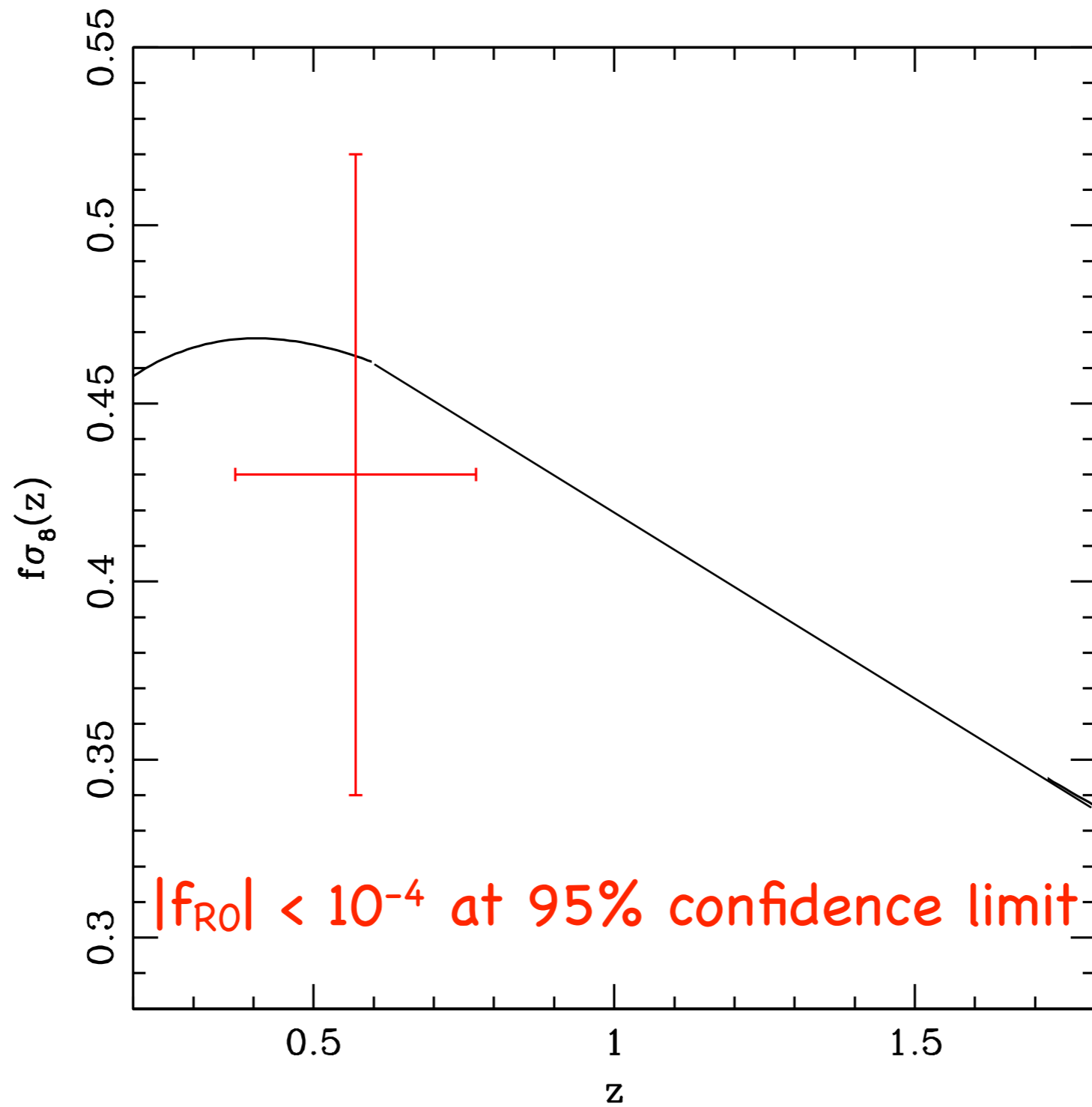
cosmic acceleration was discovered with $f(R) = -a/R$. **Ruled out**

The f(R) gravity model in this talk is given by,

$$f(R) = -2 \kappa^2 \rho_\Lambda + |f_{R0}| R_0^2 / R^2$$

Measured coherent motion

Results from BOSS maps



LSS of $f(R)$ gravity

Dynamic equations of perturbations

$$d\delta_m/dt + \theta_m/a = 0$$

$$d\theta_m/dt + H\theta_m = k^2\psi/a$$

$$k^2\phi = 3/2 H_0^2\Omega_m \delta_m/a F(\epsilon)$$

$$k^2\psi = -3/2 H_0^2\Omega_m \delta_m/a G(\epsilon)$$

which are not closed without knowing ϵ evolution

For the case of DGP, dynamics equations with extra variable are closed with a constraint equation, but for the case of $f(R)$ gravity, it is closed with an extra dynamic equation of ϵ .

$$\epsilon'' + \left(\frac{7}{2} + 4p_B \right) \epsilon' + \frac{2}{B} \epsilon = \frac{1}{B} F(\Phi_-, S, Hq)$$

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which are not closed without knowing ϵ evolution

Mass screening effect:

$$k^2\phi_{fR} = \phi_{GR} F(\epsilon)$$

Geometrical anisotropy:

$$k^2\phi_{fR} + k^2\psi_{fR} = -3H_0^2\Omega_m \delta_m/a [F(\epsilon) - G(\epsilon)]$$

Change on photon trajectory:

$$\phi_{fR} - \psi_{fR} = (\phi_{GR} - \psi_{GR})$$

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Introducing the Brans-Dicke parameter φ

$$\phi_{FR} - \psi_{FR} = \varphi$$

$$k^2\psi = -3/2 H_0^2\Omega_m \delta_m/a - 1/2 k^2\varphi$$

$$(1+w_{BD}) k^2/a^2 \varphi = 3H_0^2\Omega_m \delta_m/a - I(\varphi)$$

where $I(\varphi)$ is given by

$$I(\varphi) = M_1(k)\varphi(k) + 1/2 \int \cdots \int d^3k_1 \cdots d^3k_n M_1(k) \cdots M_n(k) \varphi(k_1) \cdots \varphi(k_n)$$

LSS of $f(R)$ gravity

Dynamic equations of perturbations

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Later time growth functions are given by,

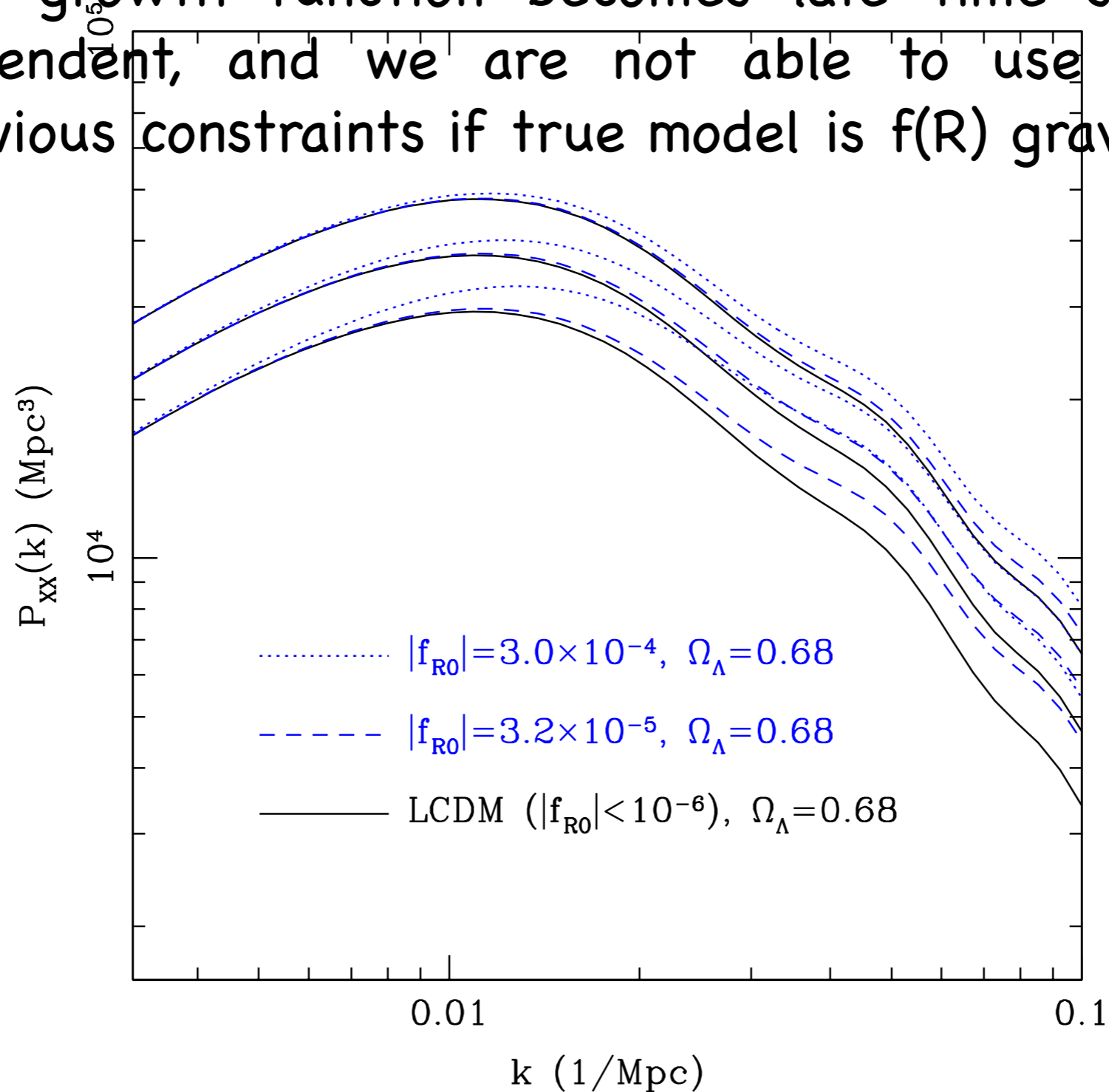
$$D^\delta(k,t) = G_\delta(t) F_\delta(k,t;M_1)$$

$$D^\theta(k,t) = G_\theta(t) F_\theta(k,t;M_1)$$

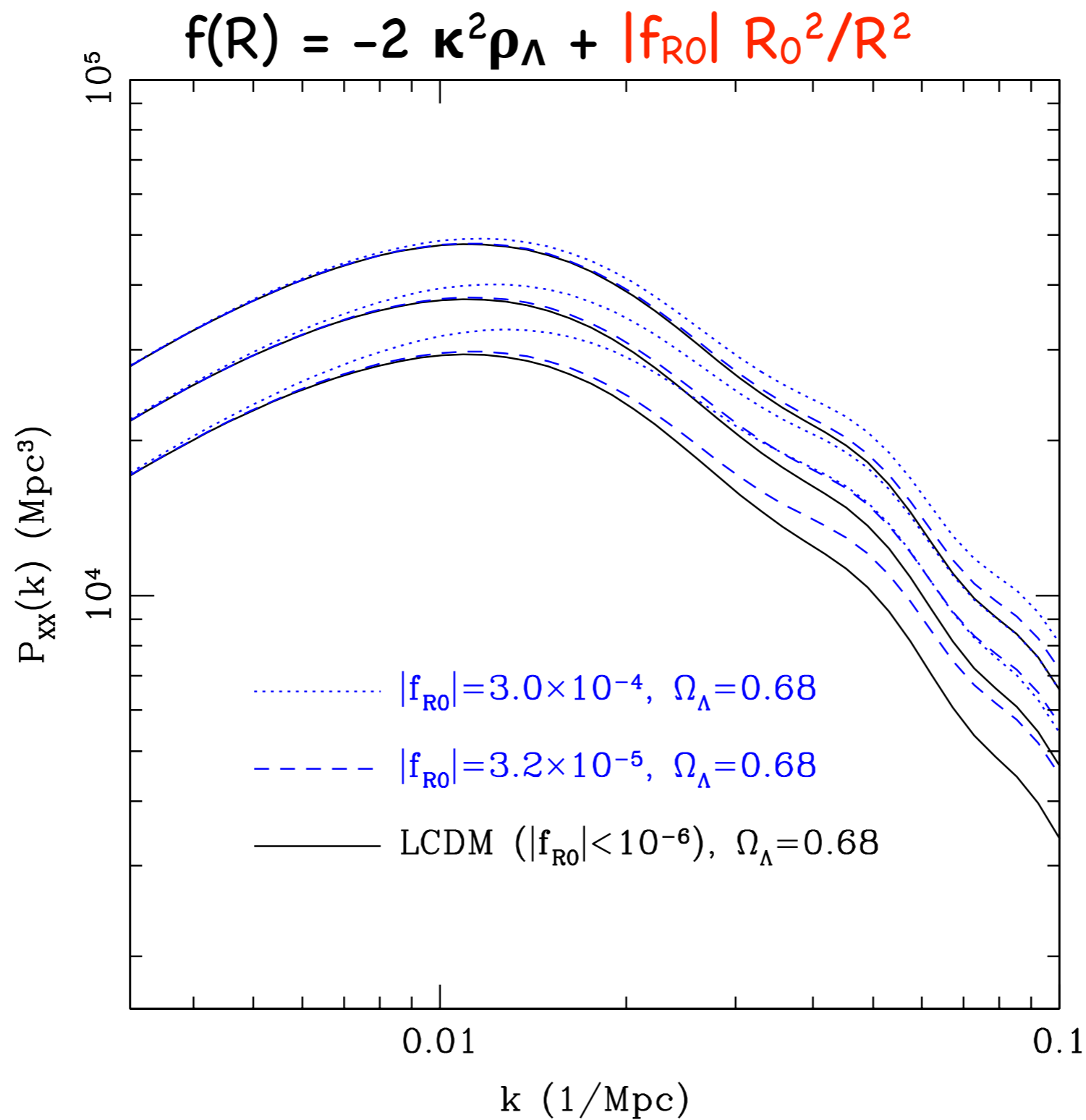
We are not able to constrain $f(R)$ gravity models using measured growth functions with the assumption of coherent growing after last scattering surface.

Linear power spectra with running $f(R)$

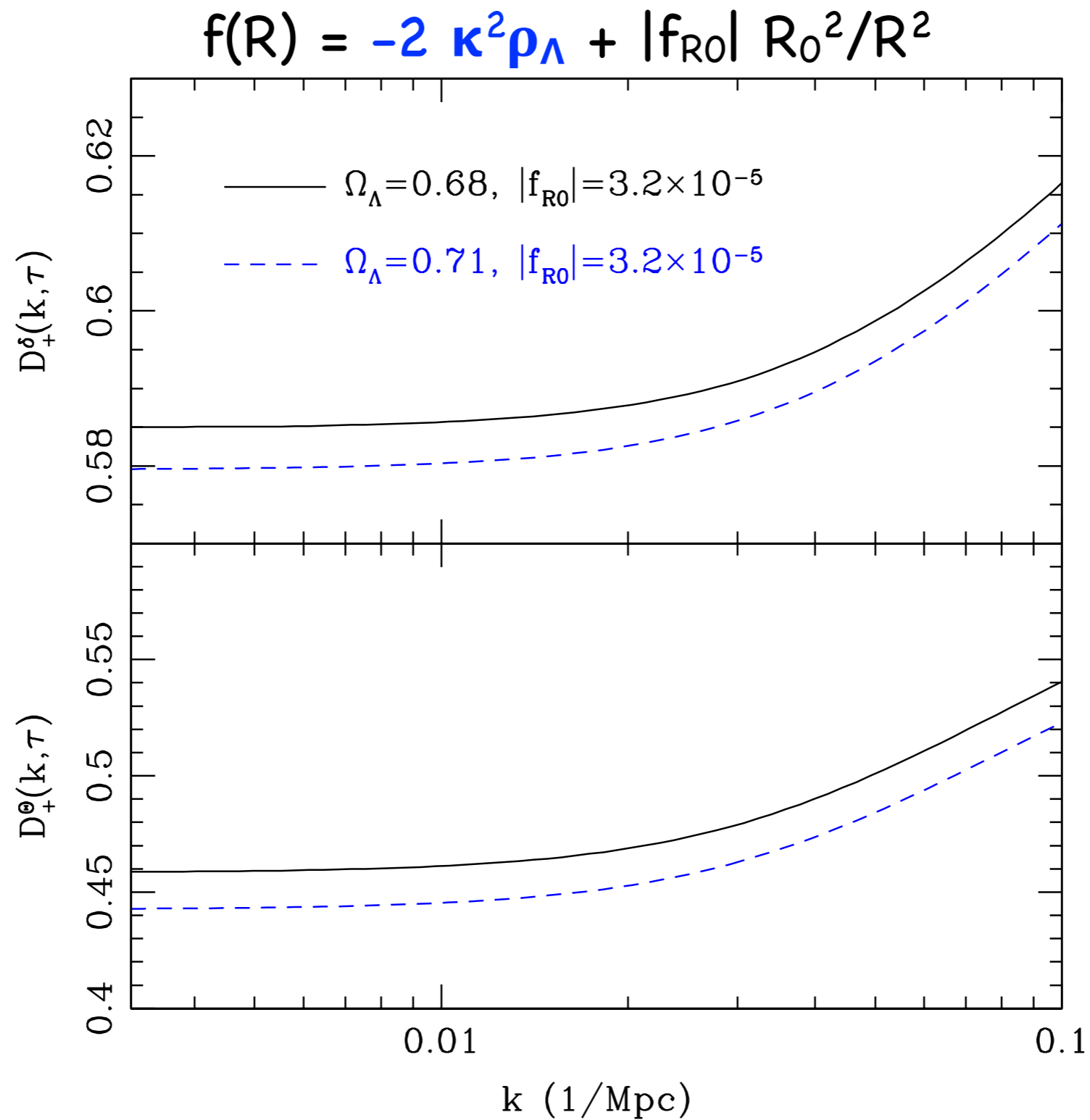
The growth function becomes late time scale dependent, and we are not able to use the previous constraints if true model is $f(R)$ gravity



Parameterisation of $f(R)$ gravity model



Parameterisation of $f(R)$ gravity model



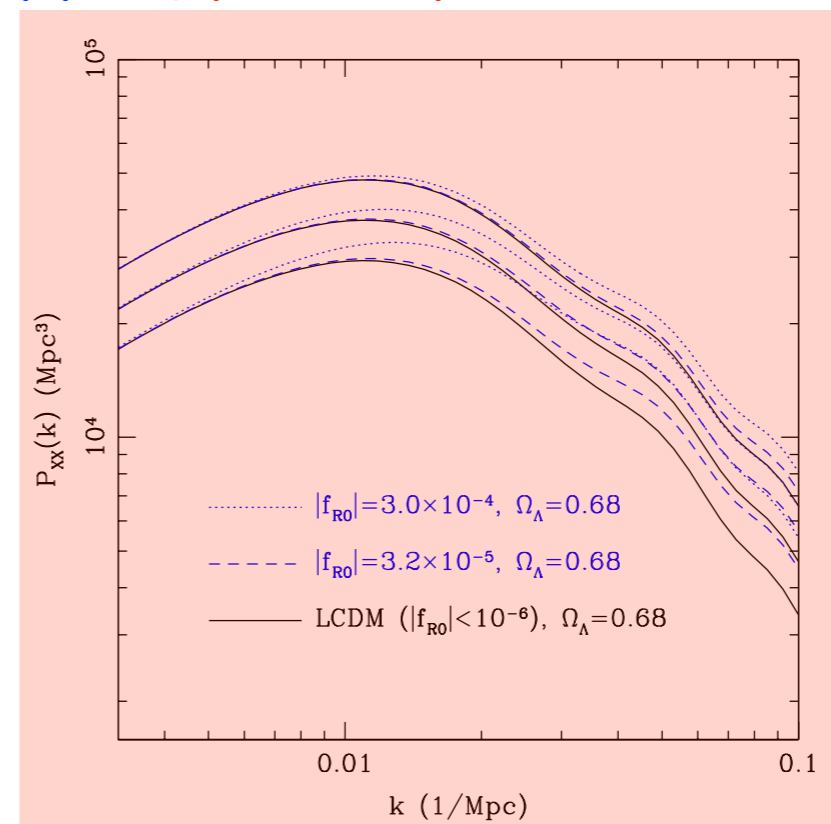
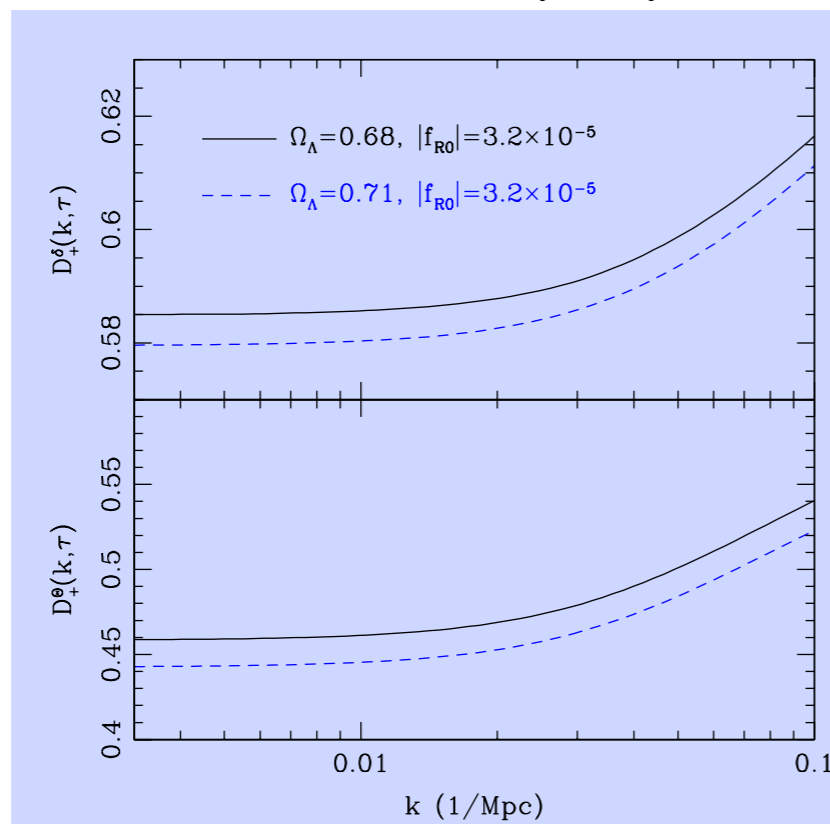
Parameterisation of $f(R)$ gravity model

$$f(R) = -2 \kappa^2 \rho_\Lambda + |f_{R0}| R_0^2 / R^2$$

We find that both coherent growth factors and scale dependent growth factors are separable in the following sense,

$$D^\delta(k, t) = G_\delta(t) F_\delta(k, t; M_1)$$

$$D^\theta(k, t) = G_\theta(t) F_\theta(k, t; M_1)$$



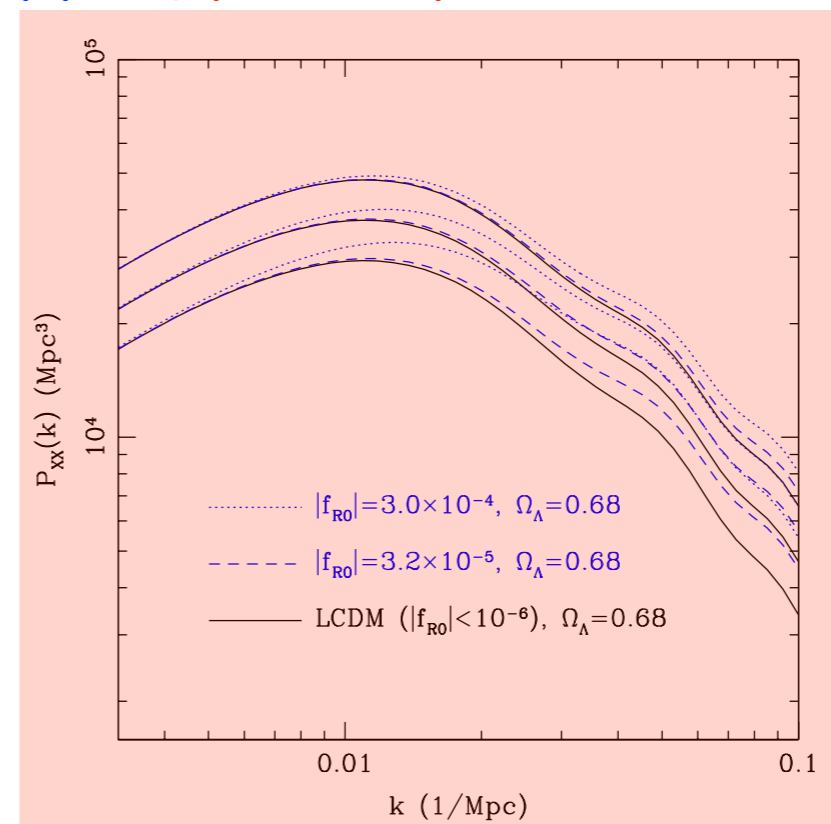
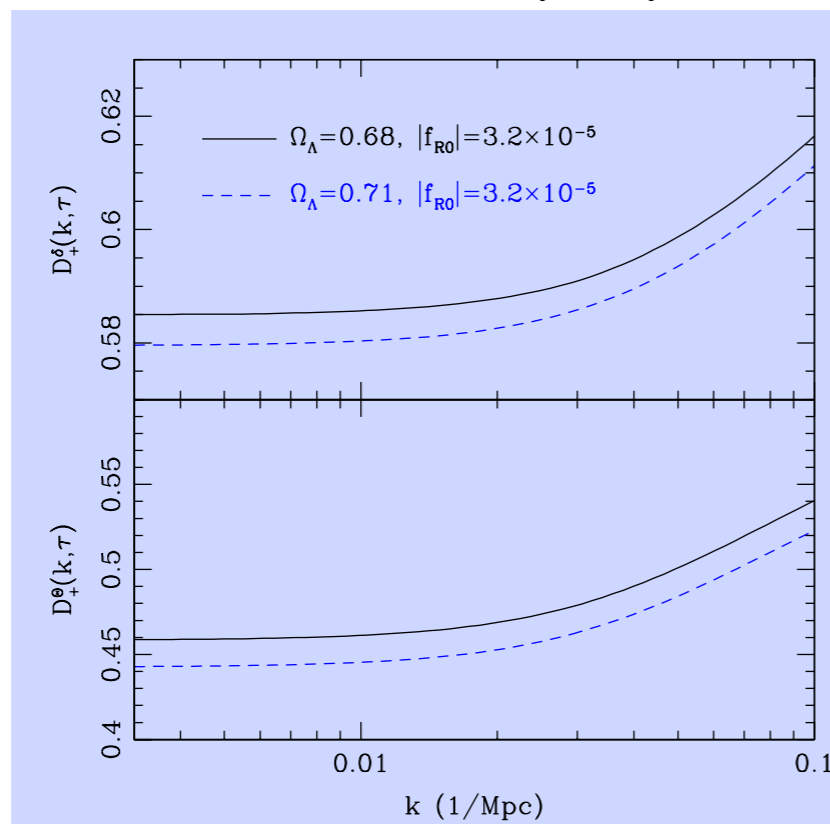
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$$D^\delta(k, t) = G_\delta(t) F_\delta(k, t; M_1)$$

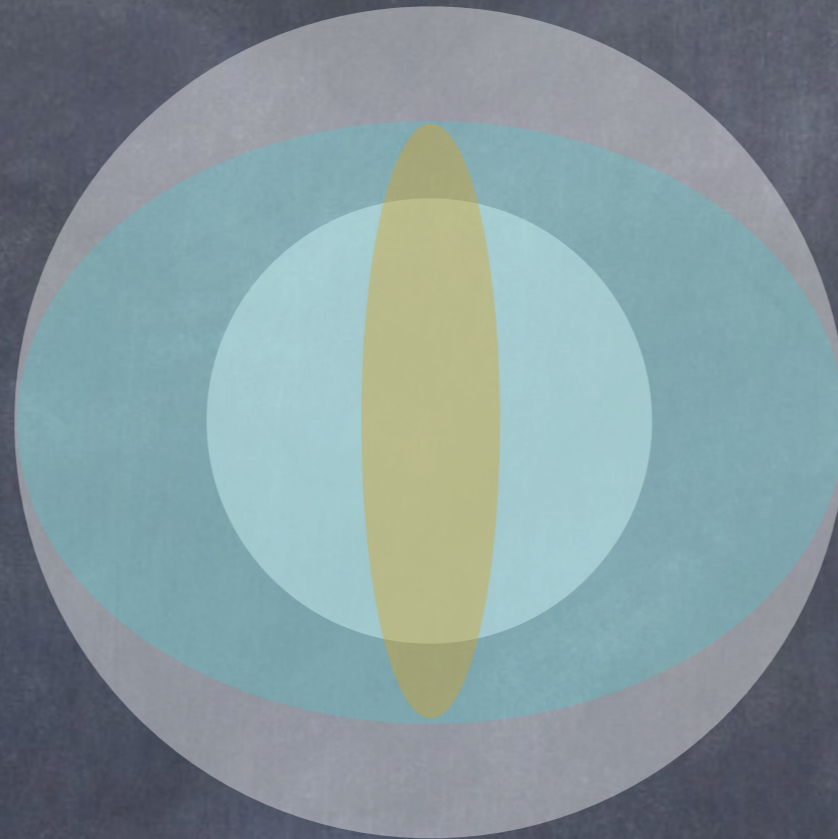
$$D^\theta(k, t) = G_\theta(t) F_\theta(k, t; M_1)$$



Structure formation of RSD

Squeezing effect
at large scales

(Kaiser 1987)



Finger of God
effect at small
scales

(Jackson 1972)

$$P_s(k, \mu) = P_{gg}(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)$$



$$P_s(k, \mu) = [P_{gg}(k) + \Delta P_{gg} + 2\mu^2 P_{g\theta}(k) + \Delta P_{g\theta} + \mu^4 P_{\theta\theta}(k) + \Delta P_{\theta\theta} + \mu^2 A(k) + \mu^4 B(k) + \mu^6 C(k) + \dots] \exp[-(k\mu\sigma_p)^2]$$

Structure formation of RSD

The non-linear solution is derived from

$$d\delta_m/dt + \nabla[(1+\delta_m)v_m]/a = 0$$

$$dv_m/dt + H v_m + (v_m \nabla)v_m/a = -\nabla\psi/a$$

$$\phi_{FR} - \psi_{FR} = \varphi$$

$$k^2 \psi = -3/2 H_0^2 \Omega_m \delta_m/a - 1/2 k^2 \varphi$$

$$(1+w_{BD}) k^2/a^2 \varphi = 3H_0^2 \Omega_m \delta_m/a - I(\varphi)$$

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Structure formation of RSD

The higher order polynomials are given by,

$$A(k,t) = b^3 \sum_n \sum_{a,b} \mu^{2n} (G_\theta/b)^{2a+b-1} \int d^3k \int dr \int dx \\ \times [A^n_{ab}(r,x) B_{2ab}(p,k-p,-k) + A^n_{ab}(r,x) B_{2ab}(k-p,p,-k)]$$

$$B(k,t) = b^4 \sum_n \sum_{a,b} \mu^{2n} (-G_\theta/b)^{2a+b-1} \int d^3k \int dr \int dx \\ \times B^n_{ab}(r,x) P_{a2}(k\sqrt{1+r^2-2rx}) P_{b2}(kr) / (1+r^2-2rx)^a$$

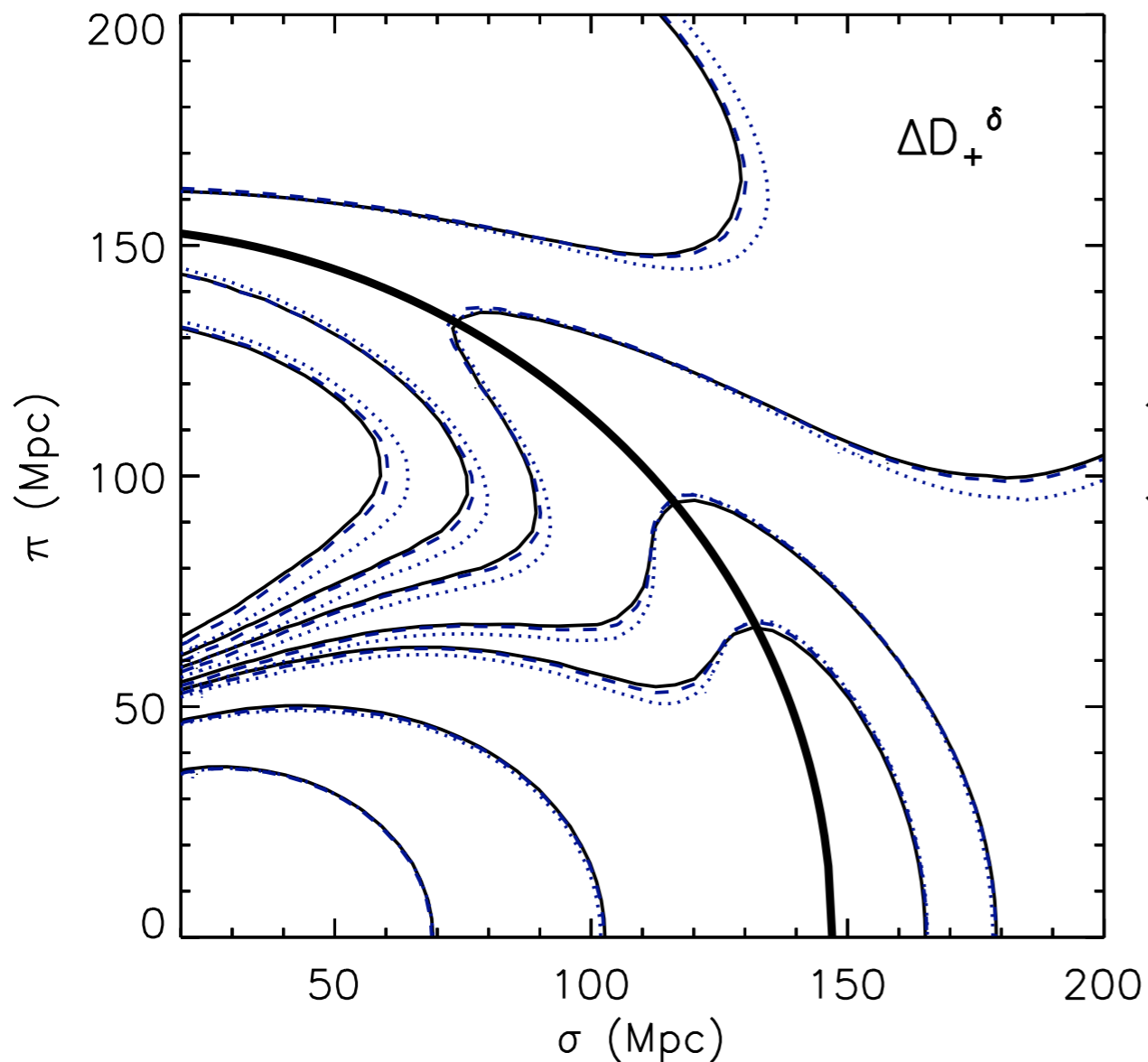
$$P_s(k,\mu) = P_{gg}(k) + 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)$$



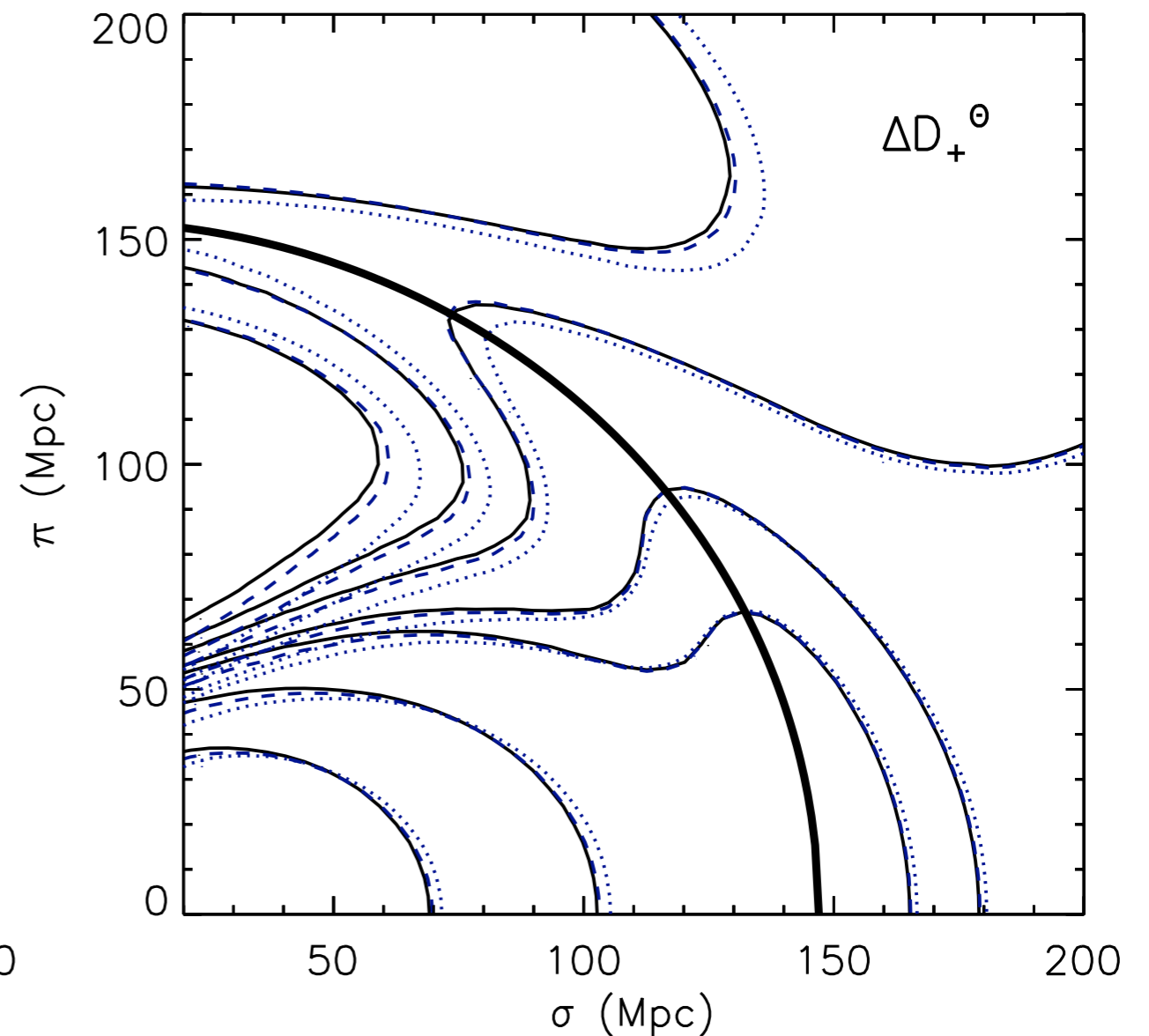
$$P_s(k,\mu) = [P_{gg}(k) + \Delta P_{gg} + 2\mu^2 P_{g\theta}(k) + \Delta P_{g\theta} + \mu^4 P_{\theta\theta}(k) + \Delta P_{\theta\theta} \\ + \mu^2 A(k) + \mu^4 B(k) + \mu^6 C(k) + \dots] \exp[-(k\mu\sigma_p)^2]$$

Correlation function of f(R) gravity model

- $|f_{R0}|=3.0\times 10^{-4}$, $\Omega_\Lambda=0.68$
- $|f_{R0}|=3.2\times 10^{-5}$, $\Omega_\Lambda=0.68$
- LCDM ($|f_{R0}|<10^{-6}$), $\Omega_\Lambda=0.68$

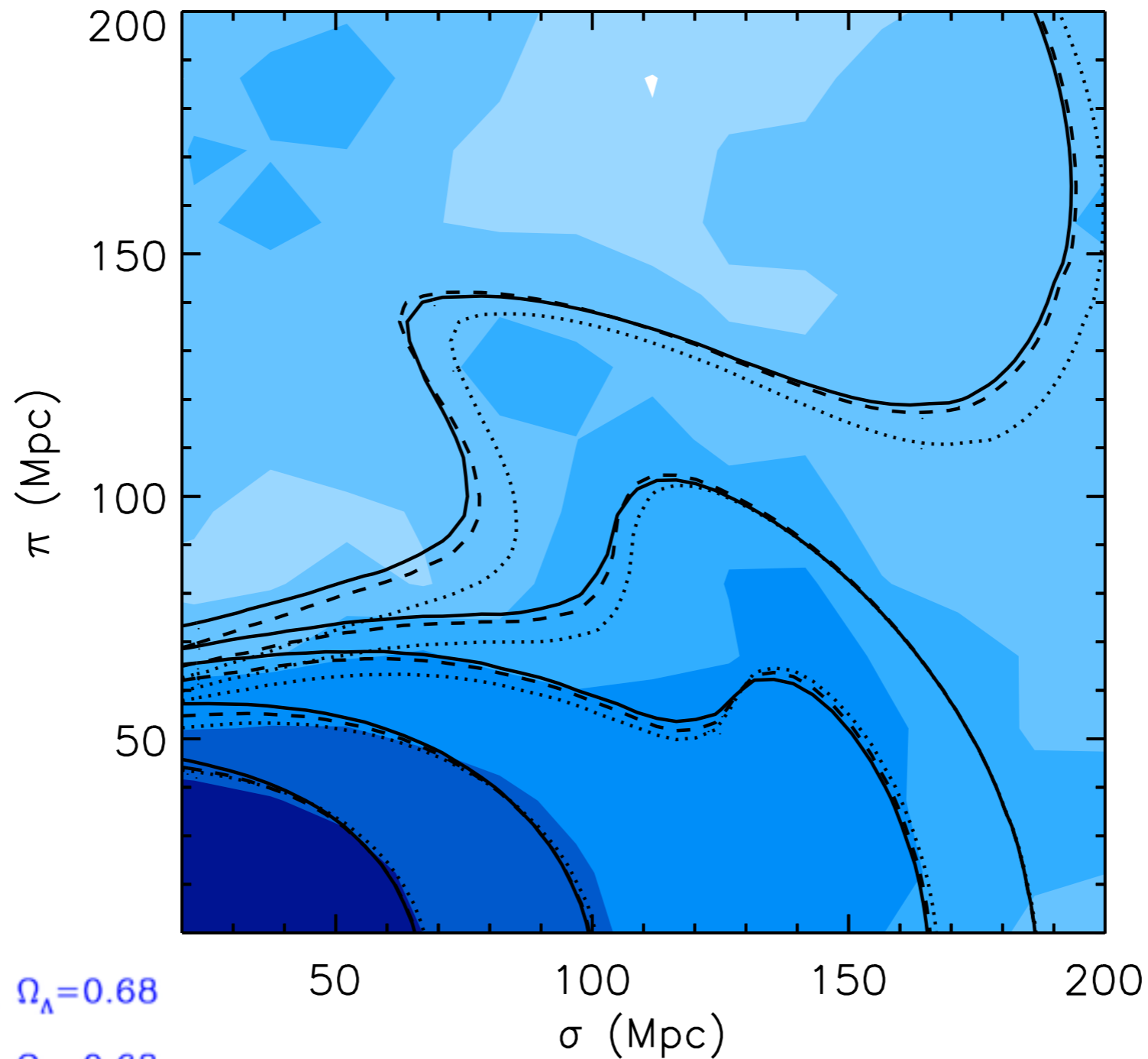


The variation of D^δ



The variation of D^θ

The measurement and best fit models



..... $|f_{R0}|=3.0 \times 10^{-4}$, $\Omega_{\Lambda}=0.68$

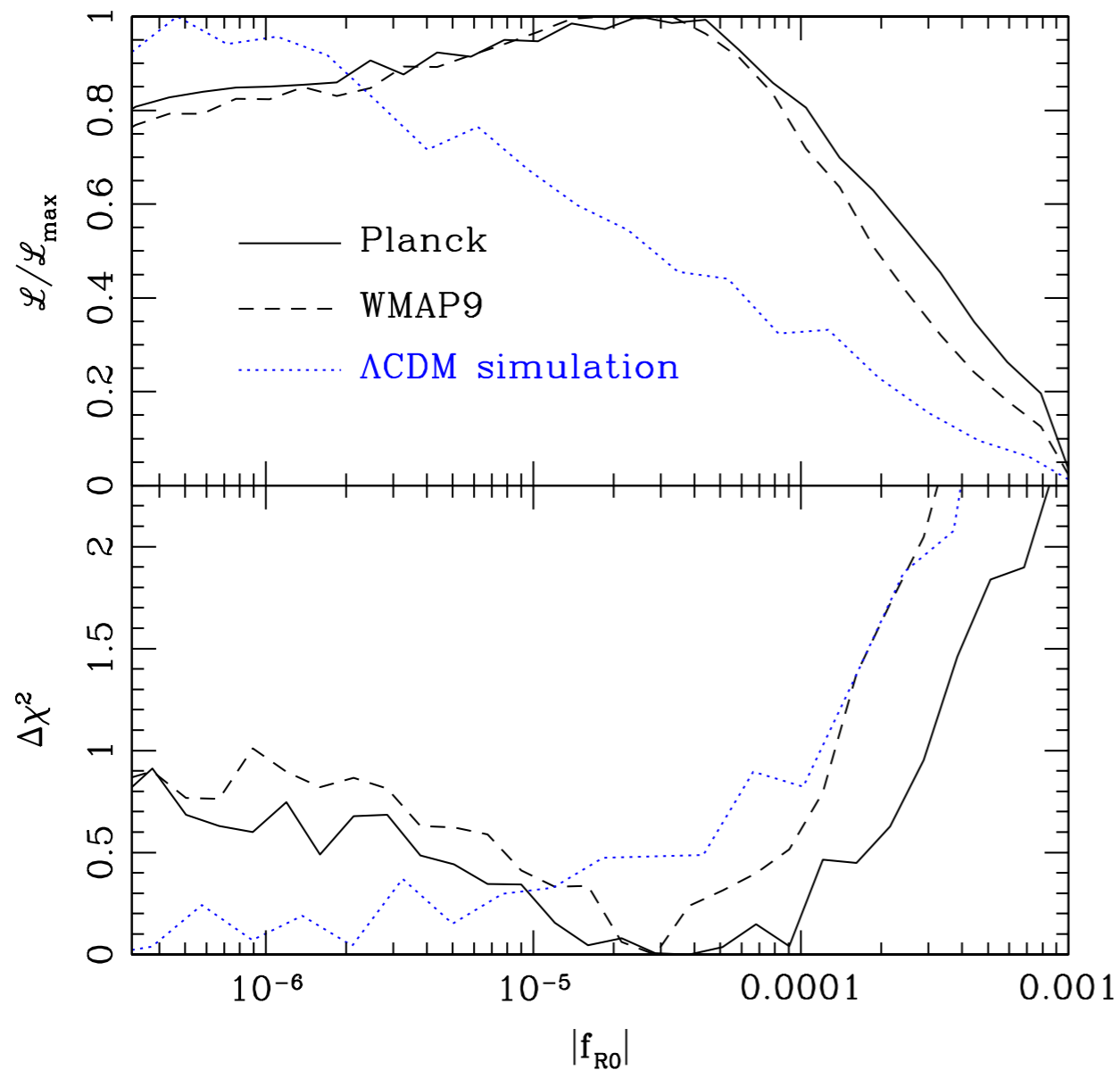
----- $|f_{R0}|=3.2 \times 10^{-5}$, $\Omega_{\Lambda}=0.68$

———— LCDM ($|f_{R0}| < 10^{-6}$), $\Omega_{\Lambda}=0.68$

Constraints on $f(R)$ gravity model

We find new constraints on $f(R)$ gravity models using BOSS DR11

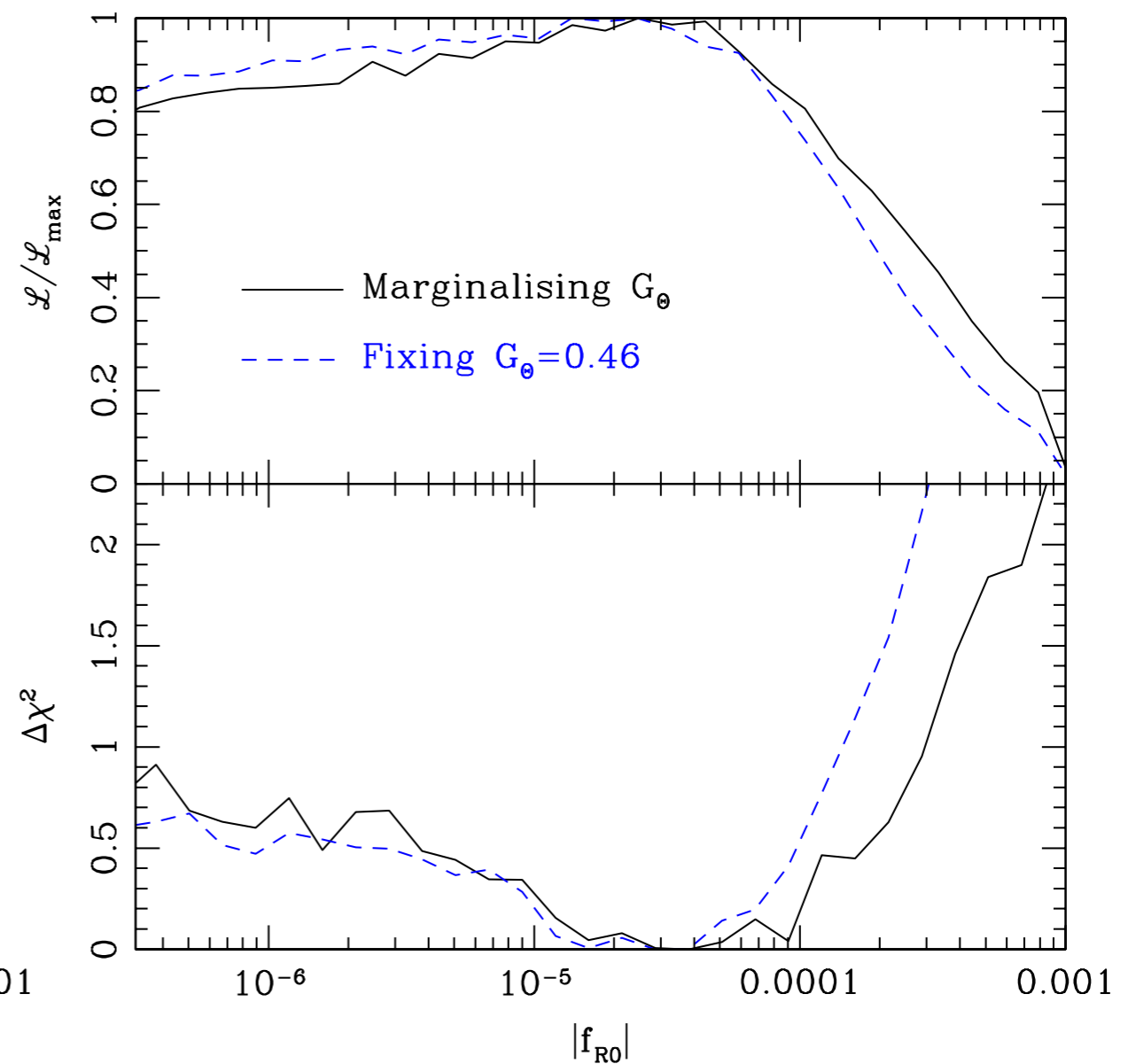
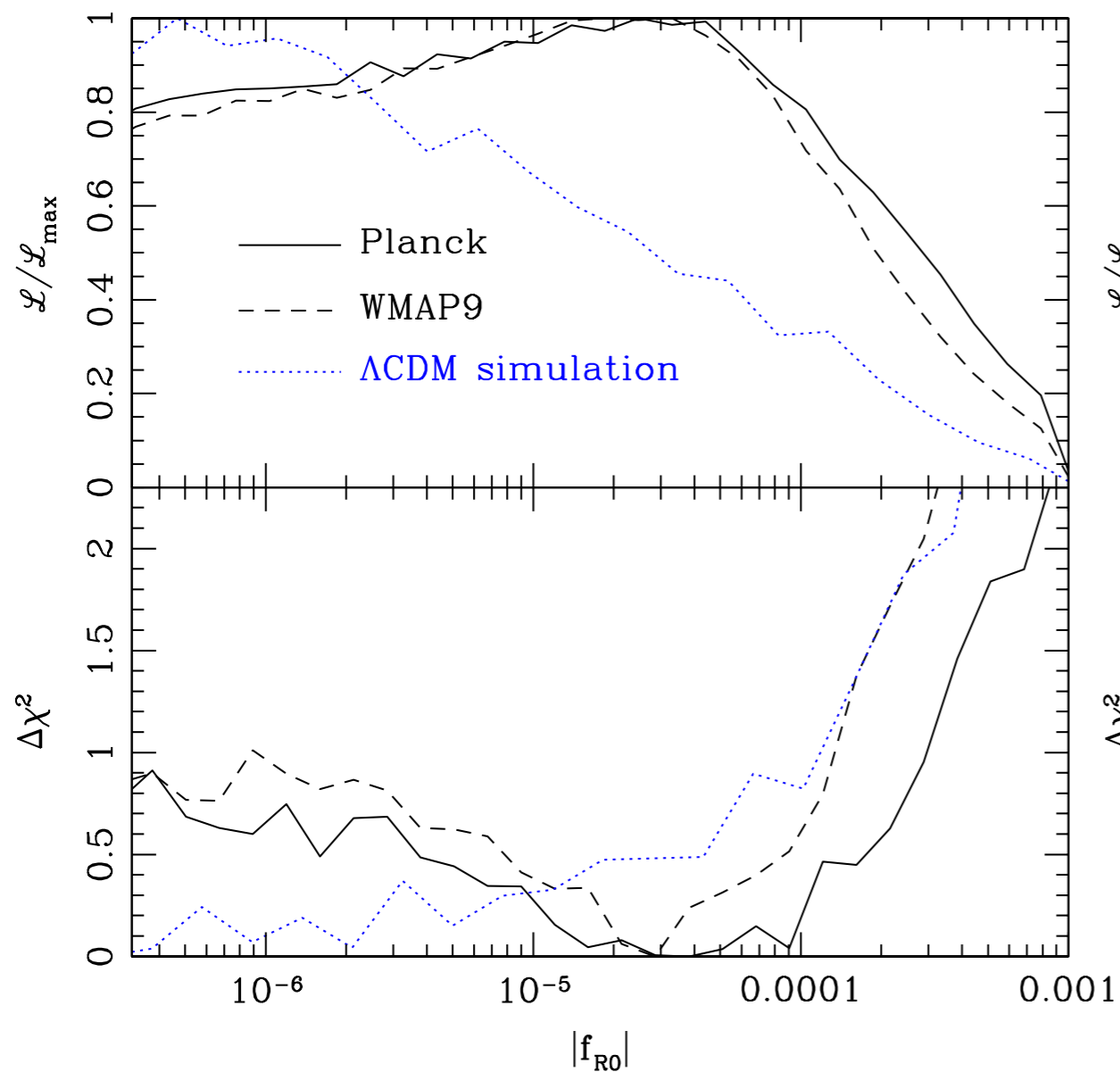
$|f_{R0}| < 8 \times 10^{-4}$ at 95% confidence limit



Constraints on $f(R)$ gravity model

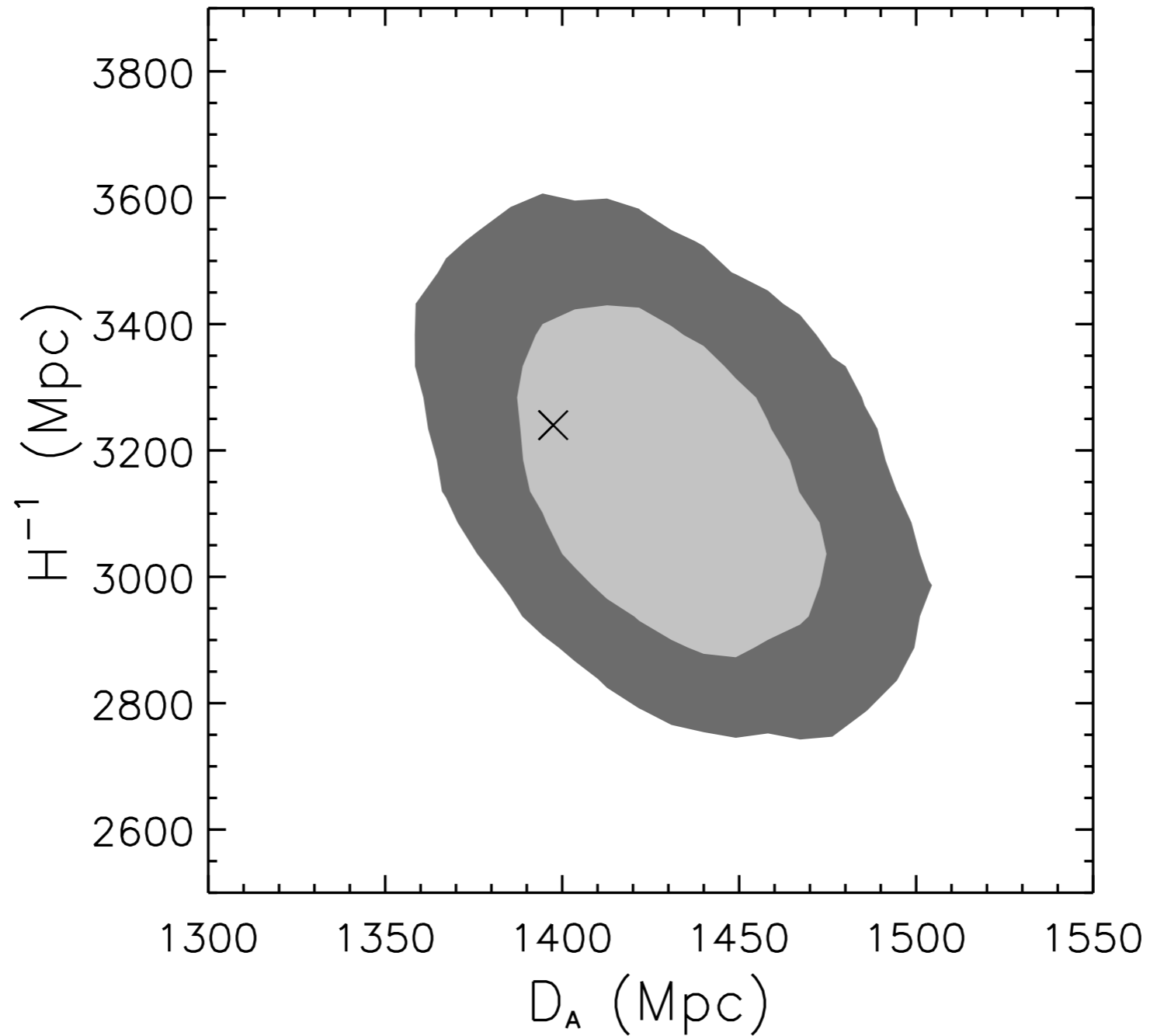
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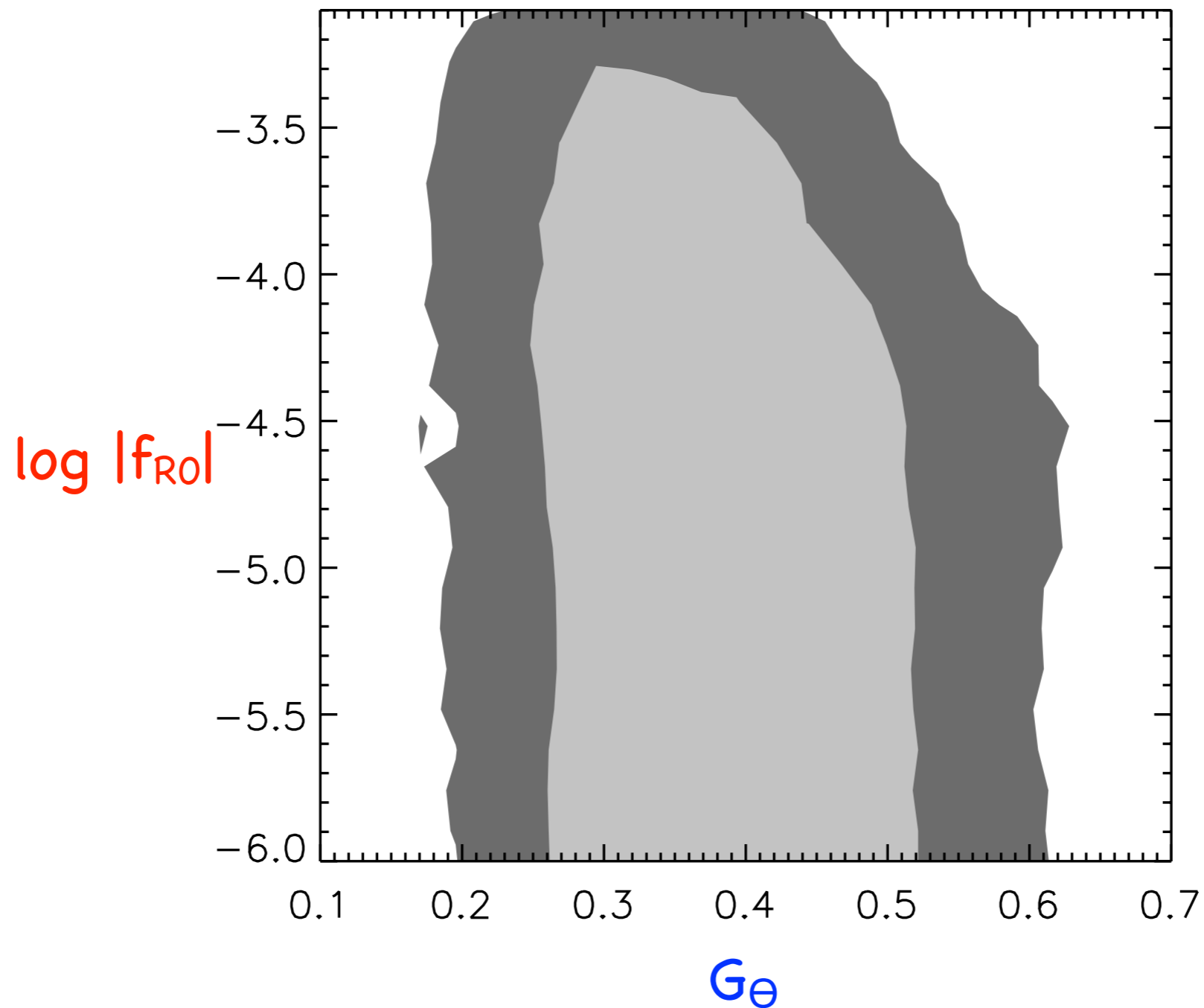
Constraints on distance measures

Measured distances are consistent with LCDM model



Constraints on growth functions

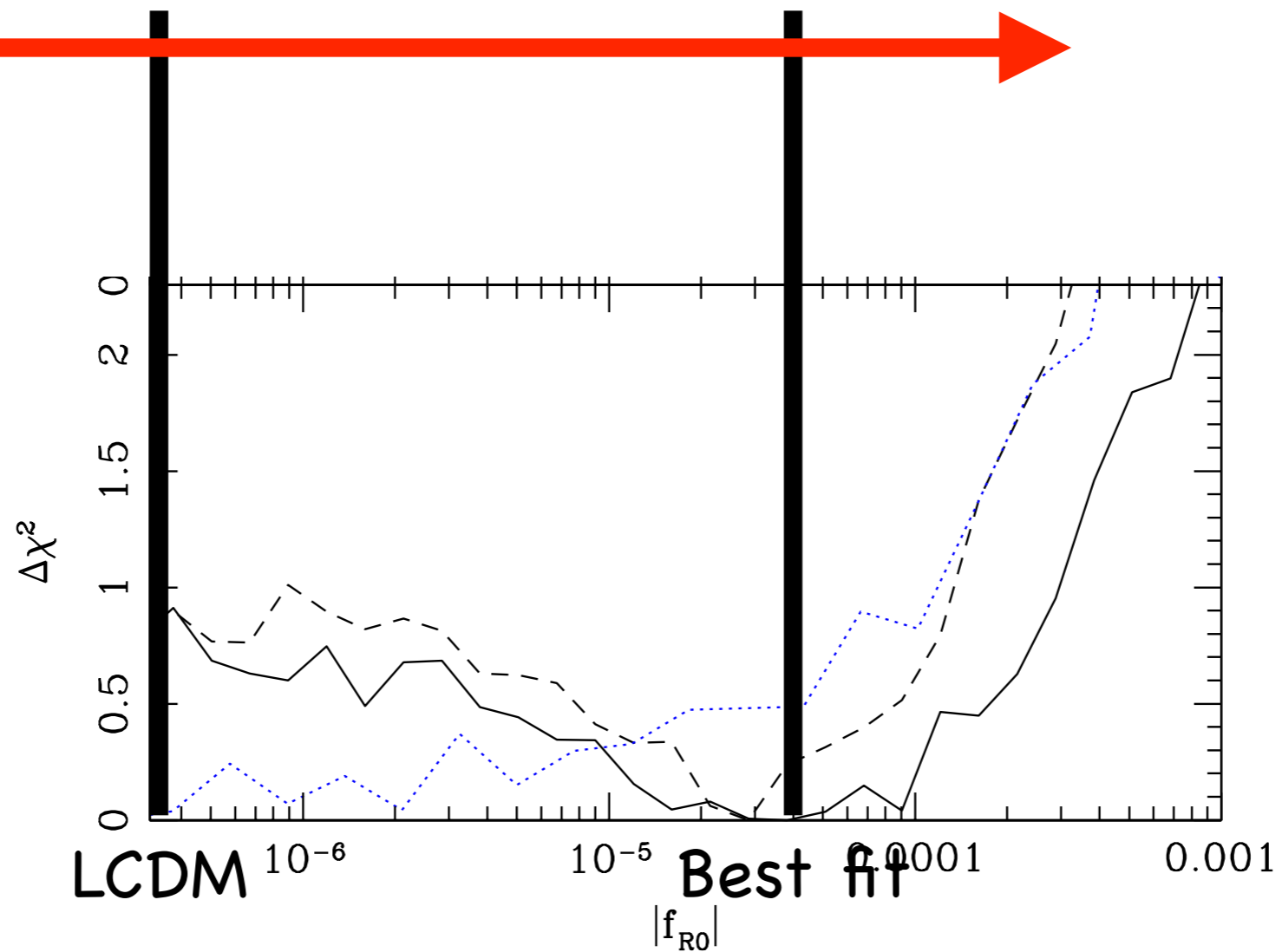
$$D^\theta(k,t) = G_\theta(t) F_\theta(k,t;M_1)$$



Constraints on $f(R)$ now and future

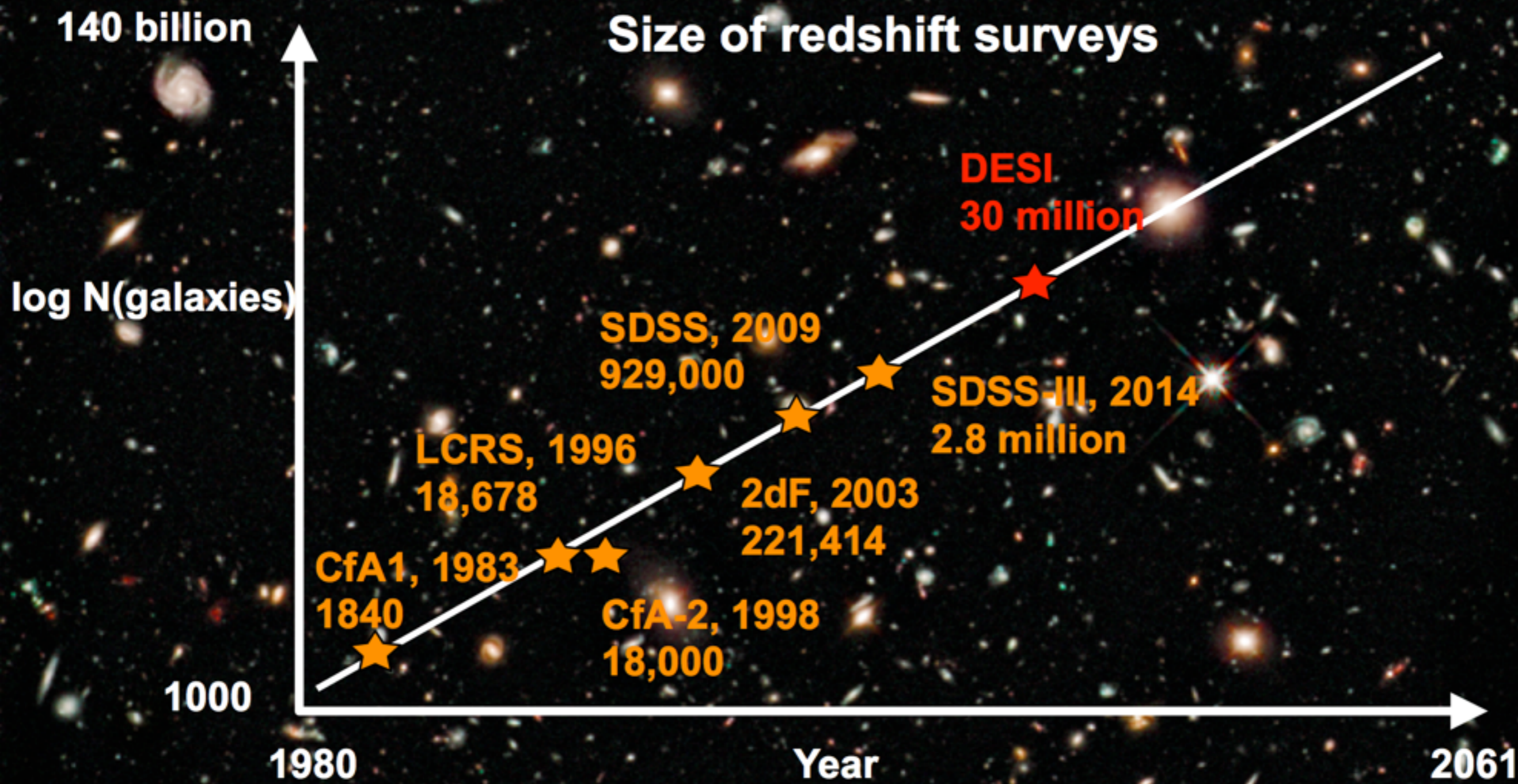
Invisible difference from LCDM model using BOSS

Need a factor of 10 improvement



Where we are, and where will we go?

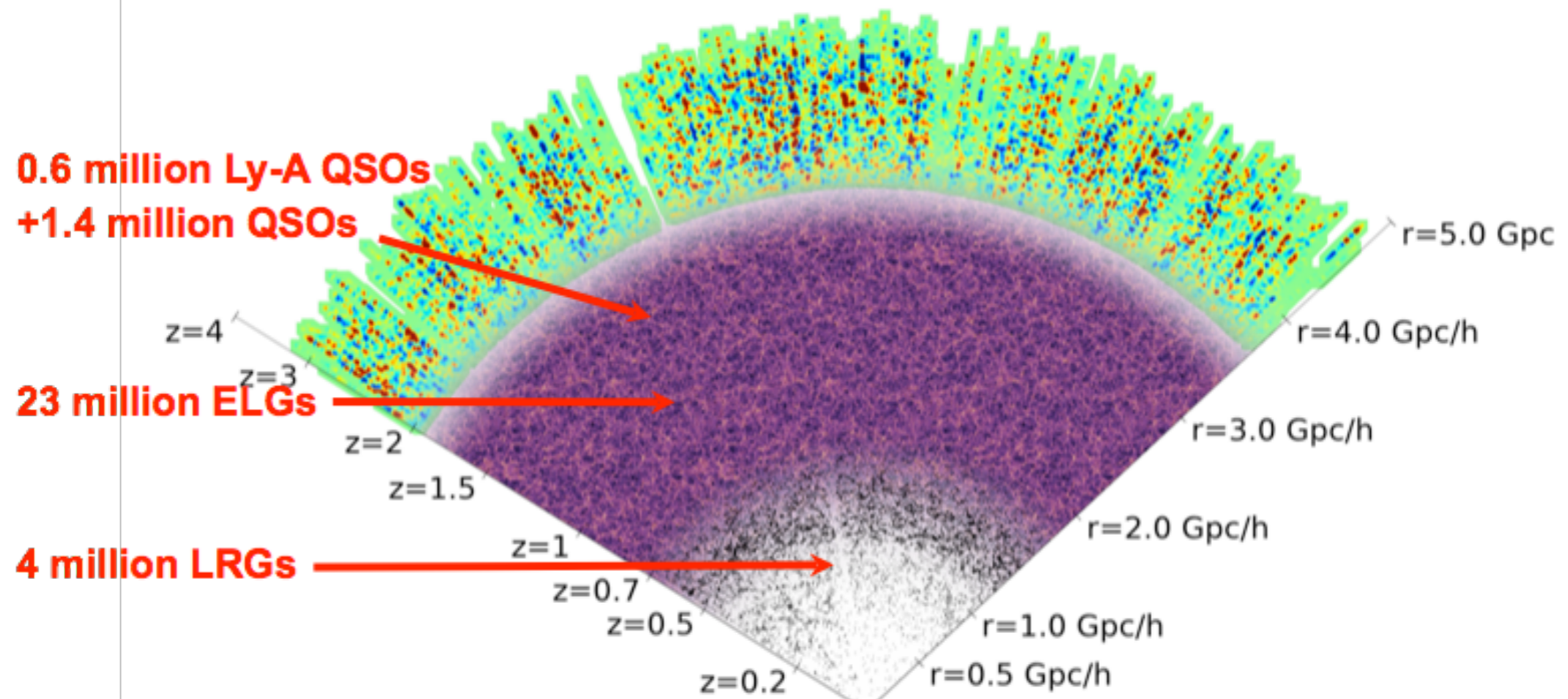
DESI ahead of the curve if completed by 2024



The targeted galaxies in next generation

Four target classes spanning redshifts $z=0 \rightarrow 3.5$

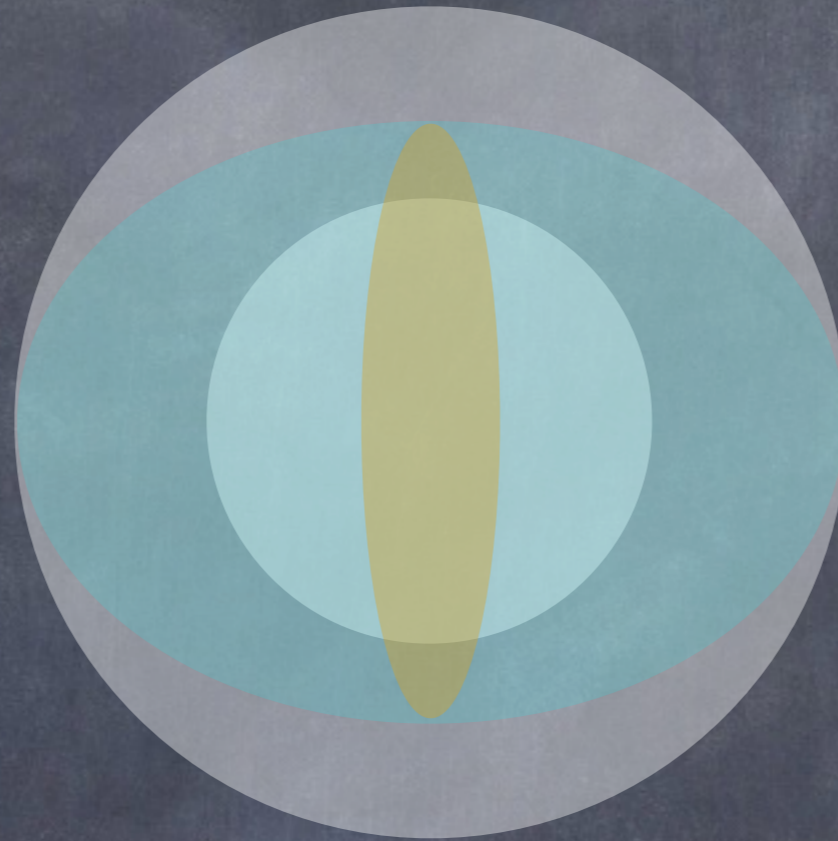
Includes all the massive black holes in the Universe (LRGs + QSOs)



Degeneracy for coherent motions

Squeezing effect
at large scales

(Kaiser 1987)



Finger of God
effect at small
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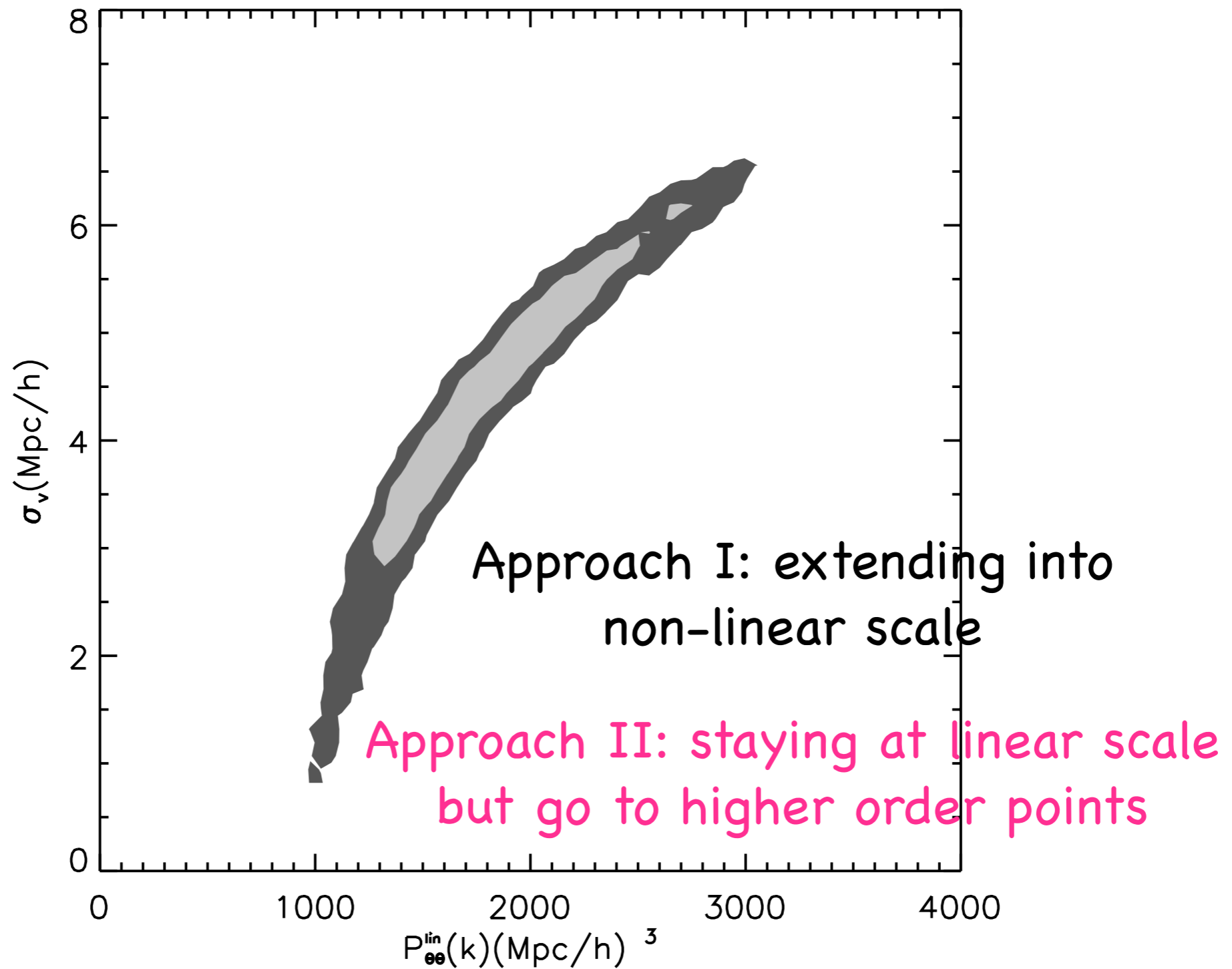
(Jackson 1972)

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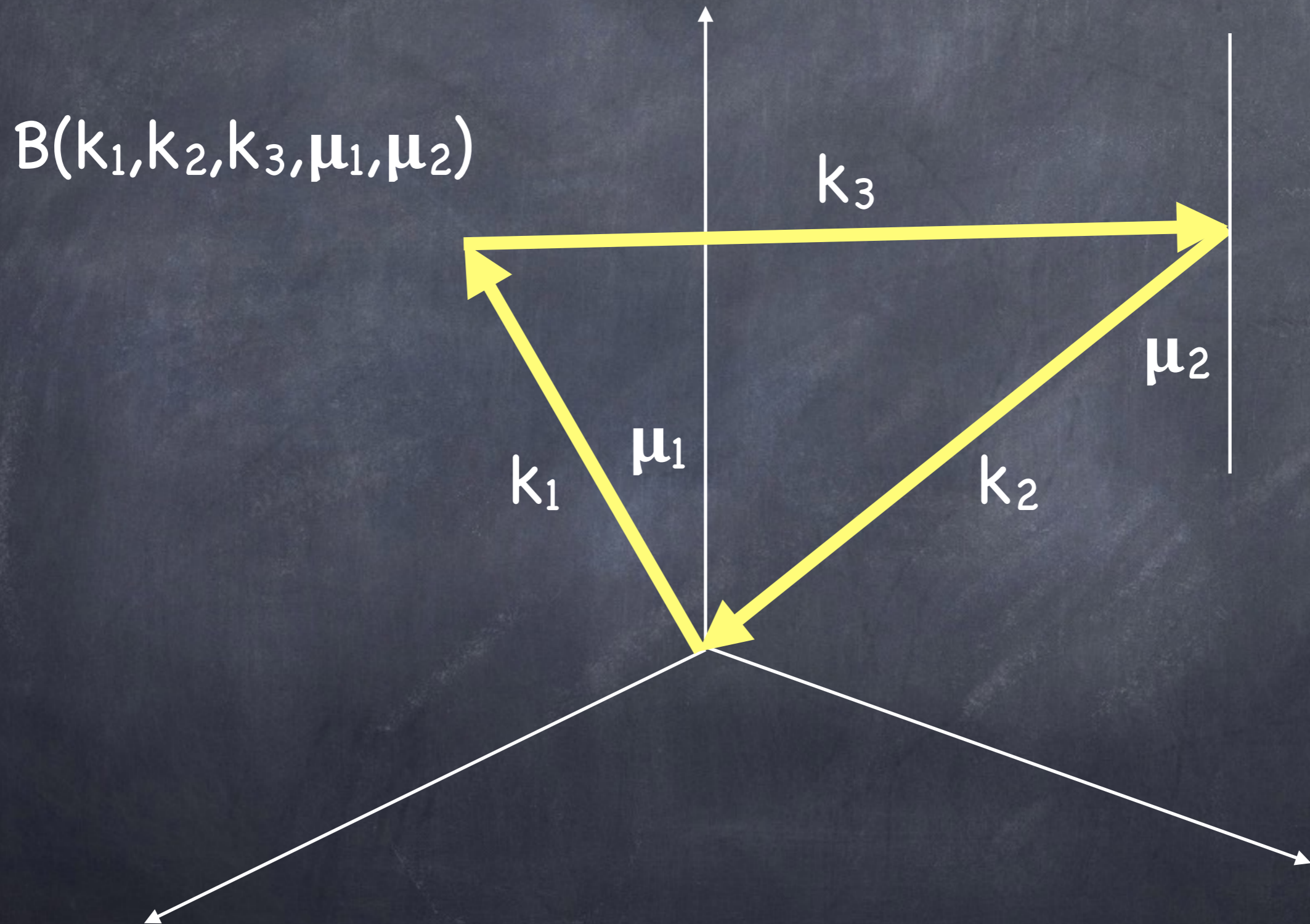
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Degeneracy for coherent motions



Bispectrum Alcock-Paczynski effect

Configuration in redshift space



Bispectrum Alcock-Paczynski effect

Definition of FoG effect

$$B(k_1, k_2, k_3, \mu_1, \mu_2) = D_{\text{FoG}}^B B^{\text{PT}}(k_1, k_2, k_3, \mu_1, \mu_2)$$

$$D_{\text{FoG}}^B = \exp[-(k_1^2 \mu_1^2 + k_2^2 \mu_2^2 + k_3^2 \mu_3^2) \sigma_p^2]$$

Bispectrum Alcock-Paczynski effect

B^{PT} in redshift space

$$B(k_1, k_2, k_3, \mu_1, \mu_2) = D_{\text{FOG}}^B B^{\text{PT}}(k_1, k_2, k_3, \mu_1, \mu_2)$$

$$B^{\text{PT}}(k_1, k_2, k_3, \mu_1, \mu_2) = 2[Z_2(k_1, k_2)Z_1(k_1)Z_2(k_2)P(k_1)P(k_2) \\ + \text{cyclic}]$$

$$Z_1(k_1) = b + f\mu_1^2$$

$$Z_2(k_1, k_2) = b_2/2 + bF_2 + f\mu_{12}G_2$$

$$+ fk_{12}\mu_{12}/2[\mu_1/k_1Z_2(k_2) + \mu_2/k_2Z_2(k_1)]$$

Bispectrum Alcock-Paczynski effect

AP projection

$$B^{\text{obs}}(k_1, k_2, k_3, \mu_1, \mu_2) = (\Delta H^{-1})^2 (\Delta D_A)^4 B(q_1, q_2, q_3, \nu_1, \nu_2)$$

$$\Delta H^{-1} = H^{-1}_{\text{fid}} / H^{-1}_{\text{true}} \quad \Delta D_A = D_{A \text{ fid}} / D_{A \text{ true}}$$

$$q_i = \alpha(\mu_i) k_i \quad \nu_i = \mu_i \Delta H^{-1} / \alpha(\mu_i)$$

$$\alpha(\mu_i) = \{ (\Delta D_A)^2 + [(\Delta H^{-1})^2 - (\Delta D_A)^2] \mu_i^2 \}^{1/2}$$

$$\nu_{ij} = (\Delta D_A)^2 \eta_{ij} / \alpha(\mu_i) \alpha(\mu_j) + [(\Delta H^{-1})^2 - (\Delta D_A)^2] \mu_i \mu_j / \alpha(\mu_i) \alpha(\mu_j)$$

Error forecast using power and bi combination

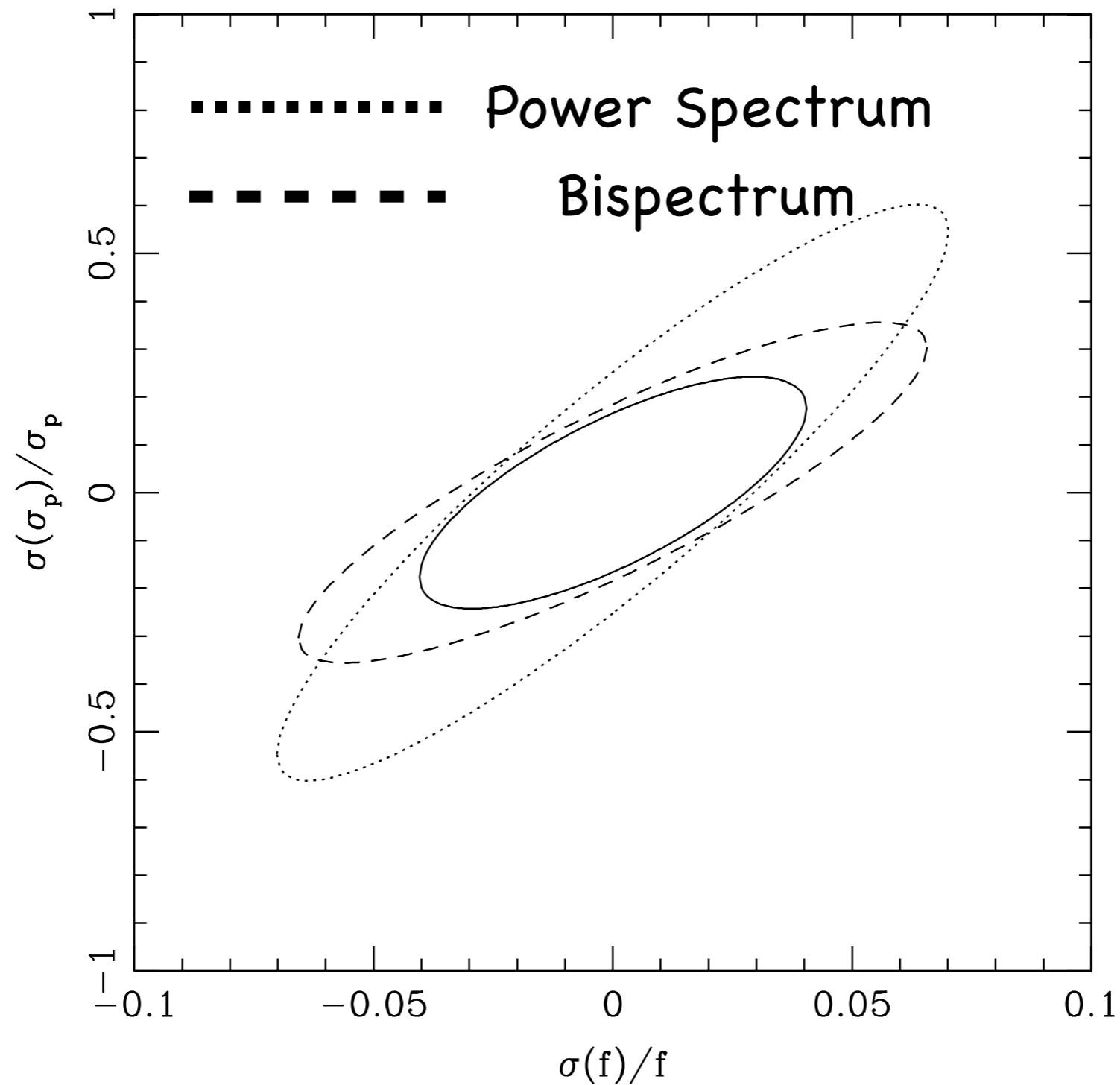
$$F_{\alpha\beta} = \sum_k \sum_{k_1 k_2 k_3} (\partial S / \partial p_\alpha) C^{-1} (\partial S / \partial p_\beta)$$

$$S = \begin{pmatrix} P(k, \mu) \\ B(k_1, k_2, k_3, \mu_1, \mu_2) \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} M & -M C_{PB} C_{BB}^{-1} \\ -C_{BB}^{-1} C_{BB}^{-1} M & C_{BB}^{-1} + C_{BB}^{-1} C_{BP} M C_{PB} C_{BB}^{-1} \end{pmatrix}$$

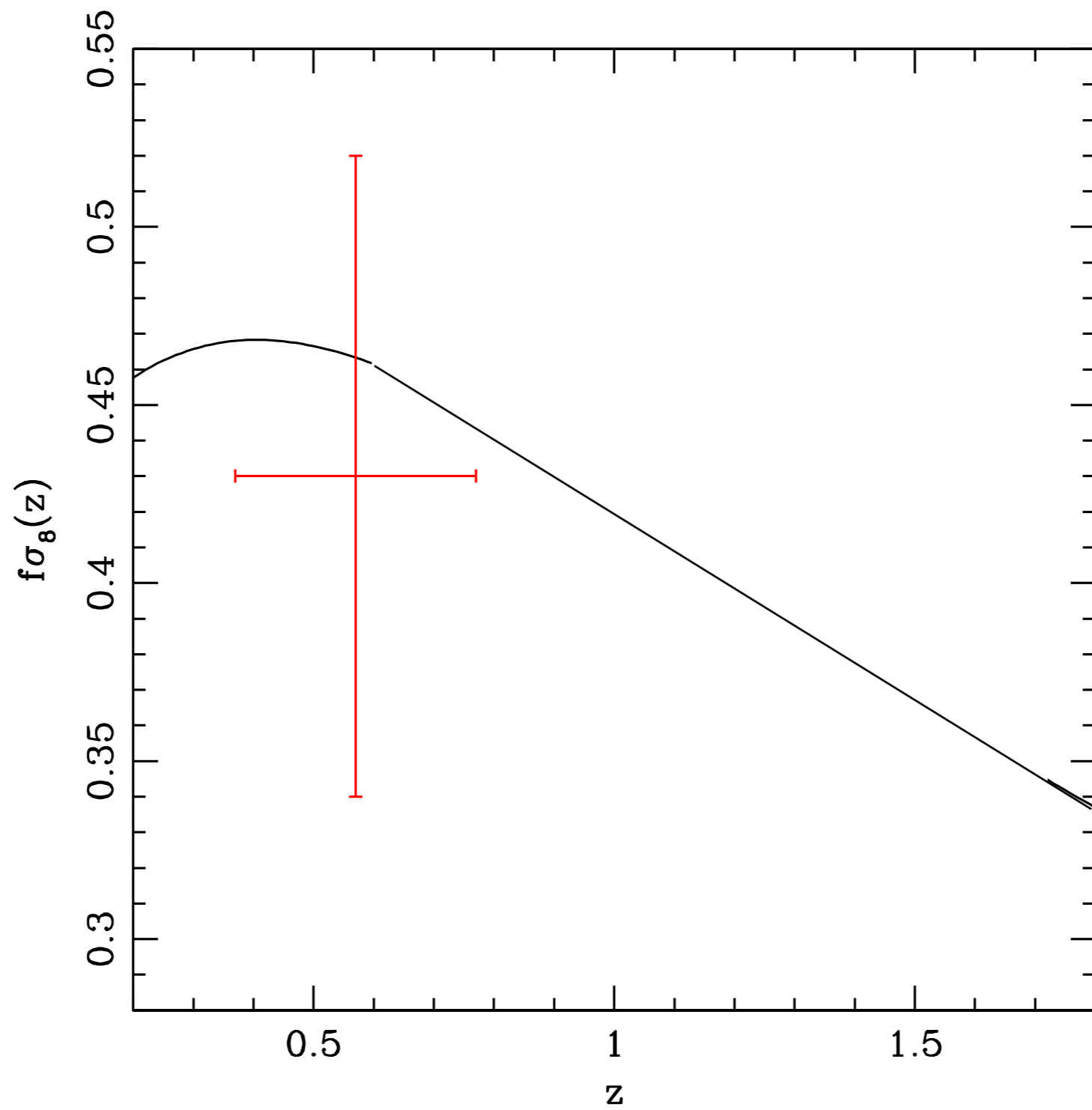
$$M = (C_{pp} - C_{PB} C_{BB}^{-1} C_{BP})^{-1}$$

Degeneracy in coherent and random motions



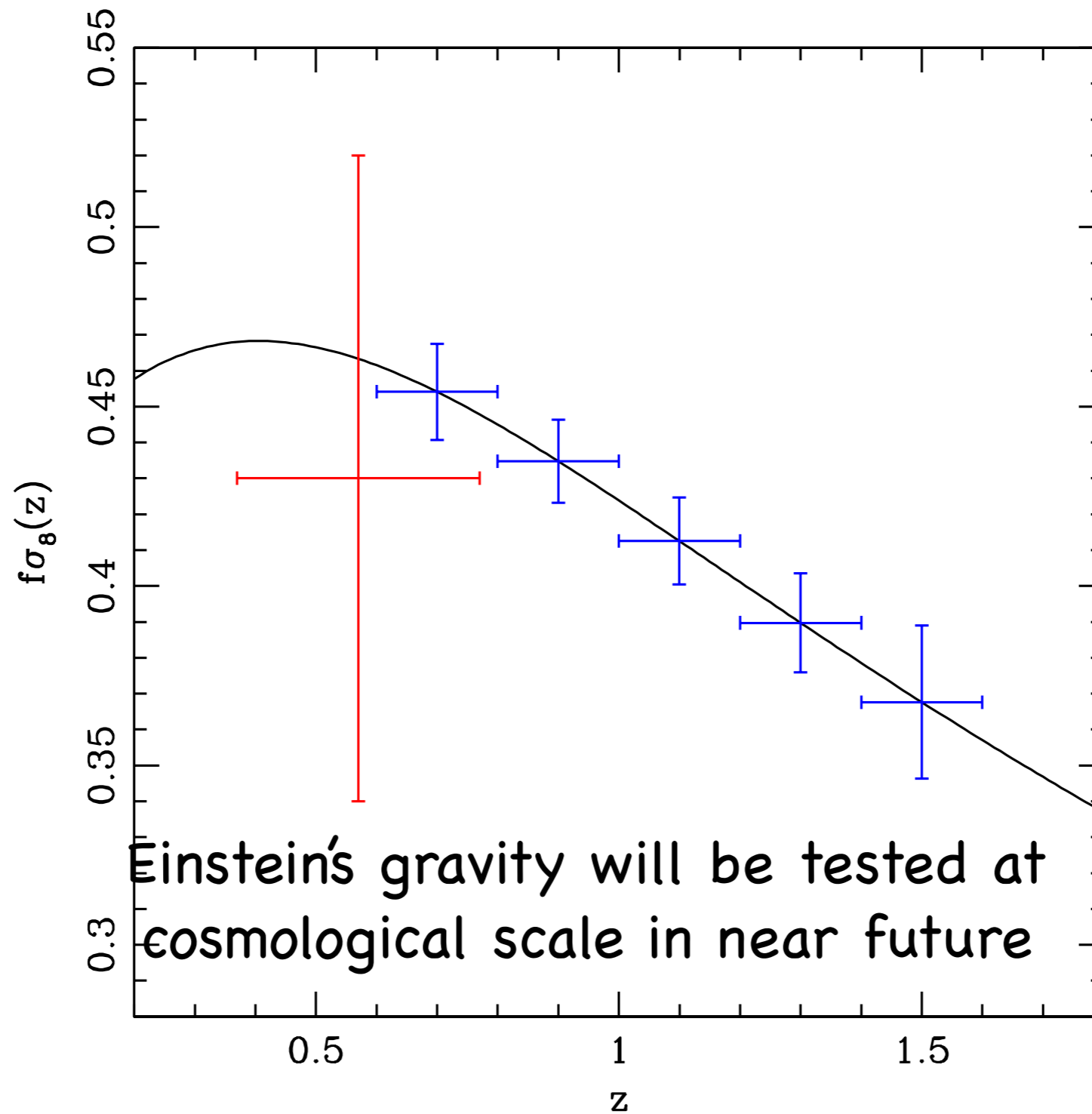
Measured coherent motion

Results from BOSS maps



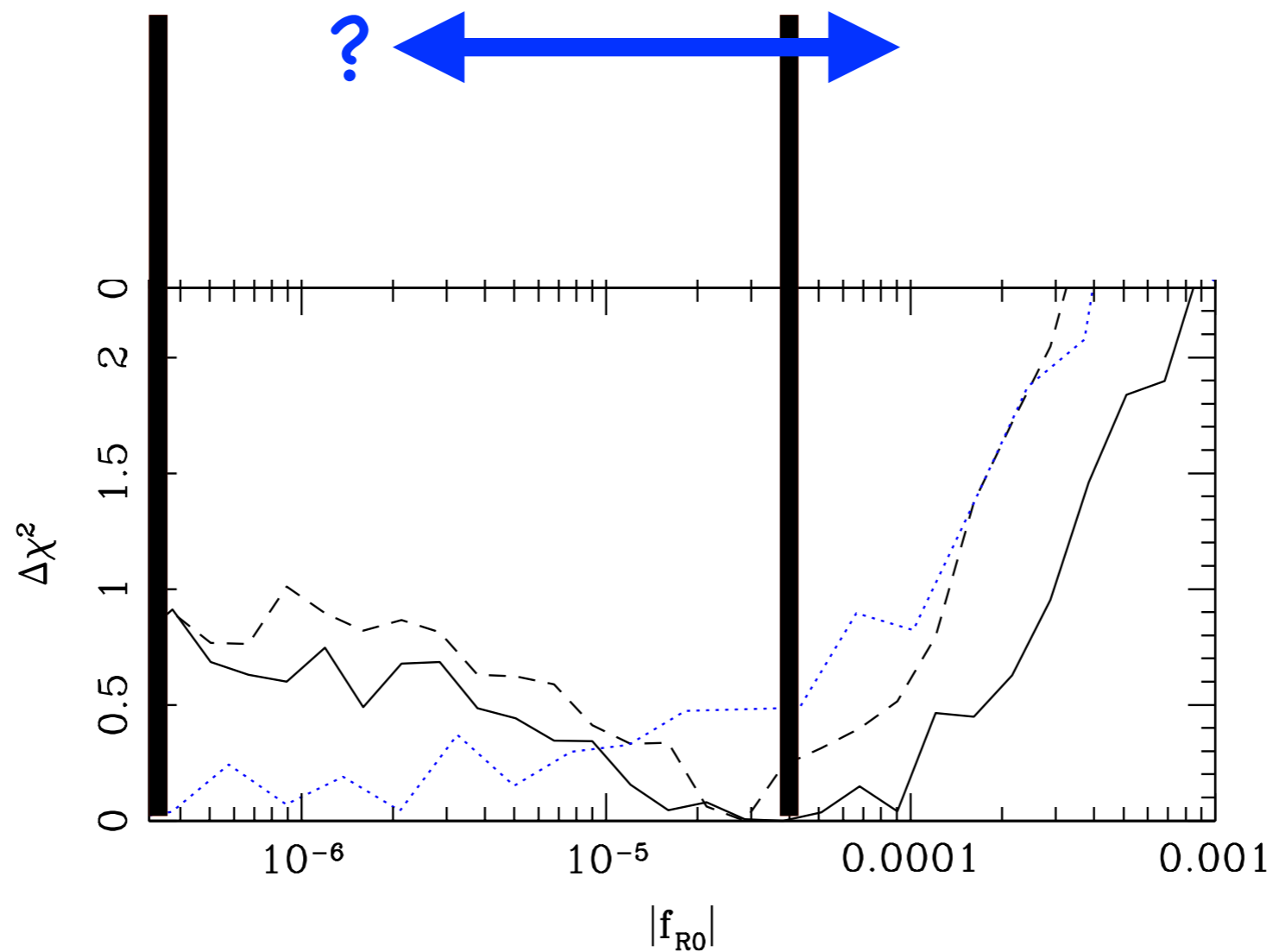
Future constraints

Expectation from DESI



Future work

Invisible difference from LCDM model using BOSS
then can we tell the difference in future?



Conclusion

- We succeed in measuring both distances and growth function simultaneously using RSD, and ready to test Einstein's gravity at cosmological scales through duality between distances and growth functions.
- We understand all systematics due to non-linear physics, and the perturbative description works fine the resolution of current experiment, at least two point correlation level.
- Now we face new challenge to meet the precision level of the high resolution experiment like DESI.
- We work out the Alcock-Paczynski effect on bispectra, and find that the combined constraint of power spectrum and bispectrum improves the detectability of growth function.
- We initiate new roadmap to accomplish this combination for the future experiment.