Large scale structure formation

with the

Schrödinger method

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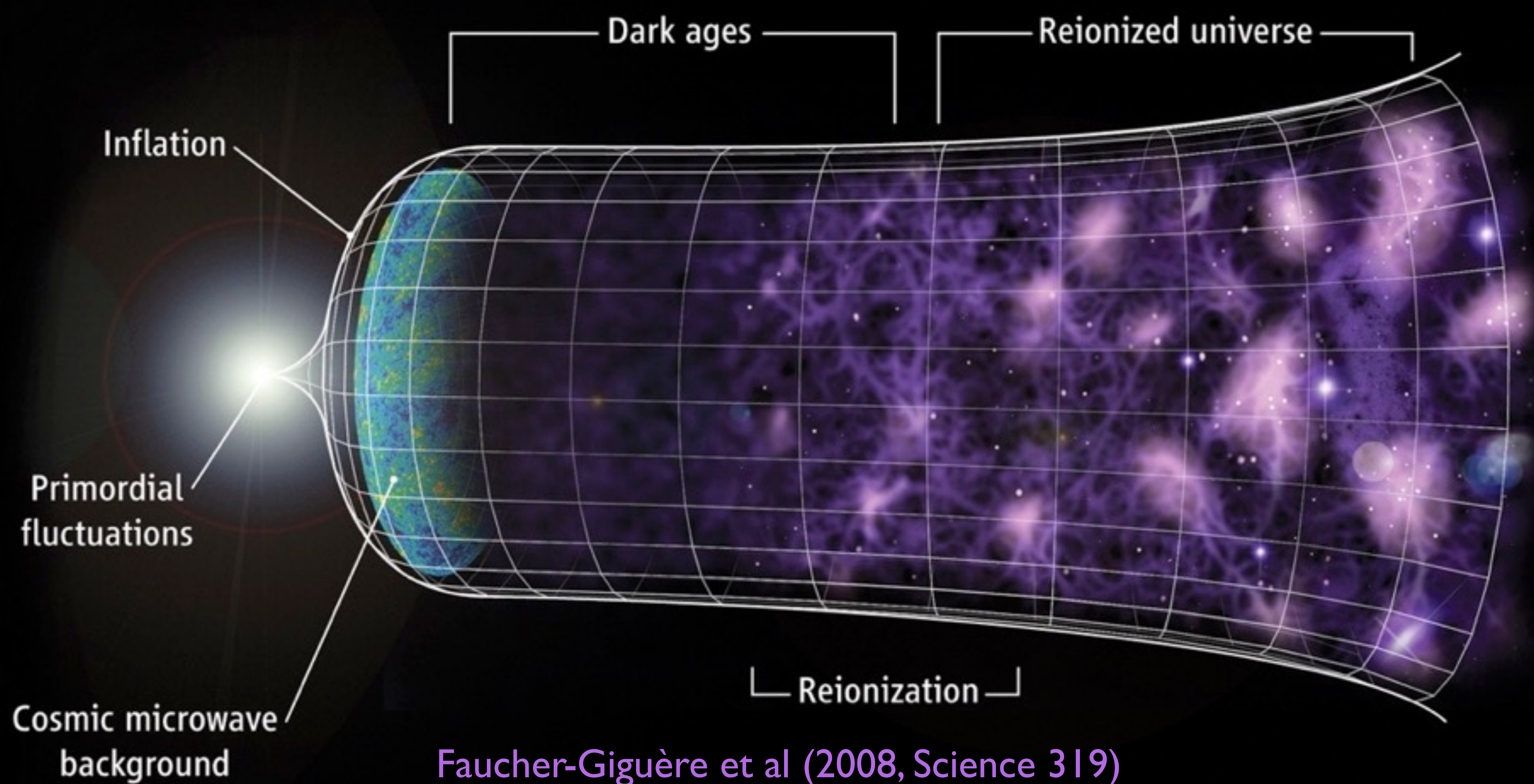
Cosmo Cruise 2015, September 7

Cosmological Structure Formation

-13.8 billion years: nearly uniform initial state

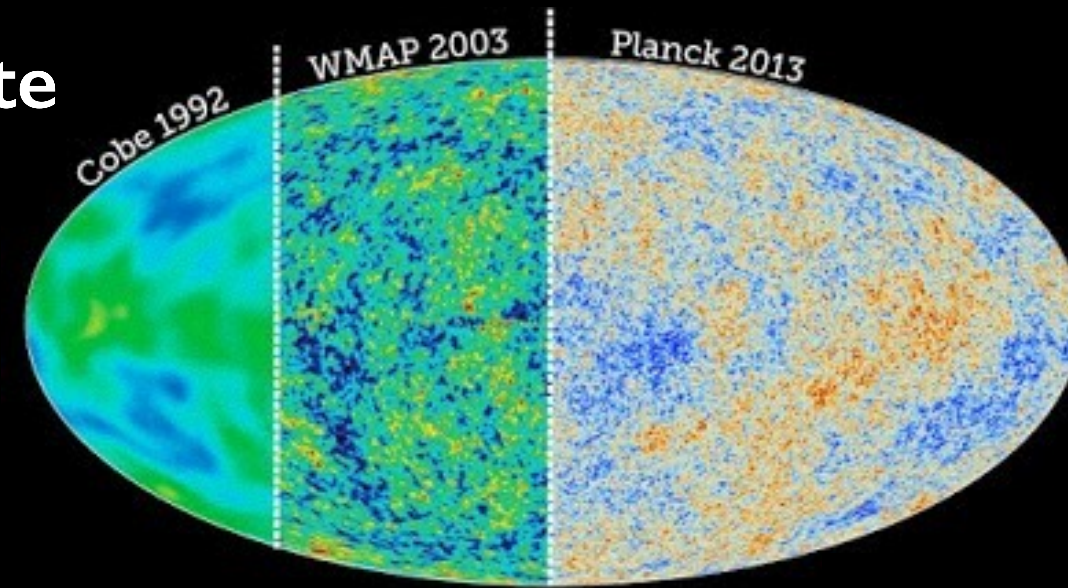


Faucher-Giguère et al (2008, Science 319)

today: rich structures in cosmic web

Cosmological Structure Formation

-13.8 billion years: nearly uniform initial state



Inflation

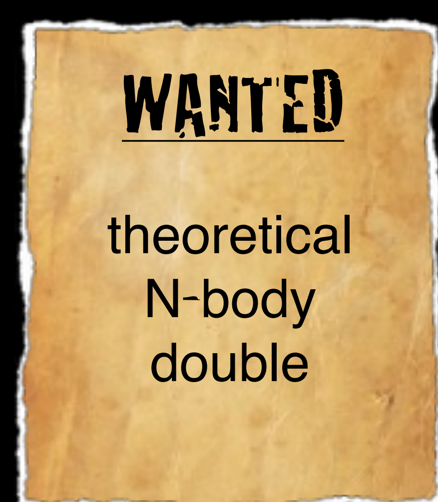
- established `boring` initial conditions
  - quantum fluctuations get amplified
  - primordial plasma cools → recombination → **CMB**

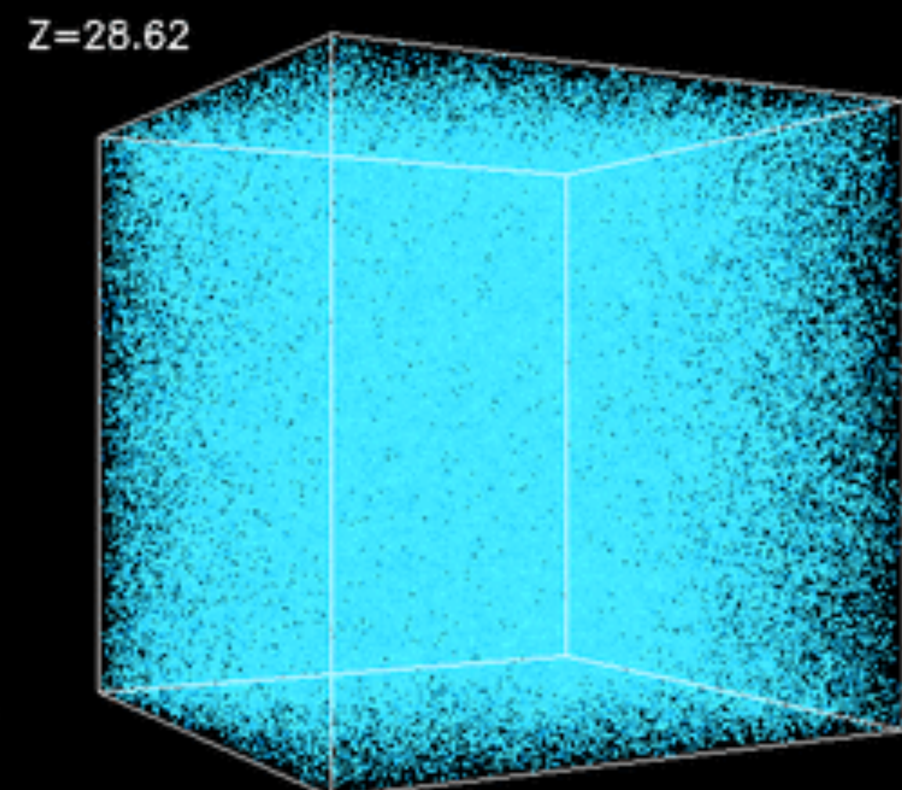
Structure formation

- hierarchical
- tiny over-densities act as seeds
  - congregation via gravitational instability
  - collapse into bound structures

Large scale structure: Cold Dark Matter

- linear regime
  - ✓ analytically understood
- nonlinear stage
  - ⁈ **N-body simulations inevitable**





Kravtsov & Klypin (simulations @NCSA)

today: rich structures in cosmic web

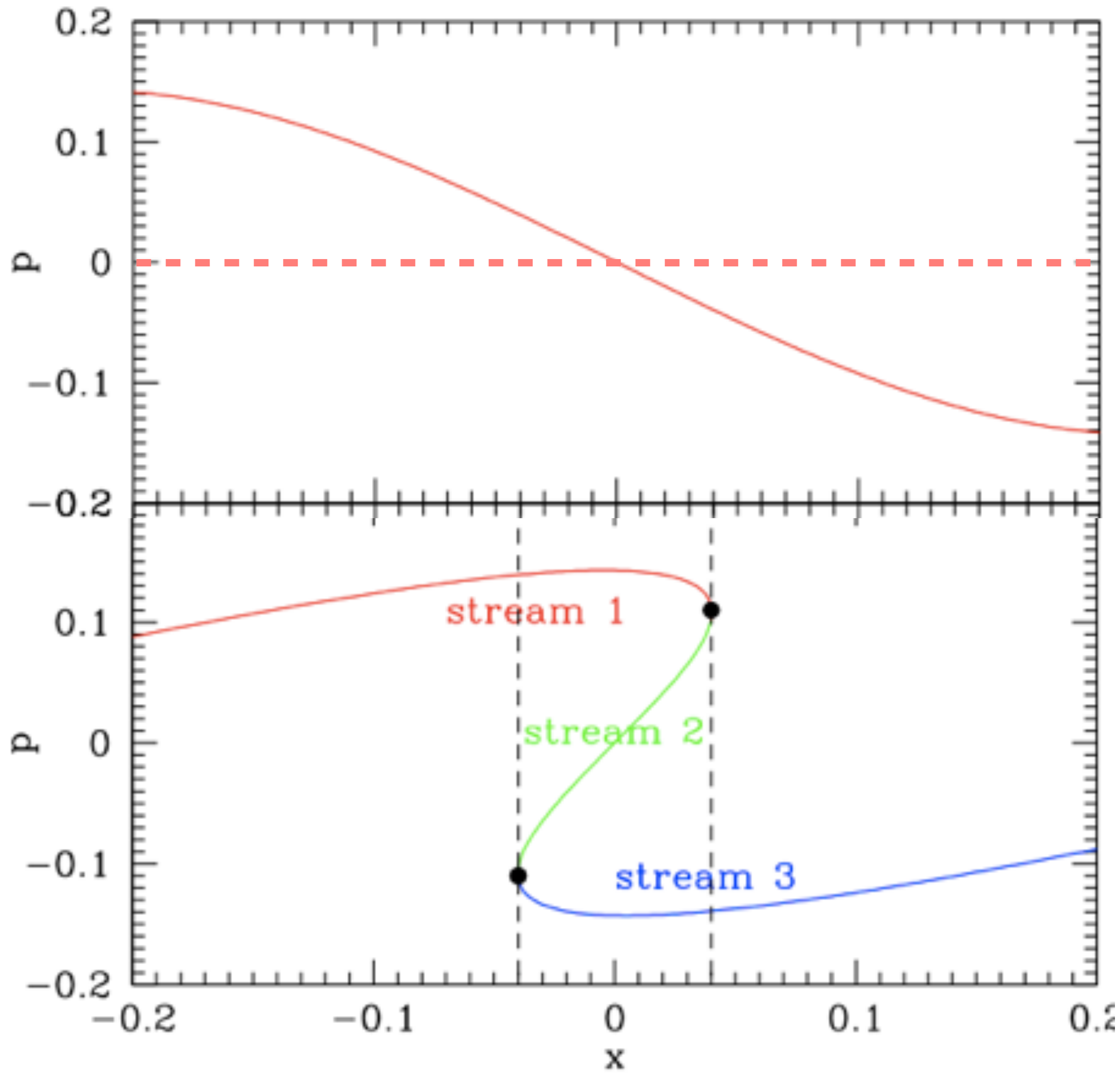
Describing Cold Dark Matter with the Schrödinger method

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

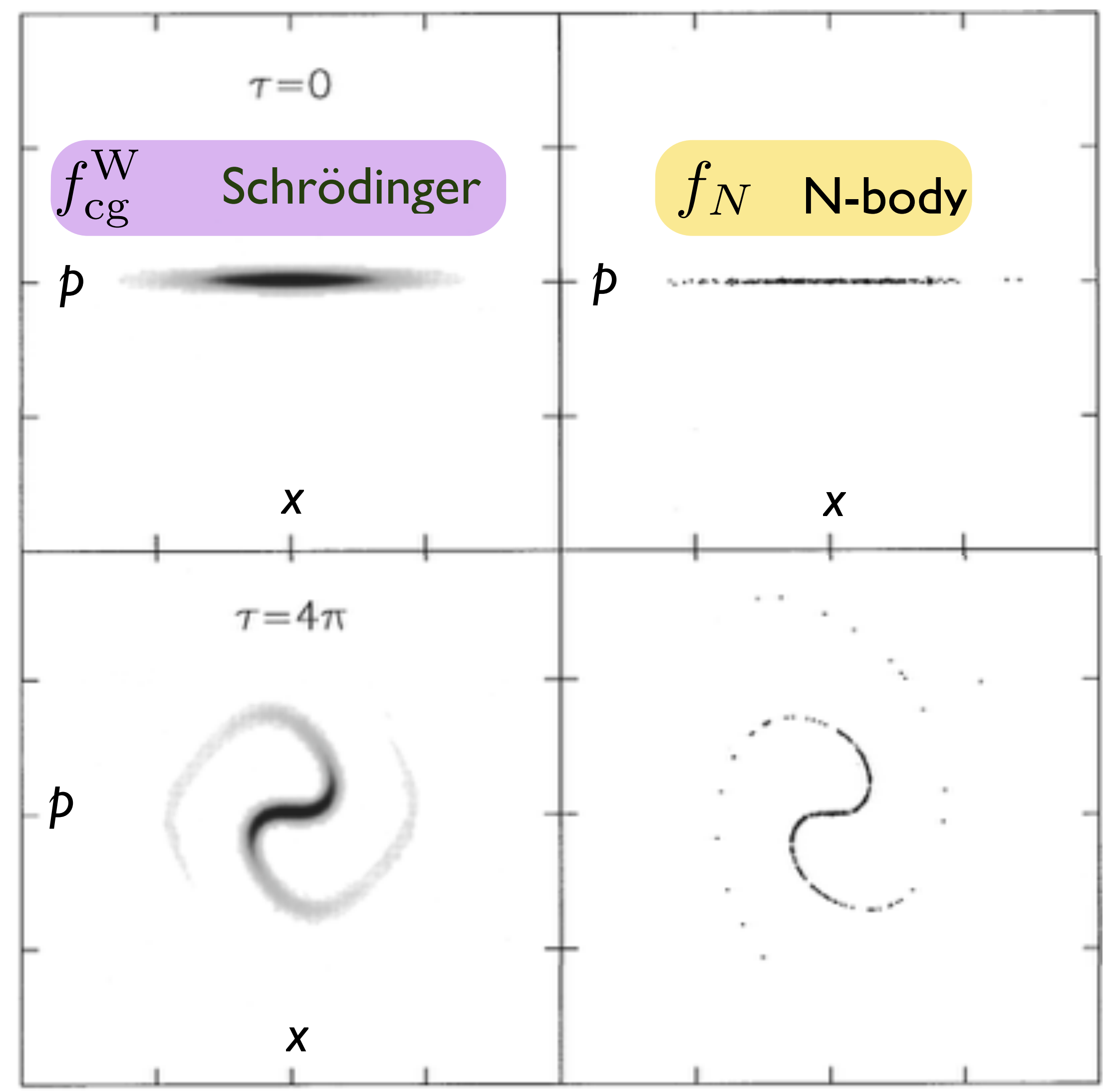
- describes number density & distribution of momenta *p*

theoretical expectation



Pueblas & Scoccimarro (2009, PRD 80)

numerical realization



Schrödinger method: Widrow & Kaiser (1993, ApJ 416)
Widrow (1997, PRD 55)

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- **N-body**: non-relativistic, only gravitationally
- **continuous**: ensemble average, no collisions

$$f_N = \sum_i \delta_D(\boldsymbol{x} - \boldsymbol{x}_i)\delta_D(\boldsymbol{p} - \boldsymbol{p}_i)$$

$$f$$

Vlasov - Poisson equation

gravitational potential

$$\partial_\tau f(\boldsymbol{x}, \boldsymbol{p}, \tau) = -\frac{\boldsymbol{p}}{am}\boldsymbol{\nabla}_x f + am\boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

$$\Delta V(\boldsymbol{x}, \tau) = \frac{4\pi G m}{a}\left(n(\boldsymbol{x}, \tau) - \langle n \rangle\right)$$

partial

3+3+1
variables

nonlinear

integro

number density $n = \int \mathrm{d}^3p \;\; f$

Solving is hard!
have to choose a special ansatz
for phase space distribution f(x,p)

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- **N-body**: non-relativistic, only gravitationally
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$$f_N \to f$$

Vlasov - Poisson equation

$$\partial_\tau f(\boldsymbol{x},\boldsymbol{p},\tau) = -\frac{\boldsymbol{p}}{am}\boldsymbol{\nabla}_x f + am\boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

gravitational potential

$$\Delta V(\boldsymbol{x},\tau) = \frac{4\pi G m}{a}\left(n(\boldsymbol{x},\tau) - \langle n\rangle\right)$$

Hierarchy of Moments $M^{(n)}(\boldsymbol{x}) = \int d^3p\, p_{i_1}\ldots p_{i_n} f$

- density n(x): $M^{(0)} = n(\boldsymbol{x})$, velocity **v**(x): $M^{(1)} = n\boldsymbol{v}(\boldsymbol{x})$
- velocity dispersion $\boldsymbol{\sigma}$(x): $M^{(2)} = n\,(\boldsymbol{v}\boldsymbol{v} + \boldsymbol{\sigma})\,(\boldsymbol{x}), \ldots$

cumulant

$$\partial_t M^{(n)} = -\frac{1}{a^2 m}\nabla \cdot M^{(n+1)} - m\nabla V \cdot M^{(n-1)}$$

infinite coupled hierarchy

Dust model

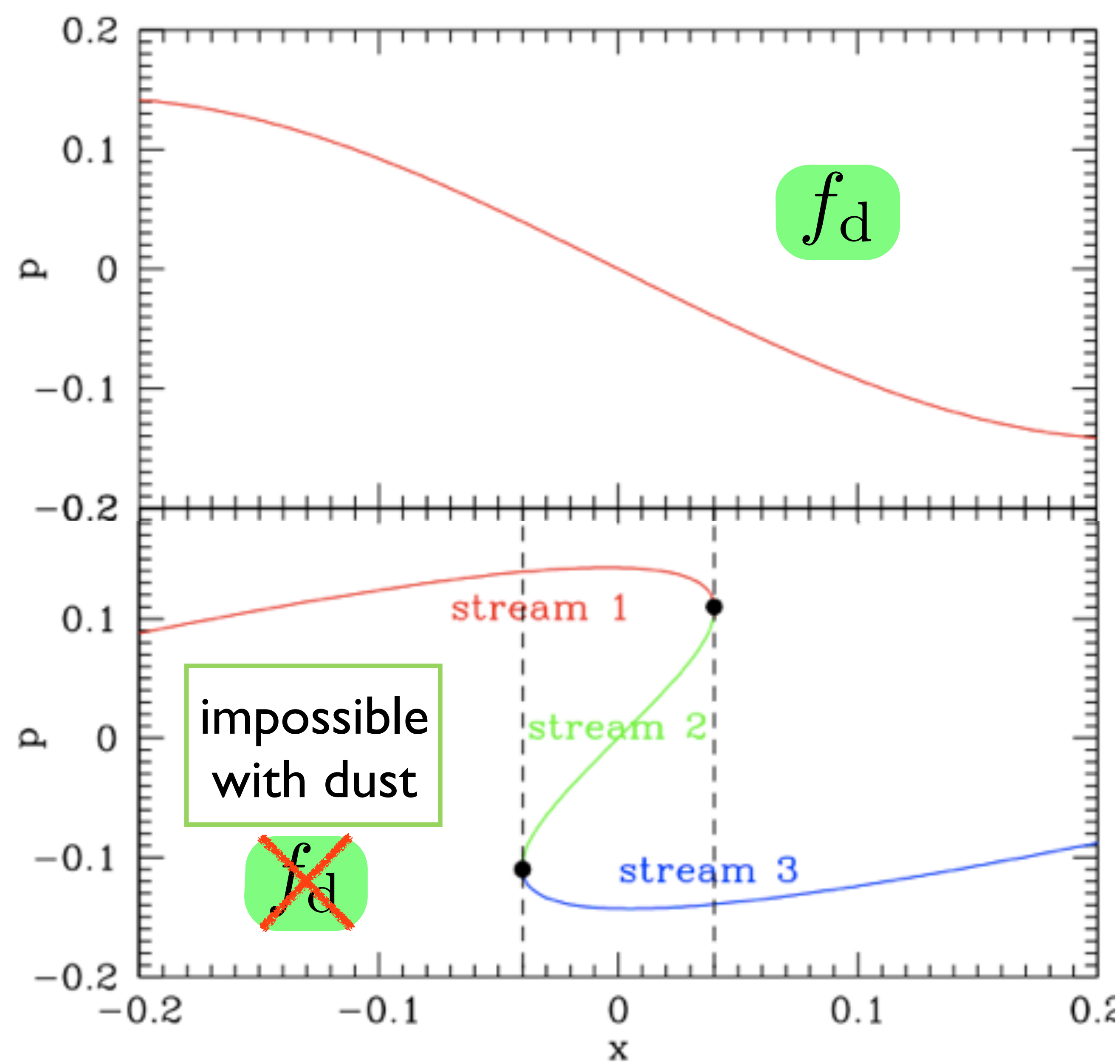
dust model

- only consistent truncation of hierarchy
- pressureless fluid: density and velocity

$$f_{\mathrm{d}}(\boldsymbol{x},\boldsymbol{p},\tau) = n(\boldsymbol{x},\tau)\delta_D^{(3)}(\boldsymbol{p} - \boldsymbol{\nabla}\phi(\boldsymbol{x},\tau))$$

Continuity $\partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

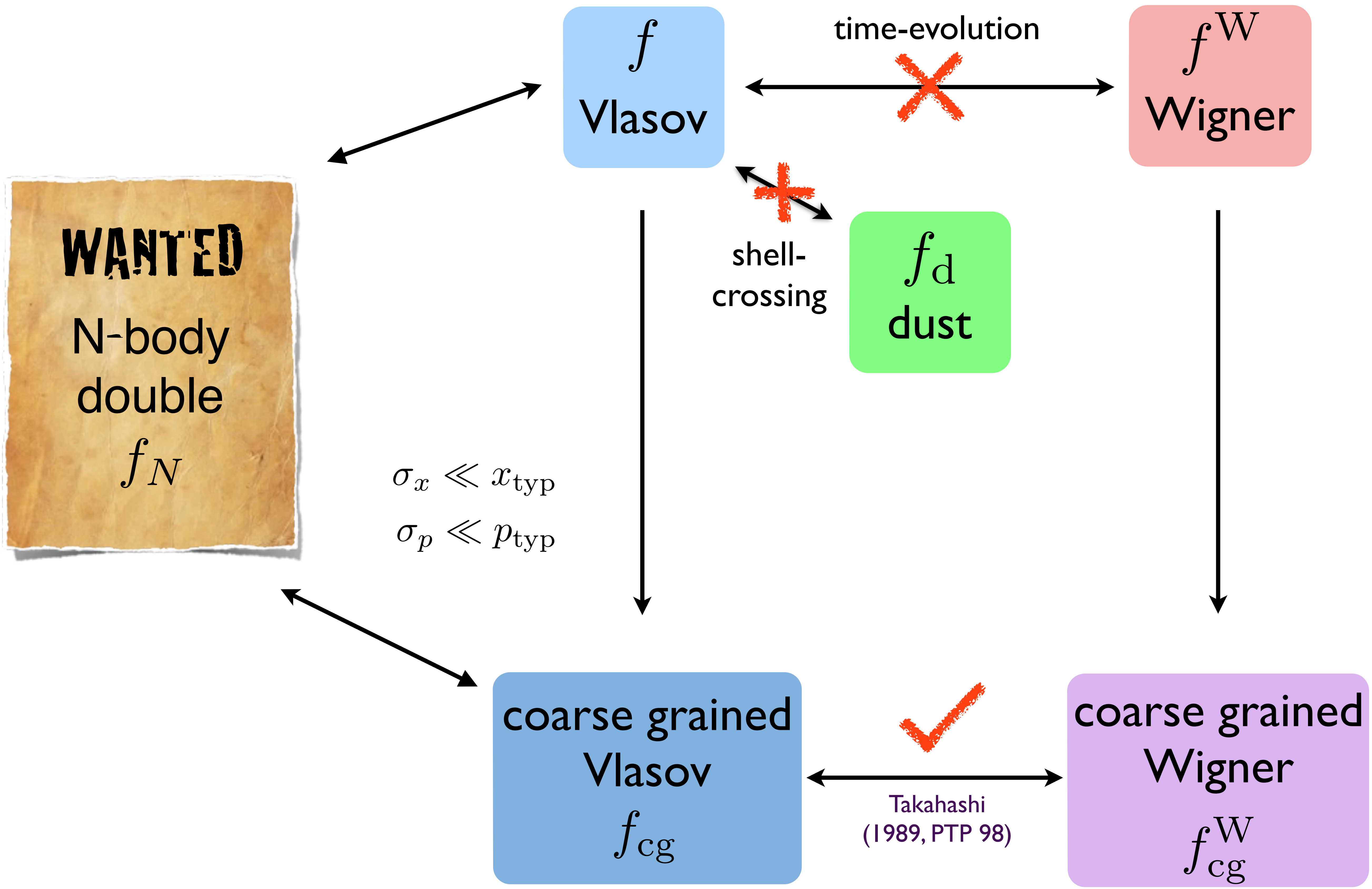
Euler $\partial_\tau \phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV$



- limited to **single-stream**
- no velocity dispersion, …

- shell-crossing singularities
- **no virialization**

Schrödinger method at a glance



Schrödinger method

Schrödinger method

- Coarse-grained Wigner function, constructed from self-gravitating field

$$\psi = \sqrt{n}\exp\left(\tfrac{i}{\hbar}\phi\right)$$

$$f_{cg}^{W}(\boldsymbol{x},\boldsymbol{p}) = \int \frac{d^3x'd^3p'}{(\pi\sigma_x\sigma_p)^3}\exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x'})^2}{2\sigma_x^2}-\frac{(\boldsymbol{p}-\boldsymbol{p'})^2}{2\sigma_p^2}\right]\int\frac{d^3\tilde{x}}{(2\pi\hbar)^3}\exp\left[2\frac{i}{\hbar}\boldsymbol{p'}\cdot\tilde{\boldsymbol{x}}\right]\psi(\boldsymbol{x'}-\tilde{\boldsymbol{x}})\bar{\psi}(\boldsymbol{x'}+\tilde{\boldsymbol{x}})$$

Schrödinger - Poisson equation

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2a^2m}\Delta + mV\right]\psi$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(|\psi|^2-1)$$

degrees of freedom

- 2: amplitude n & phase ϕ

parameters

- coarse-graining σ_x, σ_p
  - fundamental resolution $\sigma_x\sigma_p \gtrsim \hbar/2$
- Schrödinger scale $\hbar$
  - degree of restriction
  - dust as special case

Continuity $\quad \partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

Euler $\quad \partial_\tau\phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV + \frac{\hbar^2}{2am}\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)$

quantum potential

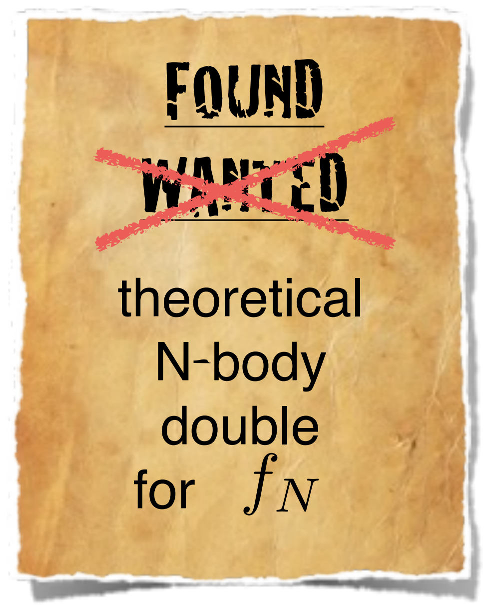
Features of Schrödinger Method

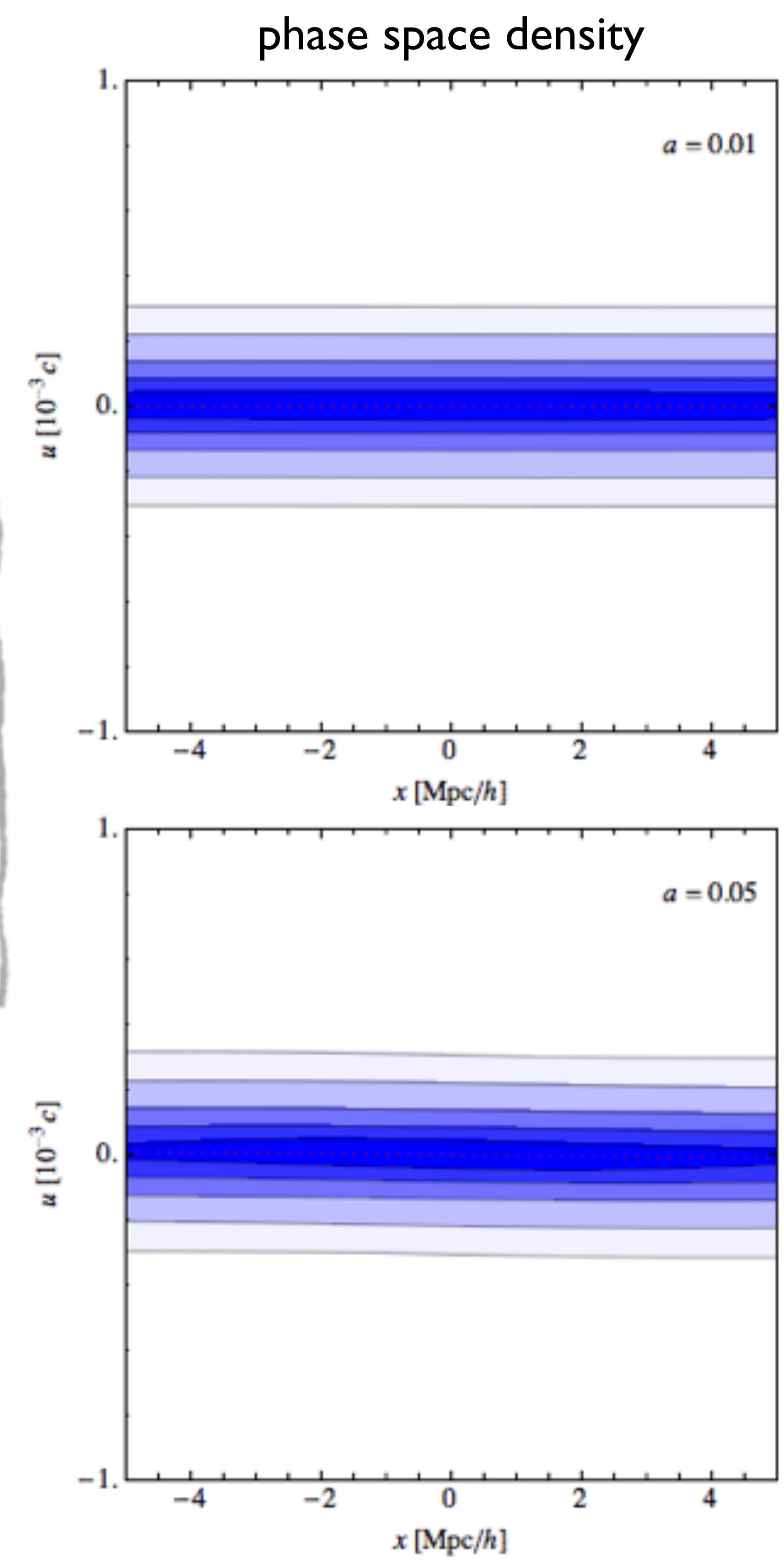
Multi-streaming

❌ dust model: fails at shell-crossing

✅ Schrödinger method: can go **beyond shell-crossing**

blue S contours: Schrödinger method
red dotted Z line: Zeldovich solution (dust model)





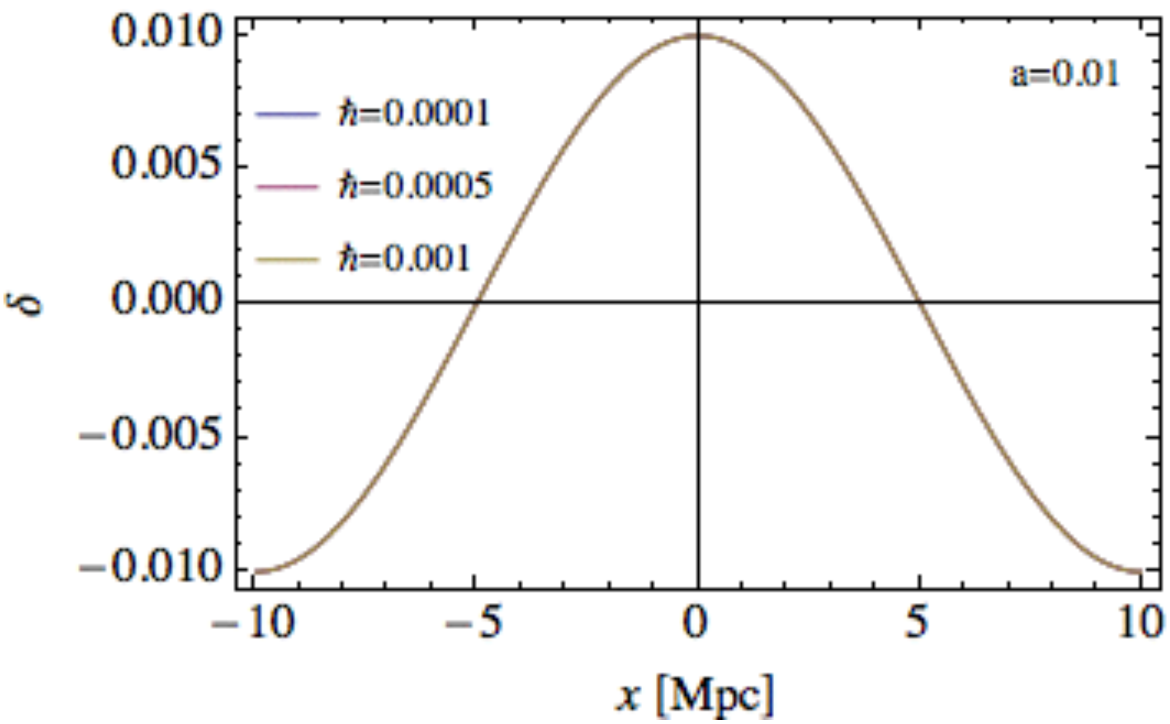
Virialization

❌ even in extended models: no **virialization**

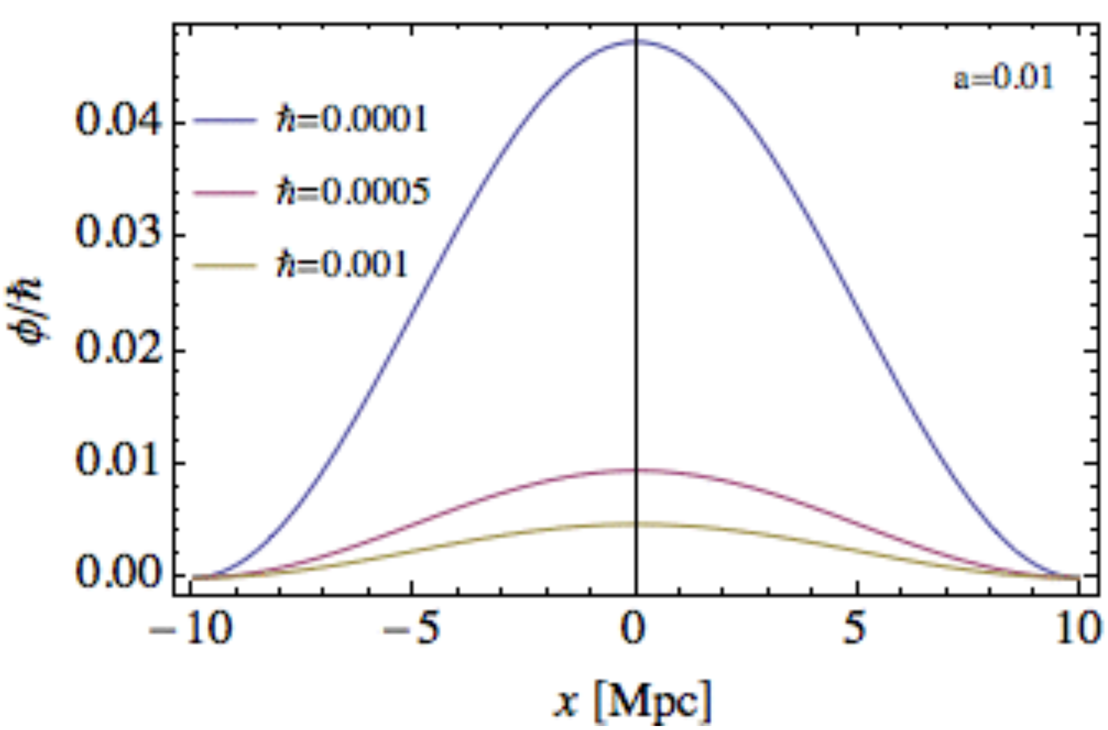
✅ Schrödinger method: **bound structures** like halos

Features of Schrödinger Method

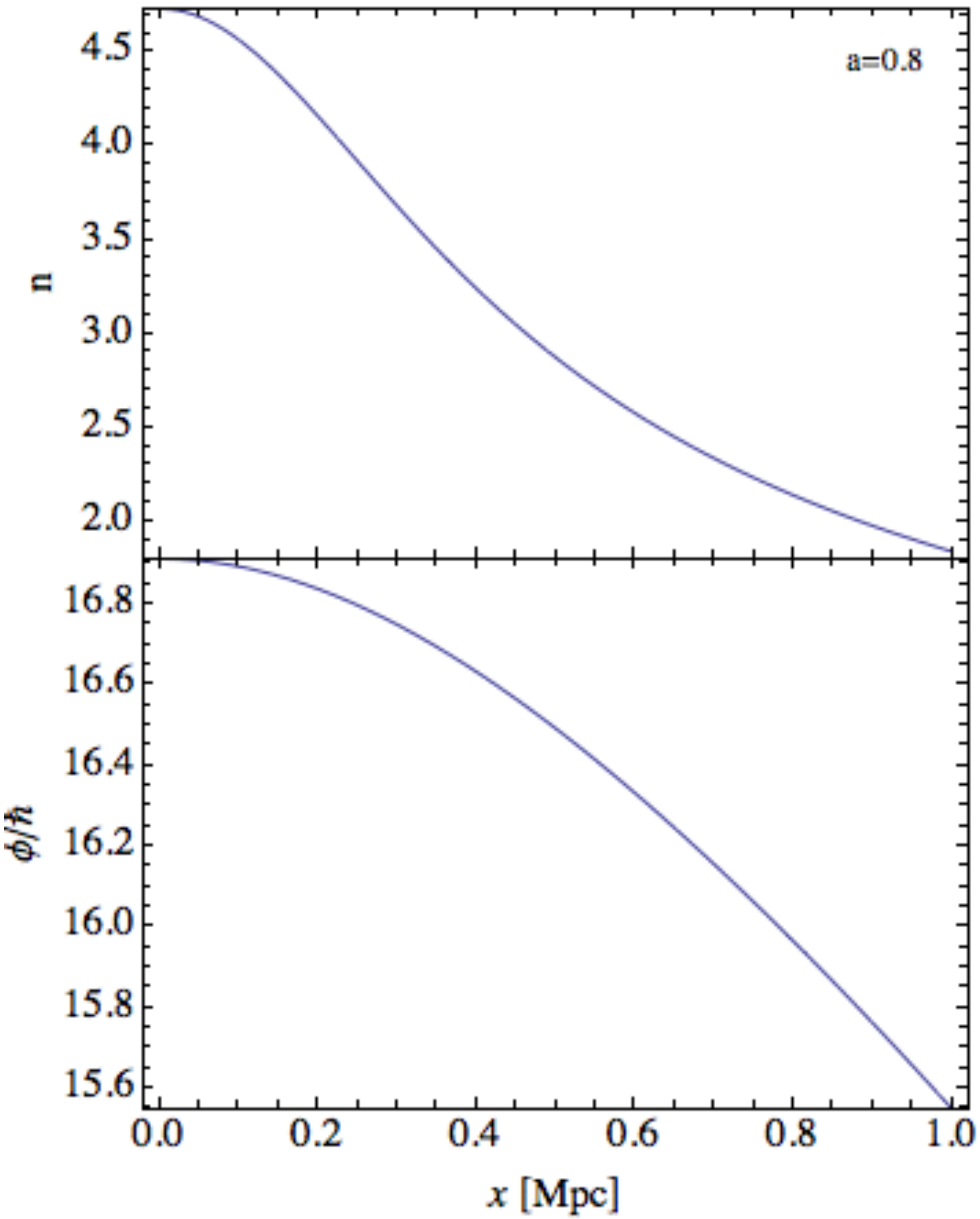
✅ **prevention of shell-crossing singularities**



⁉️ **occurrence of phase jumps**



‼️ $\psi = \sqrt{n}e^{i\phi/\hbar}$ **free of pathologies**



Features of Schrödinger Method

Schrödinger method

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special p-dependence

Cumulants

- lowest two: macroscopic density & velocity

$$\bar{n}(\boldsymbol{x}) = \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] n(\boldsymbol{x}) \qquad \bar{\boldsymbol{v}}(\boldsymbol{x}) = \frac{1}{am\bar{n}(\boldsymbol{x})} \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right] (n\boldsymbol{\nabla}\phi)(\boldsymbol{x})$$

- higher cumulants given self-consistently
  evolution equations fulfilled automatically

closure of hierarchy

CU, Kopp & Haugg (2014, PRD 90, 023517)

$$C^{(0)} = \ln n, \quad C^{(1)} = \boldsymbol{\nabla}\phi$$

$$C^{(n+2)} = -\frac{\hbar^2}{4}\boldsymbol{\nabla}\boldsymbol{\nabla}C^{(n)}$$

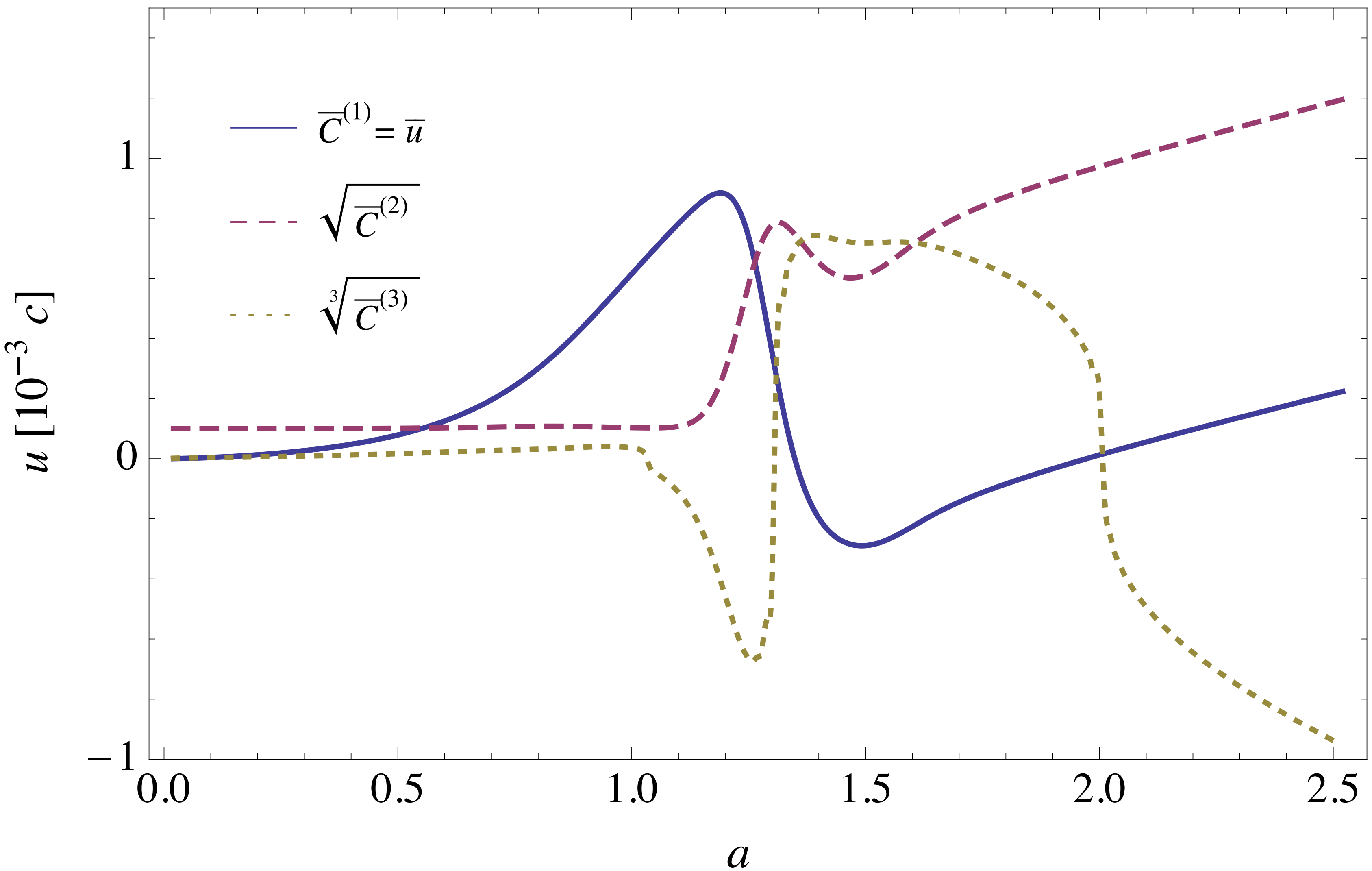
from Wigner

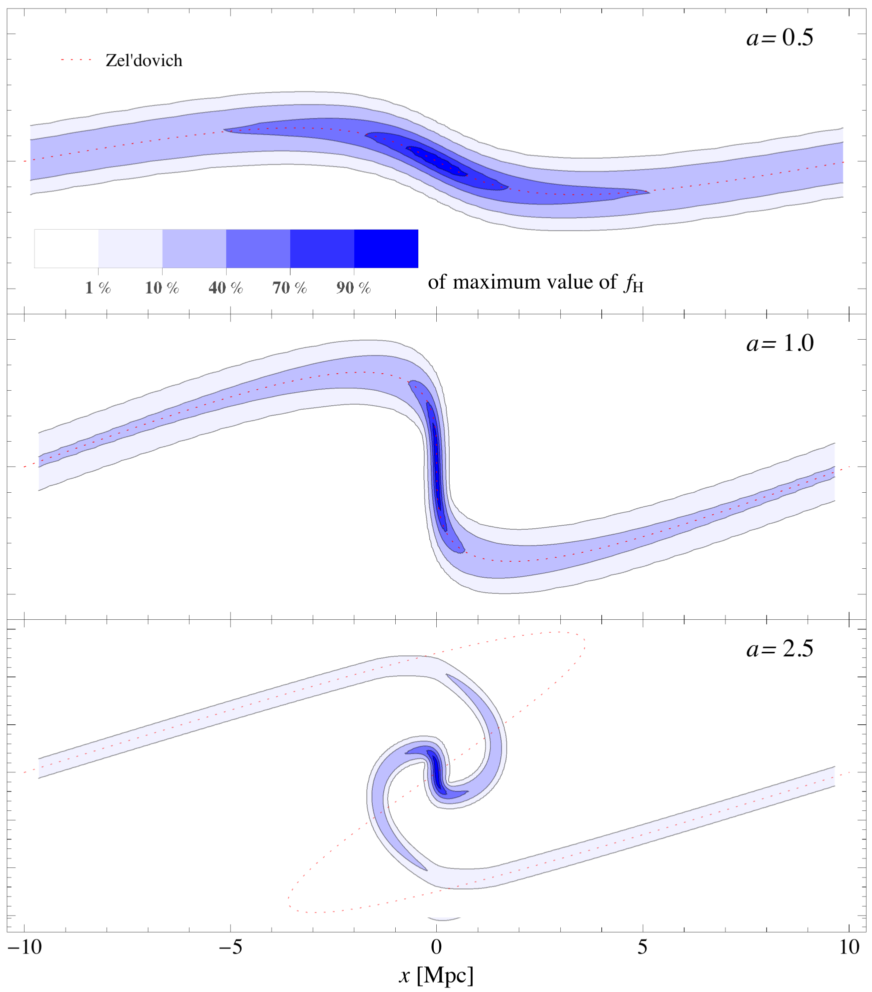
add coarse-graining to determine the moments

Features of Schrödinger Method

Multi-streaming

- higher cumulants encode **multi-streaming** effects
- during shell-crossing: **higher moments sourced dynamically**





Schrödinger method: cumulants at x = −0.5 Mpc:
all equally important after shell crossing

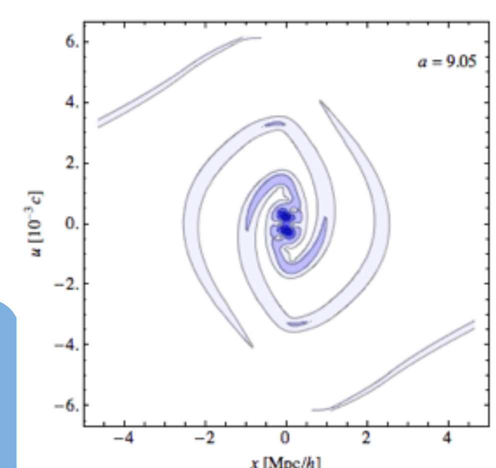
Schrödinger method at a glance

f
Vlasov

f^{W}
Wigner

f_{d}
dust

WANTED
N-body
double
f_N



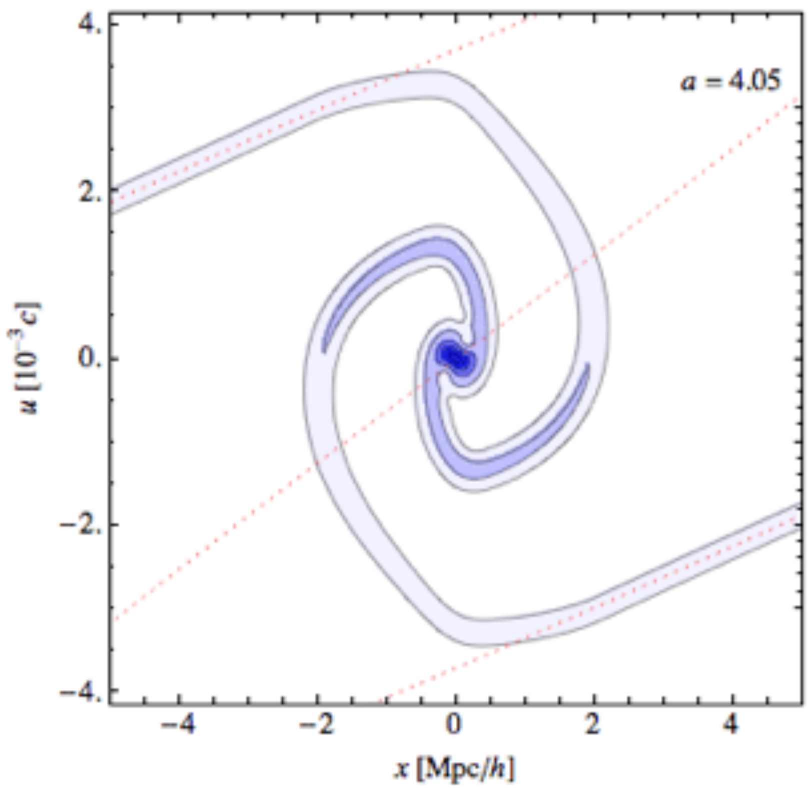
coarse grained
Vlasov
f_{cg}

coarse grained
Wigner
$f^{\mathrm{W}}_{\mathrm{cg}}$

Conclusion & Prospects

Schrödinger method
- models CDM using a self-gravitating scalar field
- analytical tool to access nonlinear stage of structure formation
  - describes multi-streaming
  - allows for virialization

CU, Kopp, Haugg
(2014, PRD 90, 023517, arXiv: 1403.5567)



Future research
- understand universal density profiles of halos (NFW)
  - search stationary solutions of gravitational collapse
- consider a flow of time or phase-space resolution $\hbar$
  - possible interpretation in terms of phase transition
- DM models: wavelike (axion), warm & (non-)relativistic neutrinos