



# Forecasts on the contamination induced by unresolved point sources in primordial NG beyond Planck

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# Overview

- Extra-galactic point sources
  - Radio sources
  - Infra-red sources
    - Late type
    - Spheroids
    - Lensed spheroids
- Point source power spectrum
  - Number counts and power spectra for different populations
  - Impact of the flux cut  $S_c$
  - Comparison with the CMB power spectrum
- Point source bispectrum
  - Bispectrum for different populations of sources (temperature and polarization)
- $f_{nl}$  estimates
  - The bias due to the point sources  $\Delta(f_{nl})$
  - The uncertainties including point sources  $\sigma(f_{nl})$

# CMB formalism

CMB anisotropies

$$X_i(\vec{n}) = \sum_{\ell m} a_{\ell m}^i Y_{\ell m}(\vec{n}) \quad X_i = \{T, E, B\}$$

Power spectrum

$$C_\ell \delta_{\ell\ell'} \delta_{mm'} = \langle a_{\ell m} a_{\ell' m'}^* \rangle$$

Bispectrum

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$$

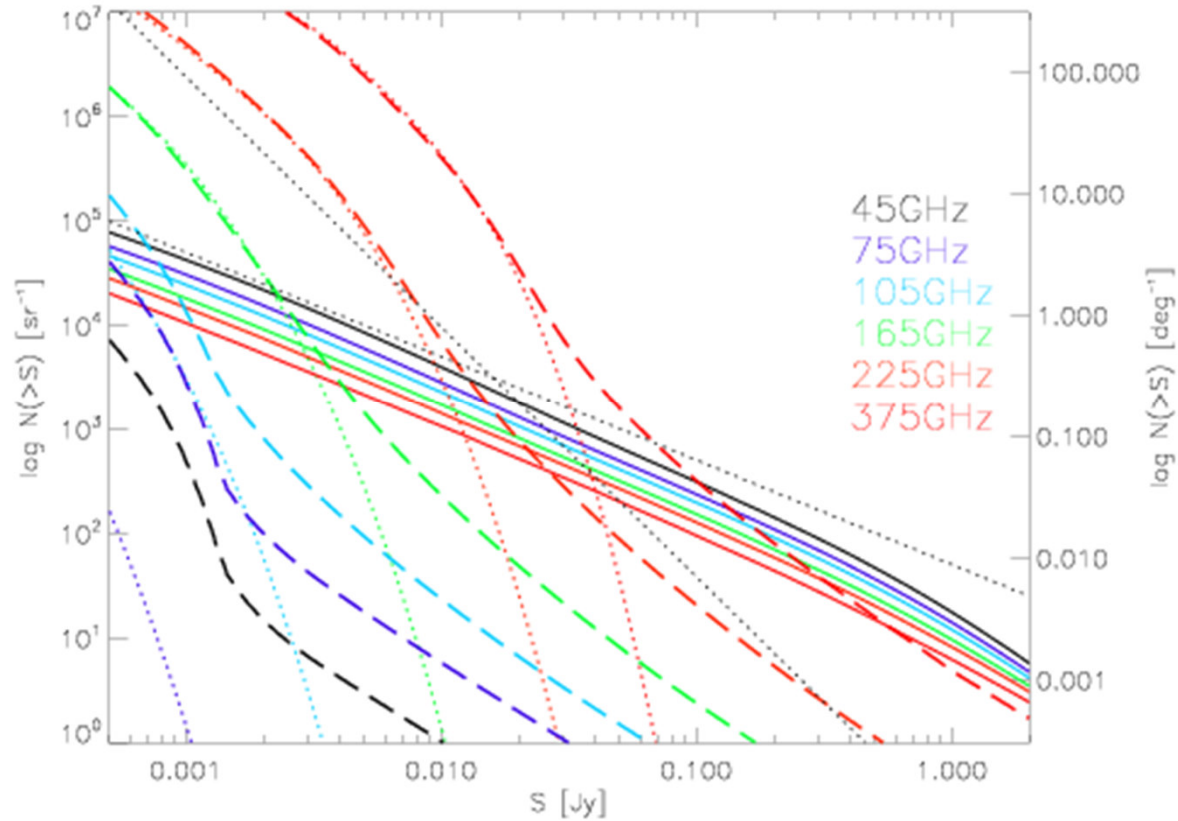
Angle averaged  
bispectrum

$$B_{\ell_1 \ell_2 \ell_3} = \sum_{m_1 m_2 m_3} B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} \begin{pmatrix} m_1 & m_2 & m_3 \\ \ell_1 & \ell_2 & \ell_3 \end{pmatrix}$$

Reduced bispectrum

$$B_{\ell_1 \ell_2 \ell_3} = b_{\ell_1 \ell_2 \ell_3} \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

# Extragalactic Point Sources



## Predicted number counts

- Radio sources: solid lines, model by Tucci et al (2011)
- Far-Infrared sources: dashed lines for total counts and dotted lines for proto-spheroidal galaxies, model by Lapi et al (2011)

# Power Spectra from Poisson distributed Point Sources

- **Temperature PS**

$$C_{T\ell} = \left( \frac{dB}{dT} \right)^2 N \langle S^2 \rangle = \left( \frac{dB}{dT} \right)^2 \int_0^{S_c} n(S) S^2 dS$$

- **Polarization PS**

$$C_{Q\ell} \approx C_{U\ell} \approx C_{E\ell} \approx C_{B\ell}$$

$$C_{Q\ell} = \left( \frac{dB}{dT} \right)^2 N \langle Q^2 \rangle = \left( \frac{dB}{dT} \right)^2 N \langle S^2 \Pi^2 \cos^2(2\varphi) \rangle = \frac{1}{2} \langle \Pi^2 \rangle C_{T\ell}$$

- **Cross PS (Temperature-Polarization)**

$$C_{TQ\ell} = \left( \frac{dB}{dT} \right)^2 \langle S^2 \Pi \cos(2\varphi) \rangle = 0$$

**Radio sources are Poissonian distributed**

**IR sources are spatially correlated**

**We use Argüeso et al (2003) prescription**

# Polarization of Point Sources

The typical linear polarization degree ( $\Pi$ ) is a **few per cent**, indication of a low order of magnetic fields.

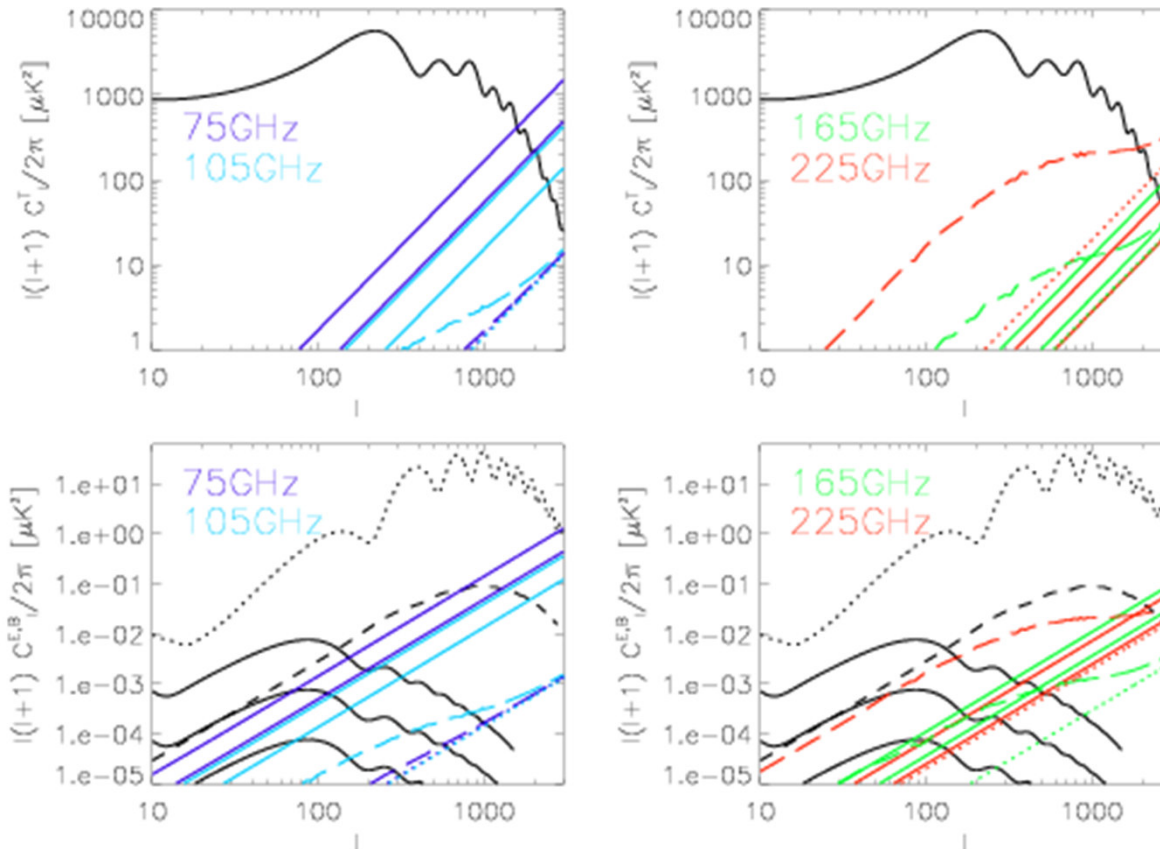
Polarization parameters for radio sources (Tucci & Toffolatti 2012):

	$\Pi_{\text{med}}$	$\langle \Pi^2 \rangle^{1/2}$
Steep	4%	6%
FSRQ	3%	3.8%
BLLac	3.6%	4.5%

Polarization parameters for far-IR sources (few data so far collected):

average polarization level of 1 %

# Point source power spectrum



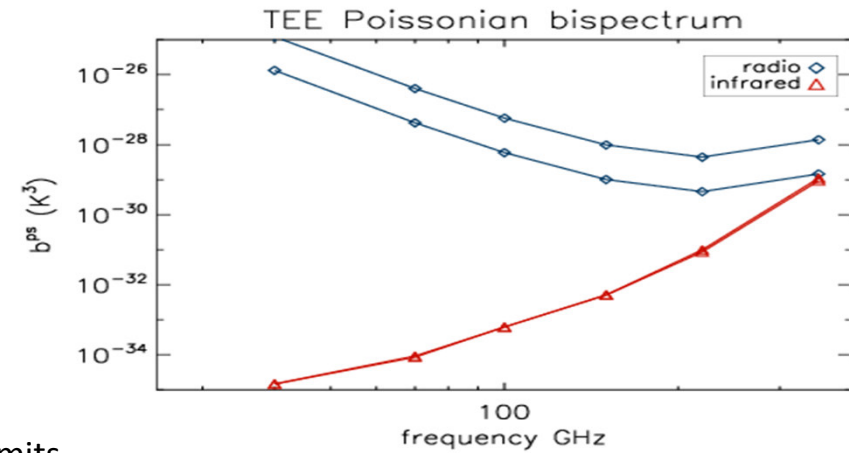
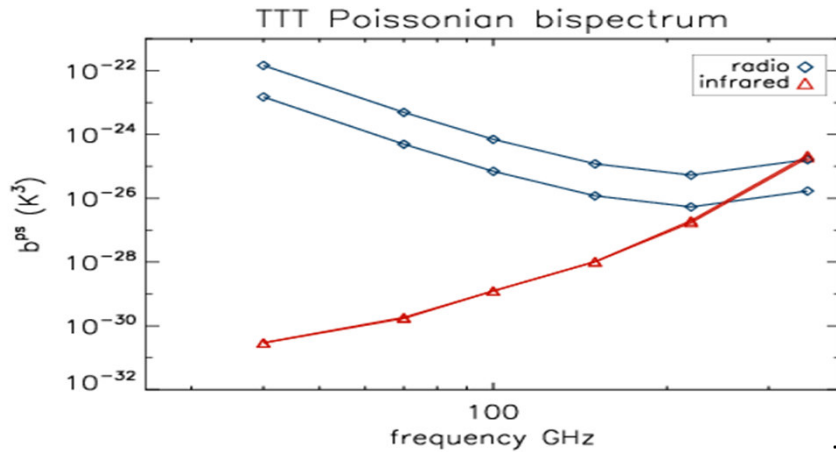
B-mode with  
 $r = 0.1, 0.01, 10^{-3}$

Two flux limits  
 $S_{lim} = 0.3 \text{ Jy}$   
 $S_{lim} = 1 \text{ Jy}$

## Power spectra

- Radio sources: solid lines, model by Tucci et al (2011)
- Far-Infrared sources: dashed lines for total counts and dotted lines for Poisson contribution, model by Lapi et al (2011)

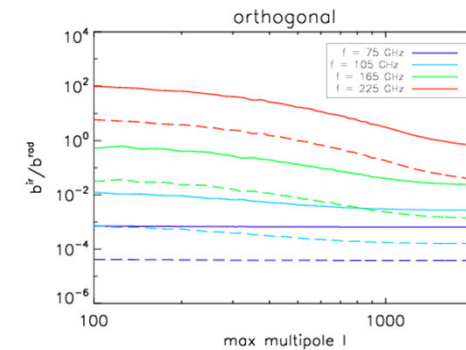
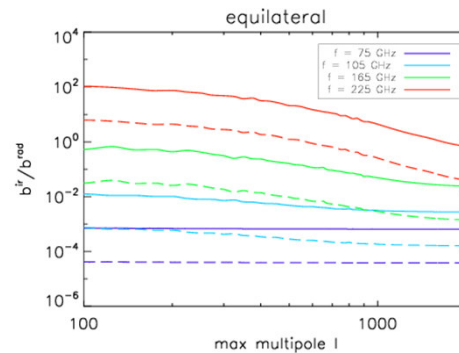
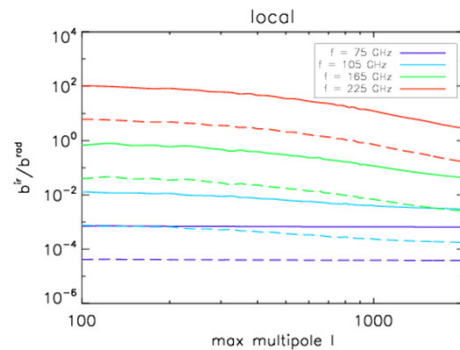
# The PS bispectrum



Two flux limits

$$S_{lim} = 0.3 \text{ Jy}$$

$$S_{lim} = 1 \text{ Jy}$$



$$B_{l_1 l_2 l_3}^{PS} = B^{radio} + B^{ir,poiss} \sqrt{\frac{C_{l_1}^{ir} C_{l_2}^{ir} C_{l_3}^{ir}}{C_{l_1}^{ir,poiss} C_{l_2}^{ir,poiss} C_{l_3}^{ir,poiss}}}$$

The bispectrum including polarization can be defined in an equivalent way (considered components **TTT**, **TEE**, **TBB**). The remaining components are equal to zero.



# Primordial non-Gaussianity

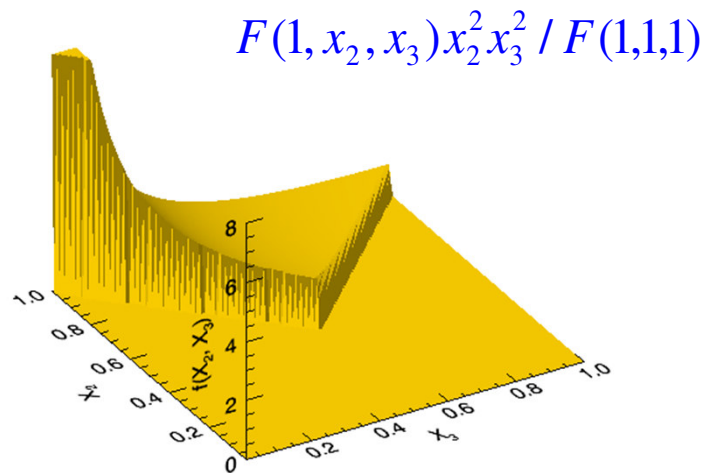
- Considered  $f_{nl}$  shapes
  - Local
  - Equilateral
  - Orthogonal
  - Flat

# Local shape

- Shape function:

$$F(k_1, k_2, k_3) = 2\Delta_{\Phi}^2 f_{nl} \left[ \frac{1}{k_1^{3-(n_s-1)} k_2^{3-(n_s-1)}} + \frac{1}{k_1^{3-(n_s-1)} k_3^{3-(n_s-1)}} + \frac{1}{k_2^{3-(n_s-1)} k_3^{3-(n_s-1)}} \right]$$

- This signal peaks in squeezed configurations:  $k_1 \approx k_2 \gg k_3$



$$x_3 = k_3 / k_1$$

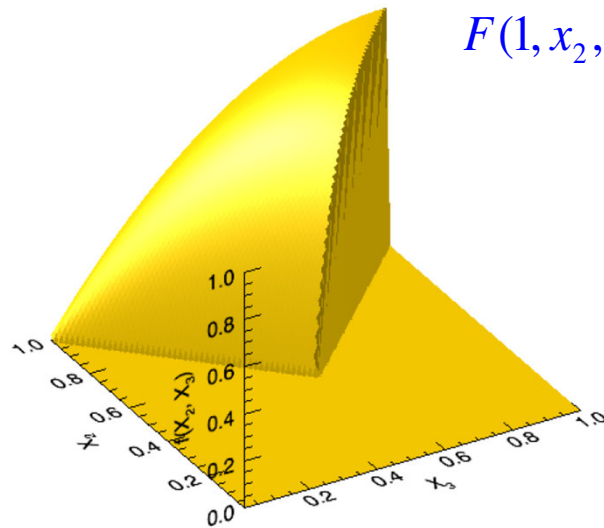
$$x_2 = k_2 / k_1$$

# Equilateral shape

- Shape function:

$$F(k_1, k_2, k_3) = 6\Delta_{\Phi}^2 f_{nl} \left[ -\frac{1}{k_1^{3-(n_s-1)} k_2^{3-(n_s-1)}} + (2 \text{ perm.}) - \frac{2}{(k_1 k_2 k_3)^{2-2(n_s-1)/3}} + \frac{1}{k_1^{1-(n_s-1)} k_2^{2-(n_s-1)} k_3^{3-(n_s-1)}} + (5 \text{ perm.}) \right]$$

- This signal peaks in *equilateral* configurations:  $k_1 \approx k_2 \approx k_3$



$$F(1, x_2, x_3) x_2^2 x_3^2 / F(1,1,1)$$

$$x_3 = k_3 / k_1$$

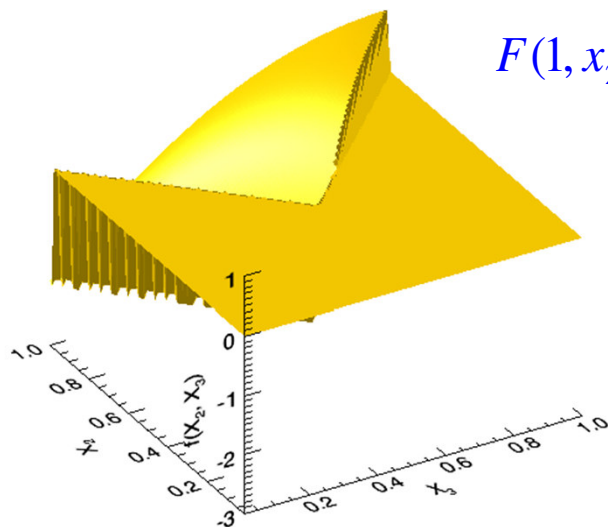
$$x_2 = k_2 / k_1$$

# Orthogonal shape

- Shape function:

$$F(k_1, k_2, k_3) = 6A f_{nl} \times \left\{ \frac{3}{k_1^{4-n_s} k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s} k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s} k_1^{4-n_s}} - \frac{8}{(k_1 k_2 k_3)^{2(4-n_s)/3}} + \left[ \frac{3}{k_1^{4-n_s} k_2^{4-n_s} k_3^{4-n_s}} + (5 \text{ perm.}) \right] \right\}$$

- This signal peaks in *equilateral*  $k_1 \approx k_2 \approx k_3$  and flat  $k_1 = k_2 + k_3$



$$F(1, x_2, x_3) x_2^2 x_3^2 / F(1,1,1)$$

$$x_3 = k_3 / k_1$$

$$x_2 = k_2 / k_1$$

# Flat shape

- Shape function:

$$F(k_1, k_2, k_3) = 6A f_{nl} \times \left\{ \frac{3}{k_1^{4-n_s} k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s} k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s} k_1^{4-n_s}} - \frac{8}{(k_1 k_2 k_3)^{2(4-n_s)/3}} + \left[ \frac{3}{k_1^{4-n_s} k_2^{4-n_s} k_3^{4-n_s}} + (5 \text{ perm.}) \right] \right\}$$

- This signal peaks in flat  $k_1 = k_2 + k_3$  configurations

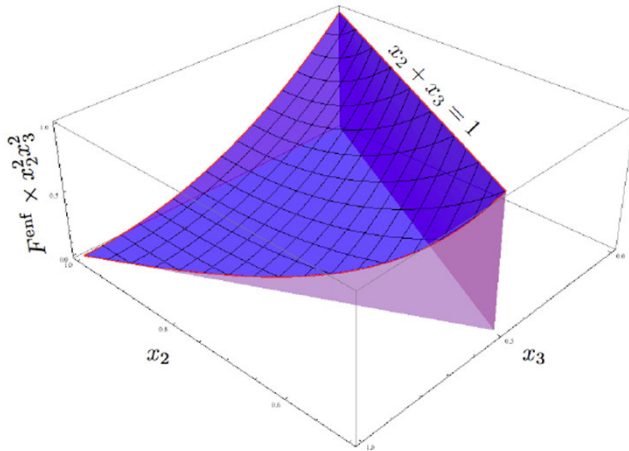


Figure 4. The enfolded template shape  $F(x_2, x_3) \times x_2^2 x_3^2$ .

$$F(1, x_2, x_3) x_2^2 x_3^2 / F(1,1,1)$$

Image from  
arXiv:0901.4044

$$x_3 = k_3 / k_1$$

$$x_2 = k_2 / k_1$$

## Primordial non-Gaussianity $f_{nl}$ bias estimator (T)

$$\Delta f_{nl} = \frac{\sum_{l_1 \leq l_2 \leq l_3 \leq l_{\max}} \frac{B_{l_1 l_2 l_3}^{ps} B_{l_1 l_2 l_3}^{prim}}{\Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}}}{\sum_{l_1 \leq l_2 \leq l_3 \leq l_{\max}} \frac{B_{l_1 l_2 l_3}^{prim} B_{l_1 l_2 l_3}^{prim}}{\Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}}} \quad \sigma^2(f_{nl}) = \frac{1}{\sum_{l_1 \leq l_2 \leq l_3 \leq l_{\max}} \frac{B_{l_1 l_2 l_3}^{prim} B_{l_1 l_2 l_3}^{prim}}{\Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}}}$$

Total power spectrum

$$C_l = \left( C_l^{CMB} + C_l^{IRPS} + C_l^{Radio PS} \right) b_l^2 w_l^2 + C_l^{noise}$$

CMB power spectrum

Infra-red point source power spectrum

radio point source power spectrum

Instrumental noise spectrum

## Primordial non-Gaussianity $f_{nl}$ bias estimator (T and E)

$$\Delta f_{nl} = \frac{\sum_{ijk, rst, l_1 \leq l_2 \leq l_3 \leq l_{\max}} B_{l_1 l_2 l_3}^{ijk, ps} C_{ijk, rst}^{-1} B_{l_1 l_2 l_3}^{rst, prim}}{\sum_{ijk, rst, l_1 \leq l_2 \leq l_3 \leq l_{\max}} B_{l_1 l_2 l_3}^{ijk, prim} C_{ijk, rst}^{-1} B_{l_1 l_2 l_3}^{rst, prim}} \quad \sigma^2(f_{nl}) = \frac{1}{\sum_{ijk, rst, l_1 \leq l_2 \leq l_3 \leq l_{\max}} B_{l_1 l_2 l_3}^{prim} C_{ijk, rst}^{-1} B_{l_1 l_2 l_3}^{prim}}$$

Indices  $i, j, k, r, s, t = \{T, E\}$

Power  
spectrum  
covariance

$$C_{ijk, pqr} = C_{l_1}^{ip} C_{l_2}^{jq} C_{l_3}^{kr} + C_{l_1}^{ip} C_{l_2}^{jr} C_{l_3}^{kq} \delta_{l_2 l_3} + C_{l_1}^{ir} C_{l_2}^{jq} C_{l_3}^{kp} \delta_{l_1 l_3} + C_{l_1}^{iq} C_{l_2}^{jp} C_{l_3}^{kr} \delta_{l_1 l_2} +$$

$$+ C_{l_1}^{iq} C_{l_2}^{jr} C_{l_3}^{kp} \delta_{l_1 l_2} \delta_{l_2 l_3} \delta_{l_1 l_3} + C_{l_1}^{ir} C_{l_2}^{jp} C_{l_3}^{kq} \delta_{l_1 l_3} \delta_{l_2 l_1} \delta_{l_2 l_3}$$

Polarization non-zero point source power spectrum  
and bispectrum (even number of E)

$$b^{TEE} = \langle p^2 \rangle \langle \cos^2(2\phi) \rangle b^{TTT}$$

$$b^{TTE} = b^{EEE} = 0$$

## Characteristic parameters for a future CMB mission

Freq (GHz)	45	75	105	165	225	375
Beam FWHM	23.3	14.0	10.0	6.4	4.7	2.8
Temp: $\sigma_{\text{noise}}$ ( $\mu\text{K}\cdot\text{arcmin}$ )	5.25	2.73	2.68	2.67	2.64	68.6
Pol: $\sigma_{\text{noise}}$ ( $\mu\text{K}\cdot\text{arcmin}$ )	9.07	4.72	4.63	4.61	4.57	119

Parameters from CORE white paper <http://arxiv.org/abs/1102.2181>

- Considered two flux limits for each frequency:

$$S_{\text{lim}} = 0.3 \text{ Jy}$$

$$S_{\text{lim}} = 1 \text{ Jy}$$

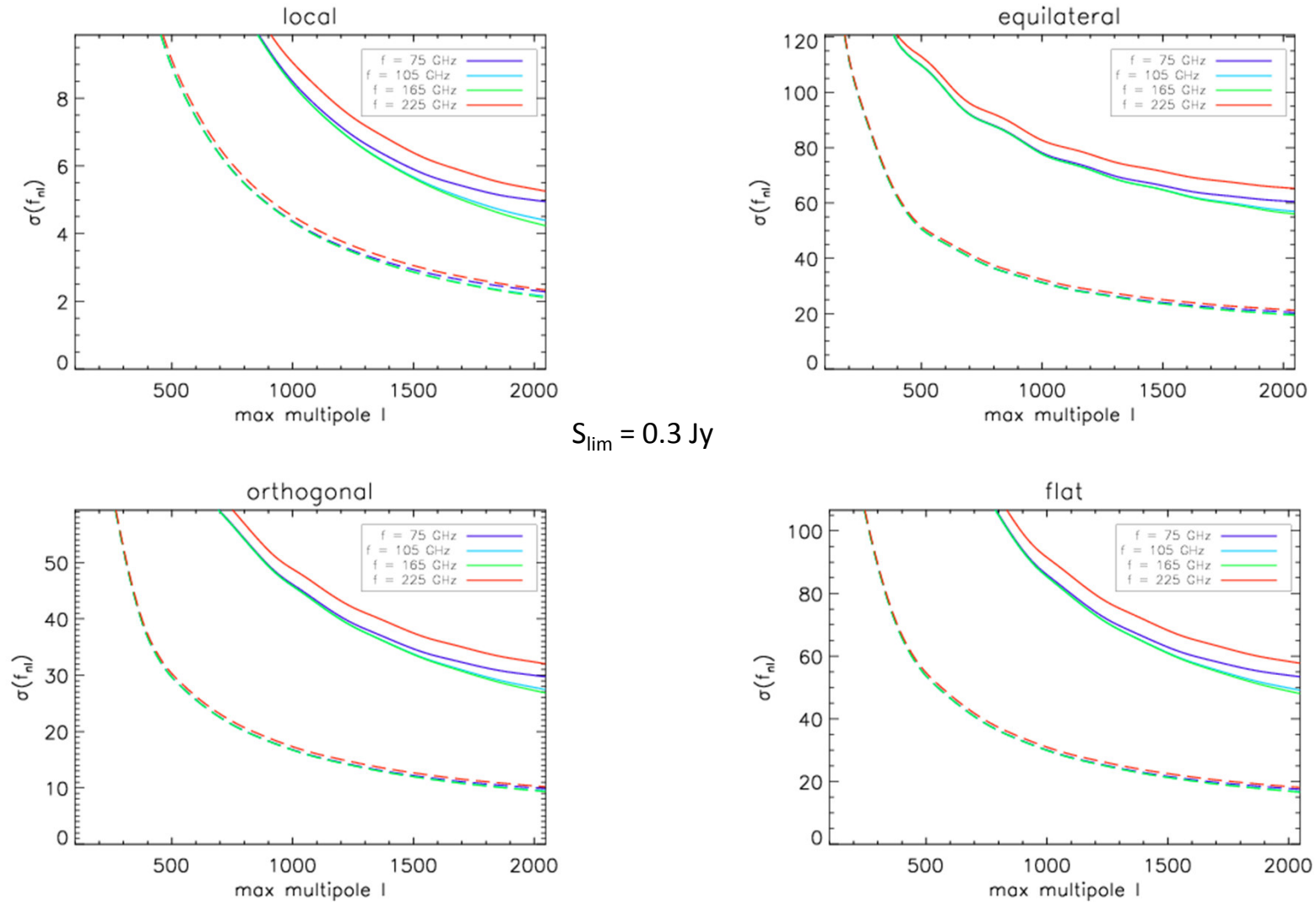
- Maximum multipole  $l_{\text{max}} = 2048$
- Considered cases:
  - Ideal conditions
  - CORE (2011) instrumental conditions



# Expected uncertainties in $f_{nl}$

(Ideal conditions)

AC, M. Tucci et al. (2013)



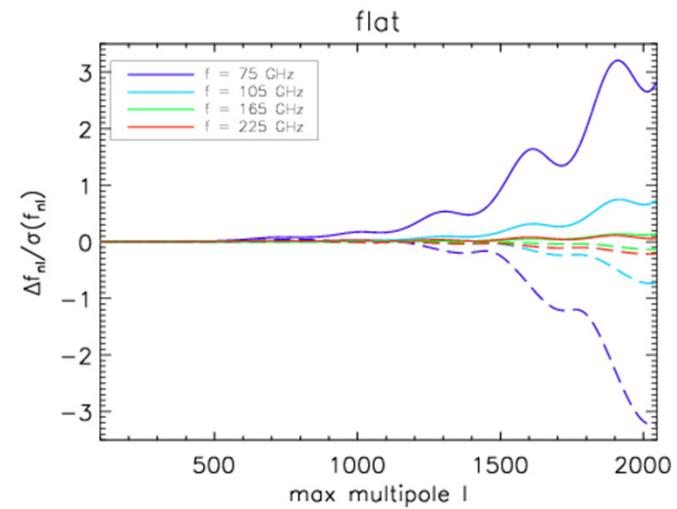
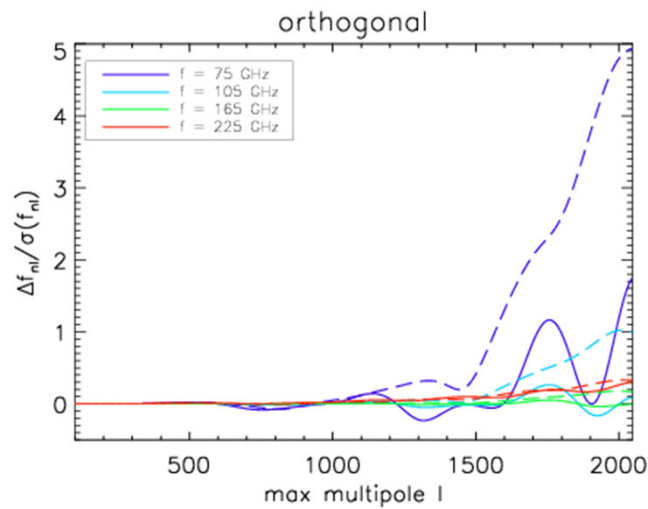
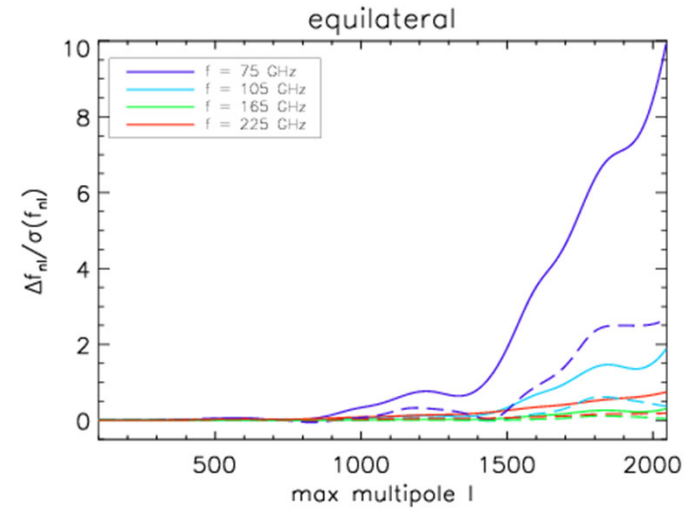
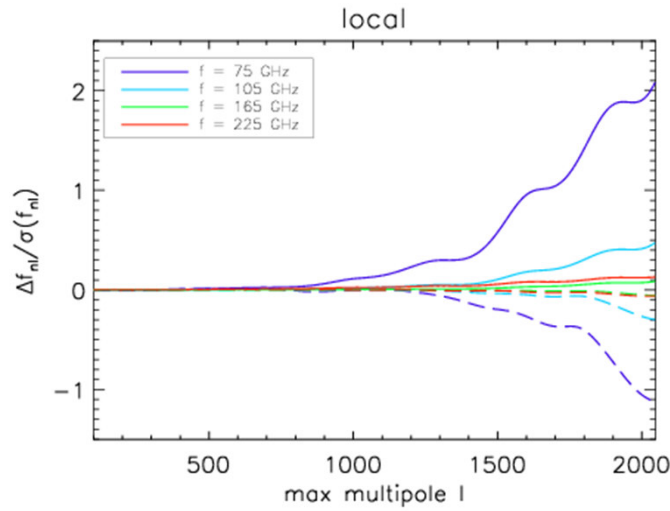
$S_{lim} = 0.3 \text{ Jy}$

Solid lines: T  
Dashed lines: T and E

# Point source contamination to $f_{nl}$

(Ideal conditions)

AC, M. Tucci et al. (2013)

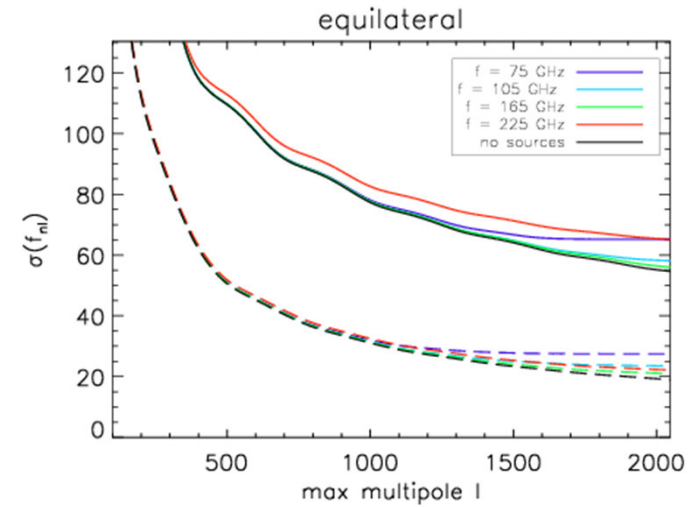
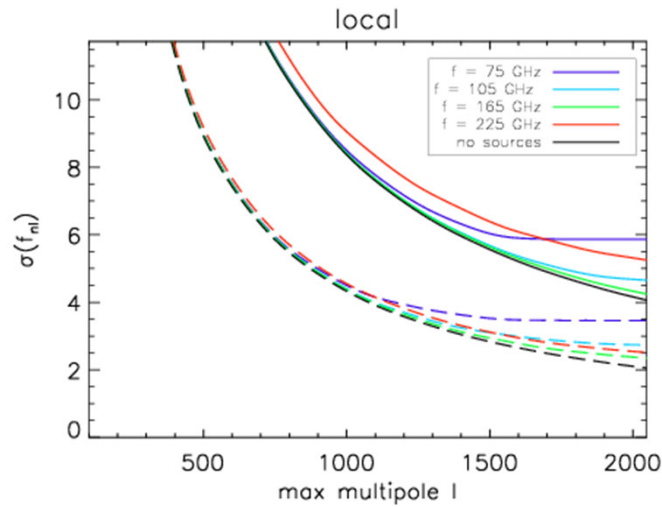


Solid lines: T  
Dashed lines: T and E

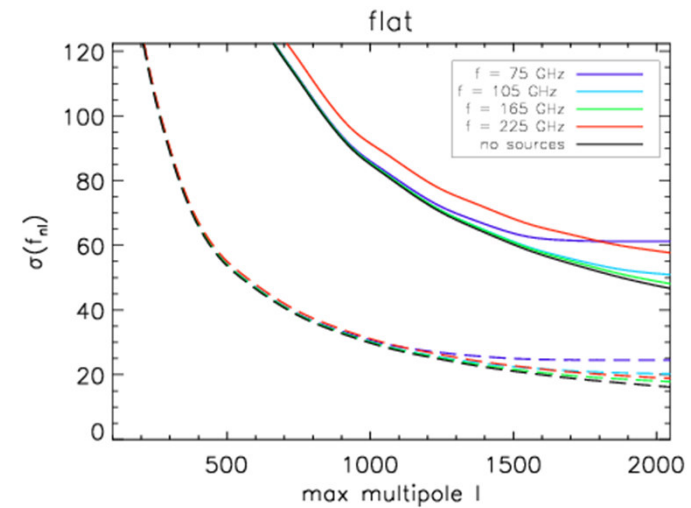
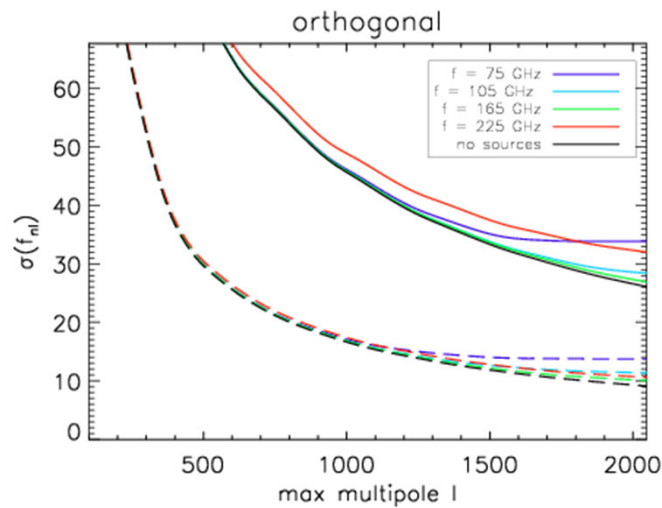
# Expected uncertainties in $f_{nl}$

(COre conditions)

AC, M. Tucci et al. (2013)



$S_{lim} = 0.3 \text{ Jy}$



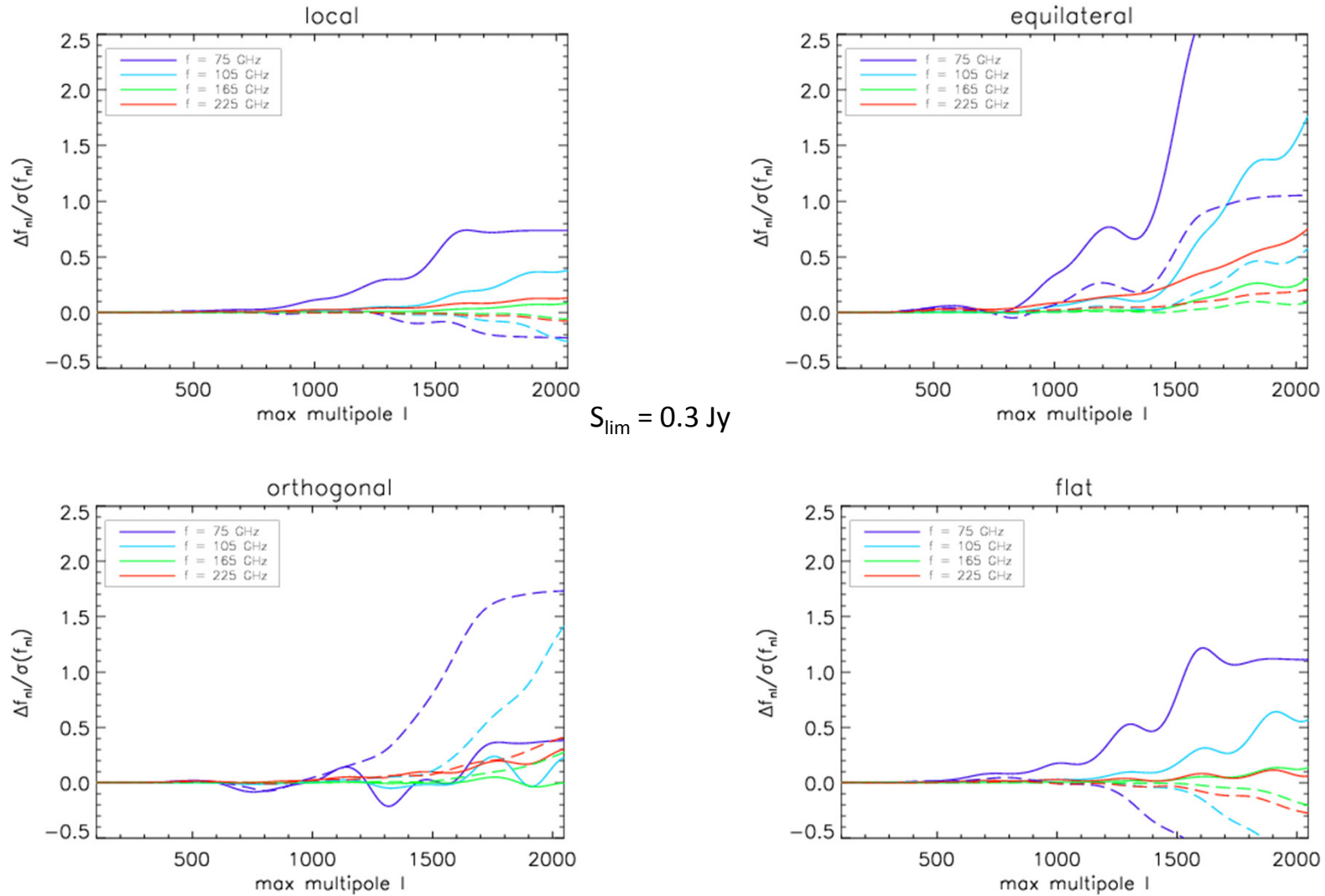
Solid lines: T

Dashed lines: T and E

# Point source contamination to $f_{nl}$

(CORe conditions)

AC, M. Tucci et al. (2013)



$S_{lim} = 0.3 \text{ Jy}$

Solid lines: T  
Dashed lines: T and E

# Expected uncertainties in $f_{nl}$

(Ideal conditions)

AC, M. Tucci et al. (2013)

**Table 2.** The expected uncertainty  $\sigma(f_{nl})$ , bias  $\Delta f_{nl}$  and relative bias  $\Delta f_{nl}/\sigma(f_{nl})$  at  $\ell_{max} = 2048$  from temperature only and temperature plus polarization for an ideal mission.  $S_c = 0.3 \text{ Jy}$  is taken.

	case	T	T	T	T	T	T	T+E	T+E	T+E	T+E	T+E	T+E
	Freq.	45	75	105	165	225	375	45	75	105	165	225	375
local	$\sigma(f_{nl})$	7.4	4.9	4.4	4.2	5.2	29.7	2.8	2.3	2.1	2.1	2.3	3.3
	$\Delta f_{nl}$	82.1	10.3	2.1	0.4	0.7	25.6	-9.7	-2.6	-0.6	-0.1	-0.1	-0.1
	$\Delta f_{nl}/\sigma(f_{nl})$	11.1	2.1	0.5	0.1	0.1	0.9	-3.5	-1.1	-0.3	-0.1	-0.1	-0.0
equilateral	$\sigma(f_{nl})$	75.4	60.4	56.8	55.9	65.1	232.4	24.0	20.3	19.5	19.2	21.1	35.3
	$\Delta f_{nl}$	4629.3	599.9	107.2	17.2	49.1	950.7	480.2	54.5	7.3	0.8	4.0	17.9
	$\Delta f_{nl}/\sigma(f_{nl})$	61.4	9.9	1.9	0.3	0.8	4.1	20.0	2.7	0.4	0.0	0.2	0.5
orthogonal	$\sigma(f_{nl})$	40.1	29.7	27.4	26.8	32.0	139.0	11.6	9.8	9.4	9.2	10.1	16.5
	$\Delta f_{nl}$	955.5	51.7	2.5	-0.2	9.7	371.4	308.7	48.3	9.4	1.6	3.4	7.2
	$\Delta f_{nl}/\sigma(f_{nl})$	23.8	1.7	0.1	-0.0	0.3	2.7	26.7	4.9	1.0	0.2	0.3	0.4
flat	$\sigma(f_{nl})$	73.7	53.3	49.0	47.9	57.6	284.7	20.6	17.4	16.6	16.4	17.9	29.1
	$\Delta f_{nl}$	598.6	150.5	36.0	6.7	3.4	-66.0	-311.6	-56.3	-12.2	-2.2	-3.9	-5.2
	$\Delta f_{nl}/\sigma(f_{nl})$	8.1	2.8	0.7	0.1	0.1	-0.2	-15.1	-3.2	-0.7	-0.1	-0.2	-0.2

$S_{lim} = 0.3 \text{ Jy}$

AC, M. Tucci et al. (2013) (arXiv:1301.1544)

# Expected uncertainties in $f_{nl}$

(CORe conditions)

AC, M. Tucci et al. (2013)

**Table 4.** The expected uncertainty  $\sigma(f_{nl})$ , bias  $\Delta f_{nl}$  and relative bias  $\Delta f_{nl}/\sigma(f_{nl})$  at  $\ell_{\max} = 2048$  from temperature only and temperature plus polarization for a CORe-like mission.  $S_c = 0.3$  Jy is adopted.

	Case Freq.	$T$	$T$	$T$	$T$	$T$	$T$	$T+E$	$T+E$	$T+E$	$T+E$	$T+E$	$T+E$
		45	75	105	165	225	375	45	75	105	165	225	375
Local	$\sigma(f_{nl})$	9.7	5.9	4.7	4.2	5.3	29.8	6.0	3.5	2.7	2.3	2.5	20.6
	$\Delta f_{nl}$	16.0	4.3	1.8	0.4	0.7	25.1	1.9	-0.8	-0.7	-0.1	-0.2	8.2
	$\Delta f_{nl}/\sigma(f_{nl})$	1.7	0.7	0.4	0.1	0.1	0.8	0.3	-0.2	-0.3	-0.1	-0.1	0.4
Equilateral	$\sigma(f_{nl})$	84.3	65.2	58.1	56.0	65.2	232.6	41.1	27.5	23.5	21.0	22.2	147.2
	$\Delta f_{nl}$	421.9	202.4	102.7	17.5	49.0	935.9	66.2	28.9	13.5	1.9	4.6	343.8
	$\Delta f_{nl}/\sigma(f_{nl})$	5.0	3.1	1.8	0.3	0.8	4.0	1.6	1.1	0.6	0.1	0.2	2.3
Orthogonal	$\sigma(f_{nl})$	49.7	33.8	28.4	26.9	32.0	139.1	21.7	13.8	11.4	10.1	10.6	82.0
	$\Delta f_{nl}$	39.9	12.9	6.5	-0.1	9.7	367.8	12.1	23.8	16.1	2.8	4.4	130.2
	$\Delta f_{nl}/\sigma(f_{nl})$	0.8	0.4	0.2	-0.0	0.3	2.6	0.6	1.7	1.4	0.3	0.4	1.6
Flat	$\sigma(f_{nl})$	93.8	61.2	50.9	48.1	57.7	285.1	39.1	24.5	20.2	17.9	18.9	148.1
	$\Delta f_{nl}$	190.3	68.1	29.0	6.5	3.4	-68.8	10.4	-26.3	-20.4	-3.7	-5.2	-38.4
	$\Delta f_{nl}/\sigma(f_{nl})$	2.0	1.1	0.6	0.1	0.1	-0.2	0.3	-1.1	-1.0	-0.2	-0.3	-0.3

$S_{\text{lim}} = 0.3$  Jy

AC, M. Tucci et al. (2013) (arXiv:1301.1544)

# Conclusions

- **Power spectrum of point sources:**
  - Temperature: contamination of radio sources is mostly located at multipoles  $l > 2000$
  - Significant contamination for the cosmological B mode at all frequencies if  $r < 10^{-2}$
- **Bispectrum of point sources:**
  - $B_{PS}$  is zero for odd polarization combinations
  - Radio  $B_{PS}$  larger than IR  $B_{PS}$  for  $\nu \leq 200$  GHz (both TTT and TEE)
- **Non-Gaussianity:  $f_{nl}$  bias:**
  - The polarization reduces the bias and the  $\sigma(f_{nl})$  on all the considered  $f_{nl}$  shapes
  - Negligible bias contamination for  $150 < \nu < 225$  GHz ( $S_{cut}=0.3$  Jy)