Constraints on Inflationary Models with *Planck*+QUIJOTE

I. Sendra, V. Salzano, R. Lazkoz, J. J. Blanco Pillado

Dpto. de Física Teórica e Historia de la Ciencia
Universidad del País Vasco - Euskal Herriko Unibertsitatea

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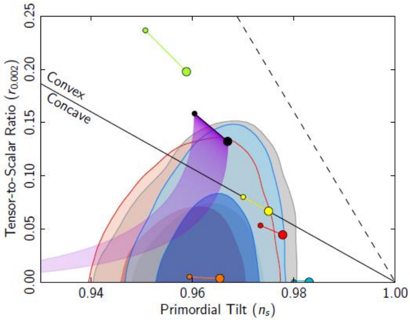
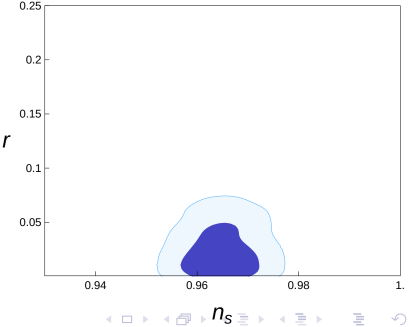
CMB simulations / mock data

- Fiducial model (Planck ΛCDM) + CAMB : $n_s = 0.9635$, $r = 0.01$
  - no running-index, B-modes and lensing included
- Errors: combining FWHM prescriptions from *Planck* and QUIJOTE
  - 3 channels from Planck: 100, 143, 217
  - 2 channel from QUIJOTE: 30, 42
- Statistical Analysis: Modified CosmoMC
  - Fitting parameters: $\Omega_b h^2$, $\Omega_c h^2$, θ, τ, n_s, r, $\log A$
- Planck+WMAP9: $n_s = 0.9624 \pm 0.0075$, $r < 0.12$ (95%)
  Planck+QUIJOTE: $n_s = 0.965 \pm 0.005$, $r < 0.06$ (95%)





Inflationary Theory essentials

Testing Inflation through measurable quantities:

- Scalar spectral index: n_s
- Tensor-to-scalar ratio: r
- Running index: $\frac{\mathrm{d}n_s}{\mathrm{d}\ln k}$

Scalar spectral index from scalar (curvature/matter) perturbations:

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3 P_{\mathcal{R}}(k)}{2\pi^2} = \Delta^2_{\mathcal{R}}(k_0) \left(\frac{k}{k_0}\right)^{n_s(k_0)-1+\frac{1}{2}\ln\left(\frac{k}{k_0}\right)\frac{\mathrm{d}n_s}{\mathrm{d}\ln k}} \tag{1}$$

Tensor perturbations (gravitational waves):

$$\Delta^2_h(k) = \frac{k^3 P_h(k)}{2\pi^2} = \Delta^2_h(k_0) \left(\frac{k}{k_0}\right)^{n_t} \tag{2}$$

With:

$$r = \frac{\Delta^2_h(k_0)}{\Delta^2_{\mathcal{R}}(k_0)} \tag{3}$$

Inflation:

- accelerate expansion period ($\ddot{a} > 0$)
- is driven by one/many spatially homogeneous scalar field ϕ:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (4)$$

- is completely determined by the potential $V(\phi)$; Friedmann equations:

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi}{3m_{Pl}^2}\left[V(\phi) + \frac{1}{2}\dot{\phi}^2\right] \\ \left(\frac{\ddot{a}}{a}\right) &= \frac{8\pi}{3m_{Pl}^2}\left[V(\phi) - \dot{\phi}^2\right] \end{aligned} \quad (5)$$

- if we write $H = H(\phi)$

$$\dot{\phi} = -\frac{m_{Pl}^2}{4\pi}H'(\phi) \qquad \left[H'(\phi)\right]^2 - \frac{12\pi}{m_{Pl}^2}H^2(\phi) = -\frac{32\pi^2}{m_{Pl}^4}V(\phi) \quad (6)$$

- the Hamilton-Jacobi can be written:

$$H^2(\phi)\left[1 - \frac{1}{3}\epsilon(\phi)\right] = \frac{8\pi}{3m_{Pl}^2}V(\phi) \qquad \epsilon \equiv \frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2 \quad (7)$$

- In the Friedmann equation:

$$\left(\frac{\ddot{a}}{a}\right) = H^2(\phi)[1 - \epsilon(\phi)] \tag{8}$$

Acceleration if $\epsilon < 1$ or inflation ends when $\epsilon \approx 1$

- lasts for N e-folds defined as:

$$N \simeq -\frac{8\pi}{m_{PL}^2} \int_{\phi}^{\phi_{end}} \frac{V(\phi)}{V'(\phi)} \mathrm{d}\phi = \ln \frac{a_{end}}{a} \tag{9}$$

- can be described (at least) by two *slow-roll* parameters (conditions for inflation $\dot{\phi}^2 \ll V(\phi)$, $\ddot{\phi} \approx 0$):

$$\epsilon \equiv \frac{m_{Pl}^2}{4\pi} \left(\frac{H'(\phi)}{H(\phi)}\right)^2 \backsimeq \frac{m_{Pl}^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)}\right)^2 \equiv \epsilon_V \tag{10}$$

$$\eta \equiv \frac{m_{Pl}^2}{4\pi} \left(\frac{H''(\phi)}{H(\phi)}\right)^2 \backsimeq \frac{m_{PL}^2}{8\pi} \left[\frac{V''(\phi)}{V(\phi)} - \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2\right] \equiv \eta_V \tag{11}$$

Measurable quantities at first order in *slow-roll* parameters:

$$n_s = 1 - 6\epsilon_V + 2\eta_V \qquad r = 16\epsilon_V \tag{12}$$

How we proceed:

- given a model $\Rightarrow$ $V(\phi)$
- given a potential $V(\phi)$ $\Rightarrow$ $\epsilon_V = \epsilon_V(\phi),\ \eta_V = \eta_V(\phi)$
- from $\epsilon_V(\phi_{end}) \approx 1$, numerically $\Rightarrow$ ϕ_{end}
- from e-folds $N = N(\phi, \phi_{end})$ $\Rightarrow$ $\phi = \phi(N)$
- tried to find a *general* analytical law for $\phi(N)$; the most general was:

$$\phi(N) = \sum_{i,0}^{\mathcal{N}} c_i (\ln N)^i$$

  Works well in the range $N = [20; 100]$ and with $\mathcal{N} = 2, 3$ (TBD)
- from $n_s = n_s(\epsilon_V, \eta_V) = n_s(\epsilon_V(\phi), \eta_V(\phi))$ $\Rightarrow$ $n_s(N)$
- from $r = r(\epsilon) = r(\epsilon_V(\phi))$ $\Rightarrow$ $r(N)$
- given $N \in [46, 60]$ we locate an inflationary model in the (n_s, r) space (square-$N = 46$; circle-$N = 60$)

Taxonomical review of Inflationary Models

Collection of Models: *"Encyclopædia Inflationaris"* *(Martin, Ringeval, Vennin 2013)*

- 64 models
- simplest models
- mainly single-field (some require a mechanism to stop inflation, e.g. a second field)
- classification: number of parameters involved
- ...and much more: 380 pages, 479 references, 167 figures...

How many models fit into $1\sigma - 2\sigma$ Quijote+Planck forecasts?

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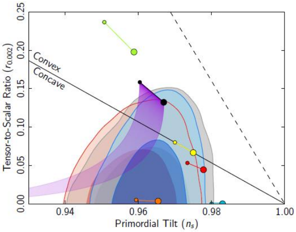
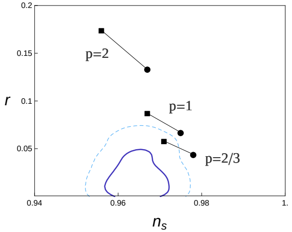
Higgs, Radiatively Corrected Higgs, Exponential SUSY, Mutated Hilltop, Kähler Moduli I and II, Pseudo Natural, Radion Gauge, ArcTan, Logarithmic, Twisted, constant n_s, Running mass, Spontaneous Symmetry Breaking

49 (77%)

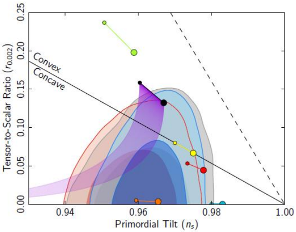
Chaotic, Natural, Power Law, Colemann-Weinberg, Mixed Large Field, Double well, Supergravity brane Loop, Intermediate, Logamediate Tip, Brane SUSY, Orientifold Valley Hybrid, etc...

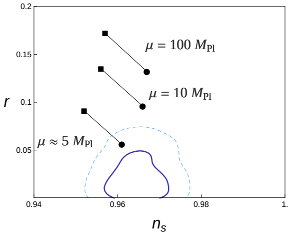
Chaotic Inflation: $\mathbf{V}(\phi) \propto \phi^{\mathbf{p}}$



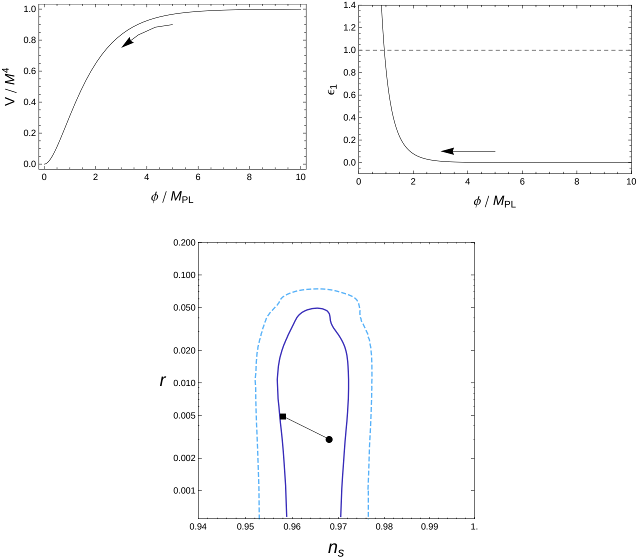


Natural Inflation: $\mathbf{V}(\phi) \propto \mathbf{1} + \cos\left(\frac{\phi}{\mu}\right)$

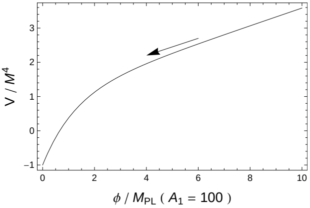


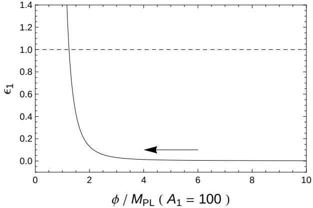


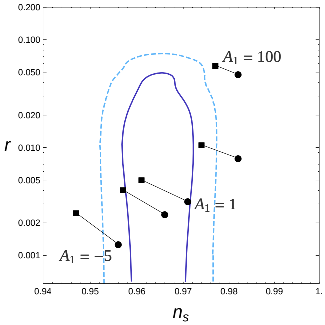
Higgs Inflation: $\mathbf{V}(\phi) \propto \left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}}\right)^2$



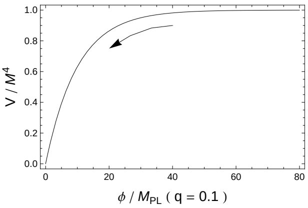
RC Higss Inflation: $V(\phi) \propto 1 - 2e^{-2\phi/(\sqrt{6}M_{Pl})} + \frac{A_1}{16\pi^2}\frac{\phi}{\sqrt{6}M_{Pl}}$

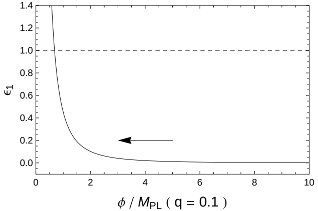


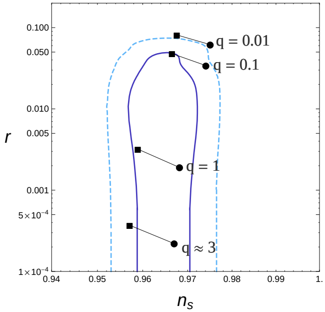




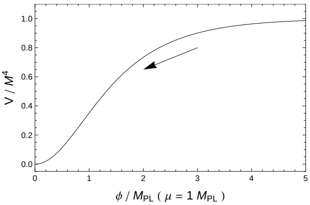
Exponential SUSY Inflation: $V(\phi) \propto 1 - e^{-q\phi/M_{Pl}}$

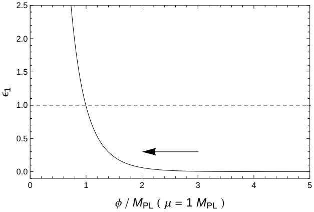


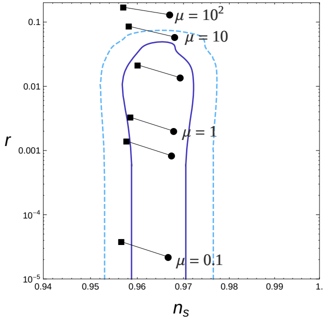




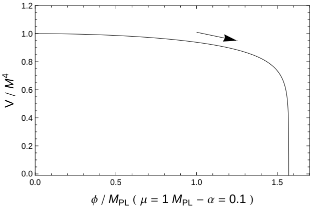
Mutated Hilltop Inflation: $V(\phi) \propto 1 - \text{sech}\left(\frac{\phi}{\mu}\right)$

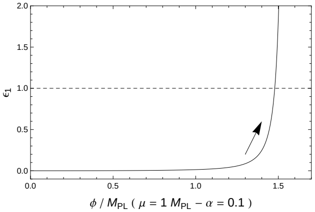


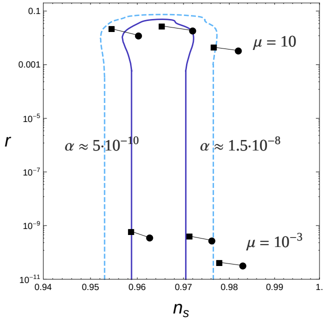




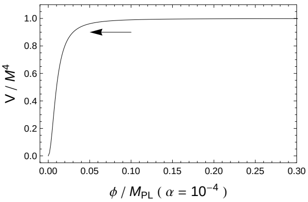
Pseudo Natural Inflation: $\mathbf{V(\phi) \propto 1 + \alpha \ln\left(\cos\frac{\phi}{\mu}\right)}$

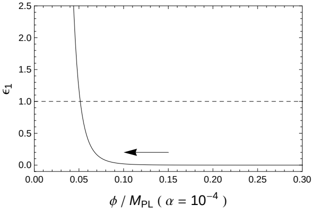


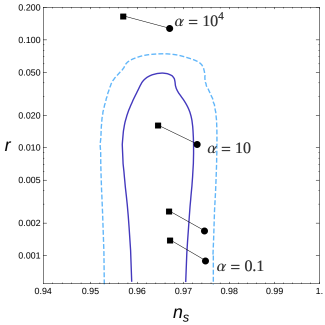




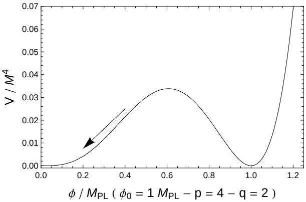
Radion Gauge Inflation: $V(\phi) \propto \frac{(\phi/M_{Pl})^2}{\alpha+(\phi/M_{Pl})^2}$

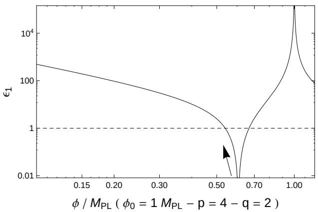


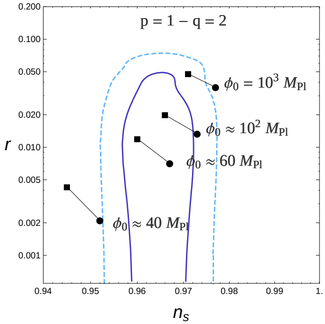




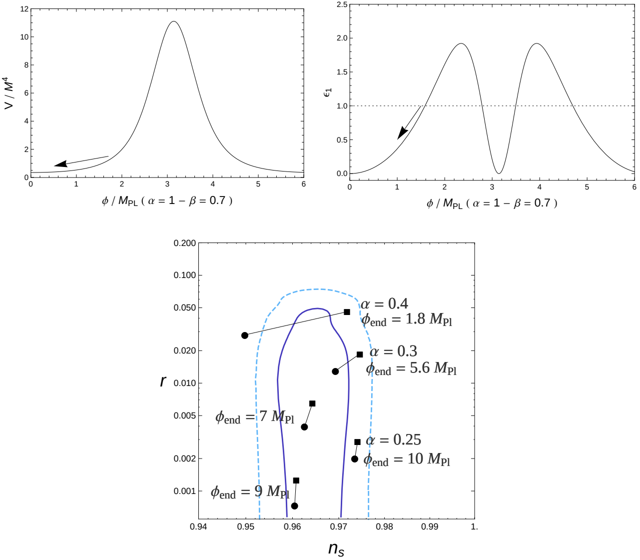
Logarithmic Potential (3) Inflation: $\mathbf{V}(\phi) \propto \left(\frac{\phi}{\phi_0}\right)^{\mathbf{p}} \left(\ln \frac{\phi}{\phi_0}\right)^{\mathbf{q}}$







Constant n_s D Inflation: $\mathbf{V}(\phi) \propto \left(1 + \beta \cos\left[\alpha\left(\frac{\phi}{\mathbf{M_{Pl}}}\right)\right]\right)^{-2}$



Monte Carlo reconstruction of the Inflationary Potential

(Kinney 2002; Easther & Kinney 2003; Kinney et al. 2006; Powell & Kinney 2007)

Slow-roll parameters: "infinite hierarchy"

$$\begin{aligned}
\epsilon &\equiv \frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2 \\
\eta &\equiv \frac{m_{Pl}^2}{4\pi}\left(\frac{H''(\phi)}{H(\phi)}\right)^2 \\
{}^2\lambda &\equiv \frac{m_{Pl}^2}{16\pi^2}\left(\frac{H'(\phi)H'''(\phi)}{H^2(\phi)}\right) \\
{}^3\lambda &\equiv \dots
\end{aligned}$$

Not constant during inflation but "flow equations" *(Liddle, Parsons, Barrow 1994)*. Introducing $\sigma \approx n_s - 1$, and truncating at some order (we choose ${}^6\lambda = 0$)

$$\begin{aligned}
\frac{d\epsilon}{dN} &= \epsilon(\sigma + 2\epsilon) \\
\frac{d\sigma}{dN} &= -5\epsilon\sigma - 12\epsilon^2 + 2({}^2\lambda) \\
\frac{d({}^\ell\lambda)}{dN} &= \left[\frac{\ell-1}{2}\sigma + (\ell-2)\epsilon\right]({}^\ell\lambda) + {}^{\ell+1}\lambda
\end{aligned}$$

Observational parameters (at second order):

$$n_s - 1 = \sigma - (5 - 3C)\epsilon^2 - \frac{1}{4}(3 - 5C)\sigma\epsilon + \frac{1}{2}(3 - C)^2\lambda \qquad (13)$$

$$r = 16\epsilon[1 - C(\sigma + 2\epsilon)] \qquad (14)$$

with $C = 4(\ln 2 + \gamma) - 5 \approx 0.0814514$.

How it works:

1. Specify a reliable window for n_s and r (from Planck+Quijote forecast)
2. Choose a random set of slow-roll parameters as initial conditions:

$$N_{ini} = [46, 60]; \quad \epsilon = [0, 0.8]; \quad \sigma = [-0.5, 0.5]; \quad {}^2\lambda = [-0.05, 0.05];$$

$${}^\ell\lambda = [-0.025 \cdot 5^{l-3}, 0.025 \cdot 5^{l-3}] \; \textit{for} \; 3 \leq \ell \leq 5; \quad {}^6\lambda = 0;$$

3. Evolve forward in time / backward in N the flow equations ($N_{ini} \to 0$)
4. 3 possible family trajectories:
   - at some $N = [0; N_{ini}] \to \epsilon > 1 \Rightarrow$ inflation ends
   - inflation never ends $\to$ at $N = 0$ we have $\epsilon = \text{const.} \neq 0$ and < 1: power-law inflation $\Rightarrow$ excluded by present data
   - inflation never ends $\to$ at $N = 0$ we have $\epsilon = 0$: associated with $n_s > 1$ $\Rightarrow$ excluded by present data
5. when/if inflation ends, go backward in time / forward in N, up to some random $N_{efold} = [46, 60]$, and evaluate n_s and r at N_{efold};
6. if $n_s(N_{efold})$ and $r(N_{efold})$ fit constraints at point 1, save trajectories $\epsilon(N)$ and... go next page...;
7. otherwise, begin again from point 2 ...

When a trajectory is accepted, goes on:

- calculate $H(N)$ from the definition of ϵ:

$$\frac{1}{H(N)}\frac{dH(N)}{dN} = \epsilon(N) \tag{15}$$

  with a "gross" normalization, $H(N_{efold}) = 10^{-5}\sqrt{\pi\epsilon(N_{efold})}$ from measured density fluctuations

- calculate the field $\phi(N)$ from Hamilton-Jacobi equation:

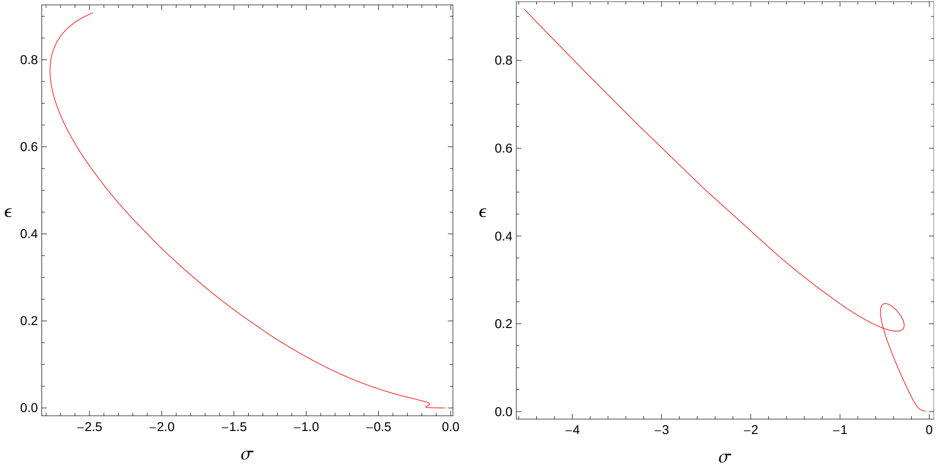
$$\frac{d\phi(N)}{dN} = \frac{m_{Pl}}{2\sqrt{\pi}}\sqrt{\epsilon(N)} \tag{16}$$

  with arbitrary normalization $\phi(N_{efold}) = 0$

- calculate the potential $V(N)$ from the Friedmann equation

$$V(\phi(N)) = \frac{3m_{Pl}^2}{8\pi}H^2(N)\left[1 - \frac{1}{3}\epsilon(N)\right] \tag{17}$$

Some examples of trajectories



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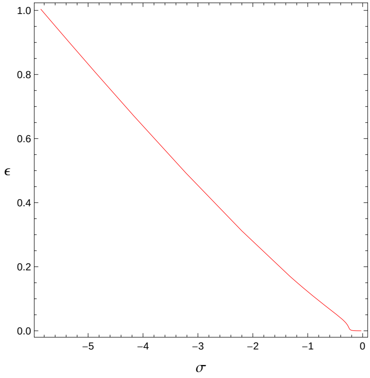
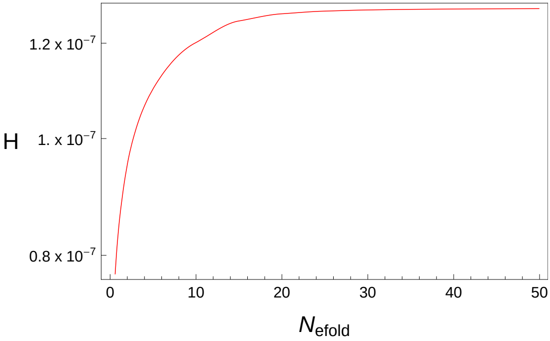
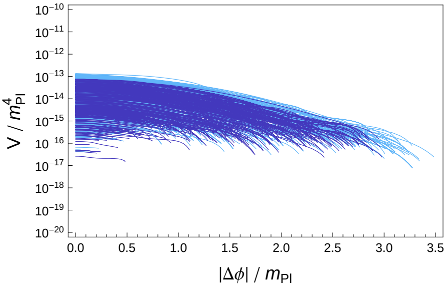
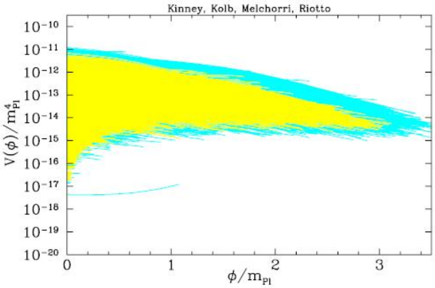




Final results:

- upper limit on potential: $V \lesssim 6 \div 8 \cdot 10^{15}$ GeV
- 2 orders of magnitude (in m_{Pl} units) lower than WMAP3+SDSS ($\approx 2 \cdot 10^{16}$ GeV)
- no inferior limit on potential $\Leftrightarrow$ no inferior limit on r
- limit on field magnitude: $\Delta\phi \lesssim 3\ \mathrm{m}_{Pl}$ (at 1σ) and $\lesssim 3.5\ \mathrm{m}_{Pl}$ (at 2σ)
- limit on $H \lesssim 8 \cdot 10^{-7}\ \mathrm{m}_{Pl}$ ($\approx 2 \cdot 10^{-5}\ \mathrm{M}_{Pl}$)





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