

Will Kinney



University at Buffalo The State University of New York

Tabula quadrorum										Tabula quadrorum														
Quadrati					Quadrati					Quadrati					Quadrati									
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12	576	2304	5184	9200	13600	18496	24640	32000	40600	50400	61400	73600	86800	101000	116200	132400	150600	170800	193000	217200	243400	270600	299800	330000
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16	1024	4096	9024	16000	23000	31504	41600	53200	66400	81200	97600	115600	135200	156400	179200	203600	229600	257200	286400	317200	349600	383600	419200	456400
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18	1296	5184	11312	20100	28500	39000	50800	64000	78800	95200	113200	132800	154000	176800	201200	227200	254800	284000	314800	347200	381200	416800	454000	492800
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20	1600	6400	14400	23000	32500	44000	57600	73200	90000	108000	128200	150600	175200	201800	230400	260800	293000	326800	362200	399200	437800	478000	519800	563200
21	1764	7056	15344	25000	35025	47000	60800	77400	95800	116000	138200	162400	188600	216800	247000	279200	313400	349600	387800	428000	469200	511400	555600	601800
22	1936	7744	16848	26800	37500	50000	64800	82000	101400	122600	145800	171000	198200	227400	258600	291800	327000	364200	403400	444600	487800	533000	579200	626400
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25	2500	10000	22500	33000	45500	60000	77600	98400	122400	148600	177000	207400	239800	274200	310600	349000	389400	431800	476200	522600	570000	619400	670800	723200



Alfonsine Tables:

Compiled in Toledo ca. 1252, financed by Alfonso X.
50 astronomer collaboration, led by Isaac ibn Sid.
First printed edition 1483.

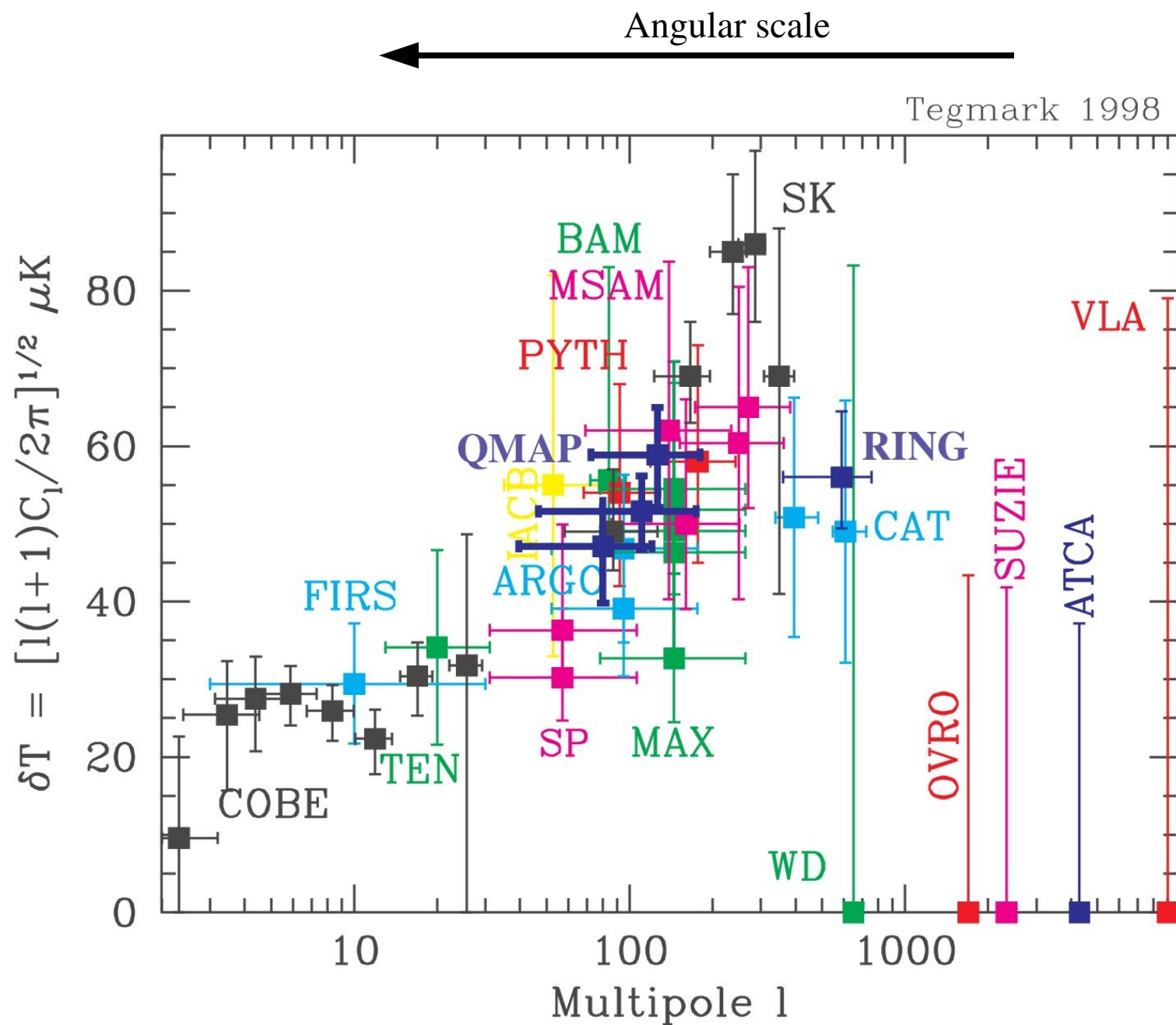
The image shows a single page from a historical manuscript, identified as the Alfonsine Tables. The page is filled with a dense grid of numbers and text written in Arabic script. The text is organized into columns and rows, typical of astronomical tables. The paper appears aged and slightly discolored, with some visible wear and tear. The handwriting is in a clear, consistent style, characteristic of the period. The page is set against a dark background, which makes the light-colored paper and the dark ink stand out.

"If the Lord Almighty had consulted me before embarking on creation thus, I should have recommended something simpler."

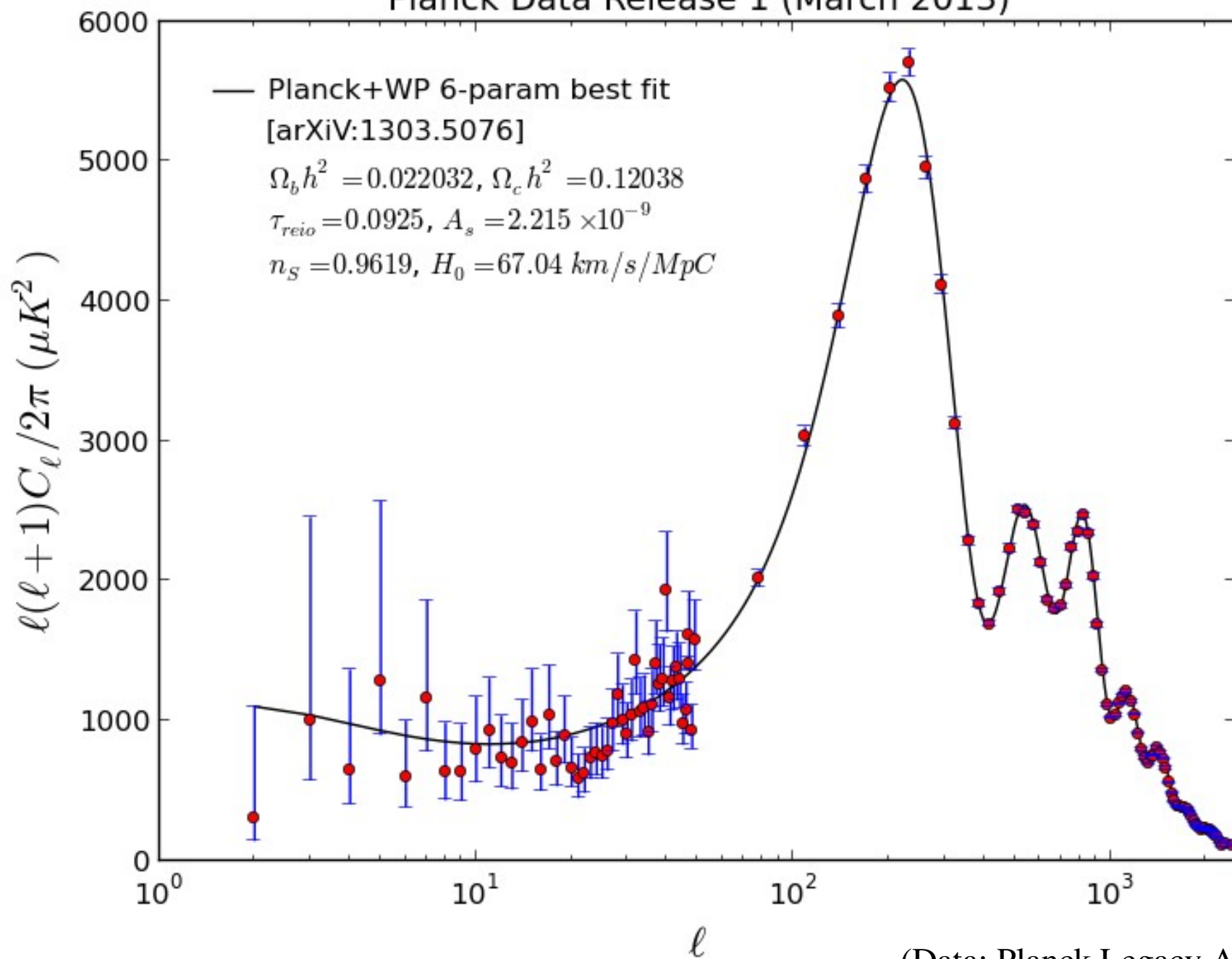
Alfonso X of Castile (r. 1252–84) on the Ptolemaic system



The CMB Angular Power Spectrum (1998)



Planck Data Release 1 (March 2013)

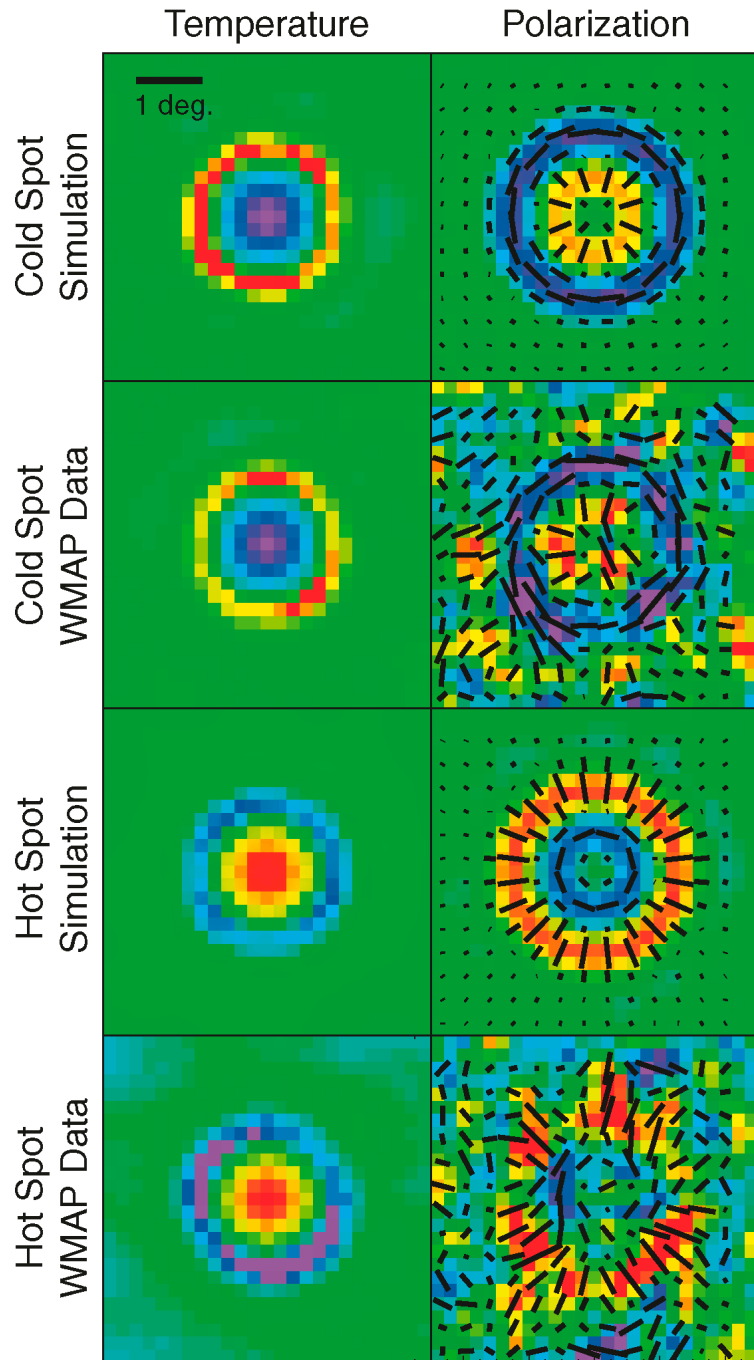


(Data: Planck Legacy Archive)

Inflation: Basic Predictions

- Adiabatic density perturbations
- Superhorizon correlations
- Gaussian statistics

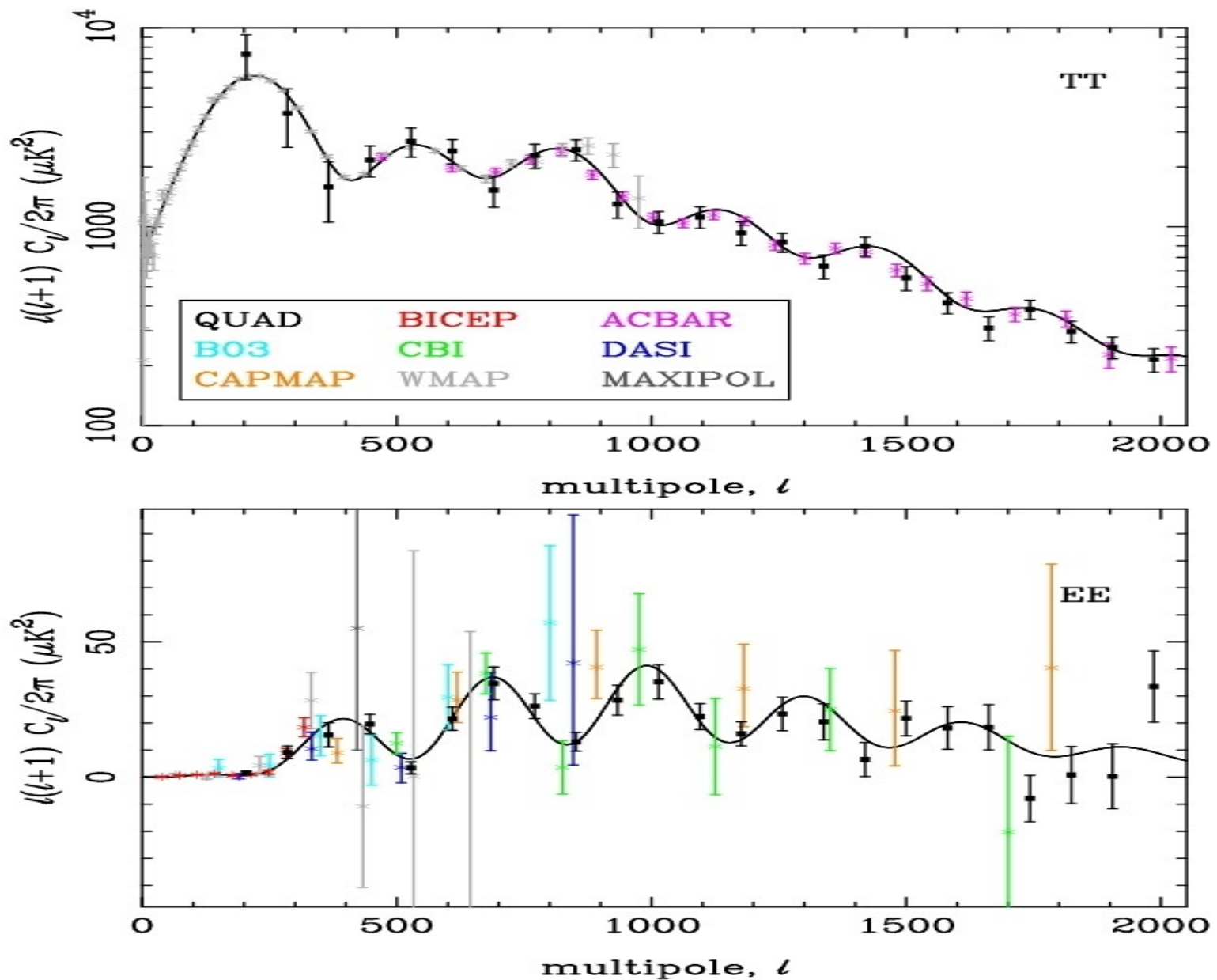
Polarization: Test of Adiabaticity



Polarization strongest along
gradients in temperature

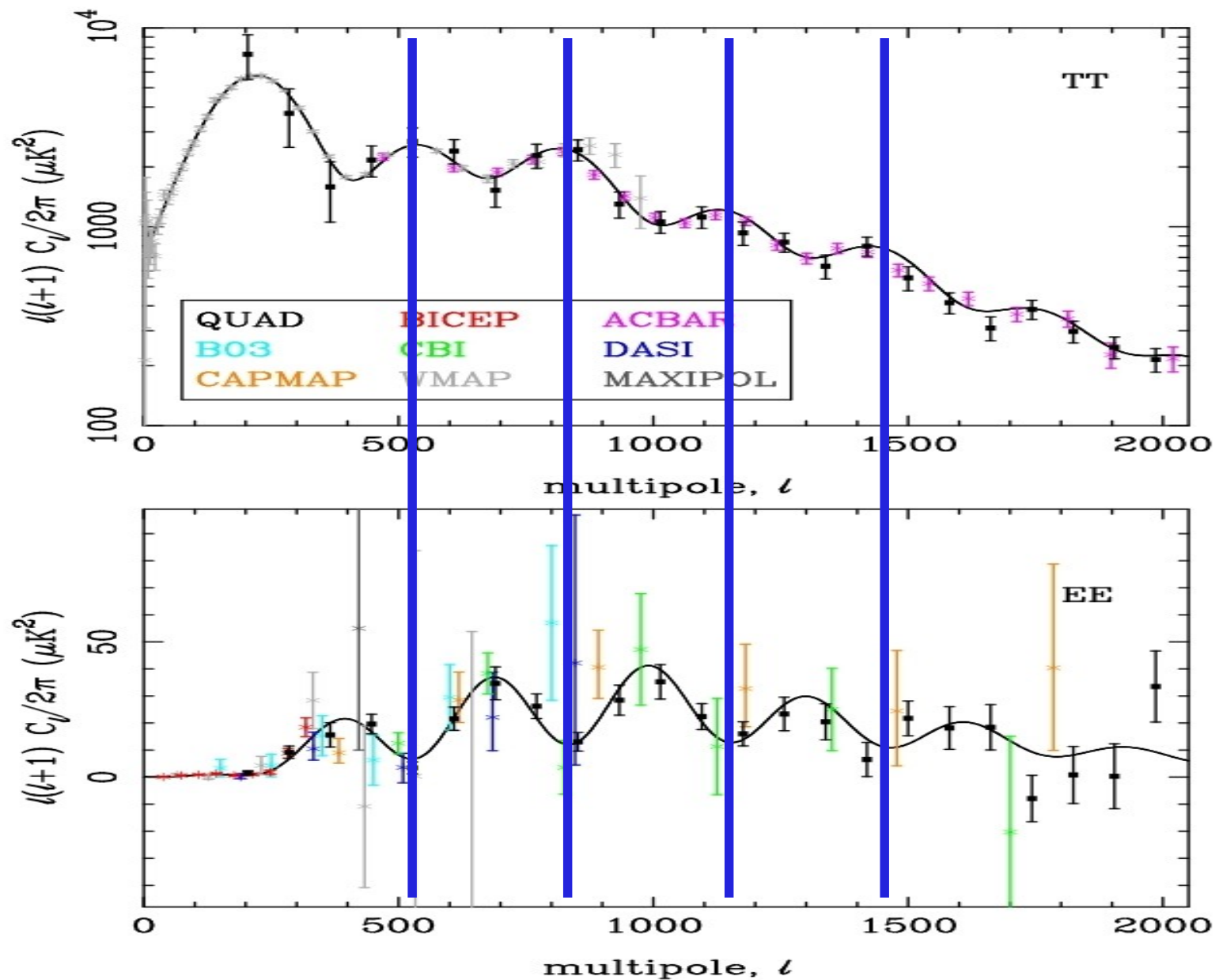
(Komatsu, *et al.*, arXiv:0912.0522)

Adiabatic Perturbations: Temperature and Polarization Spectra Anticorrelated



(QuaD Collaboration, arXiv:0906.1003)

Adiabatic Perturbations: Temperature and Polarization Spectra Anticorrelated

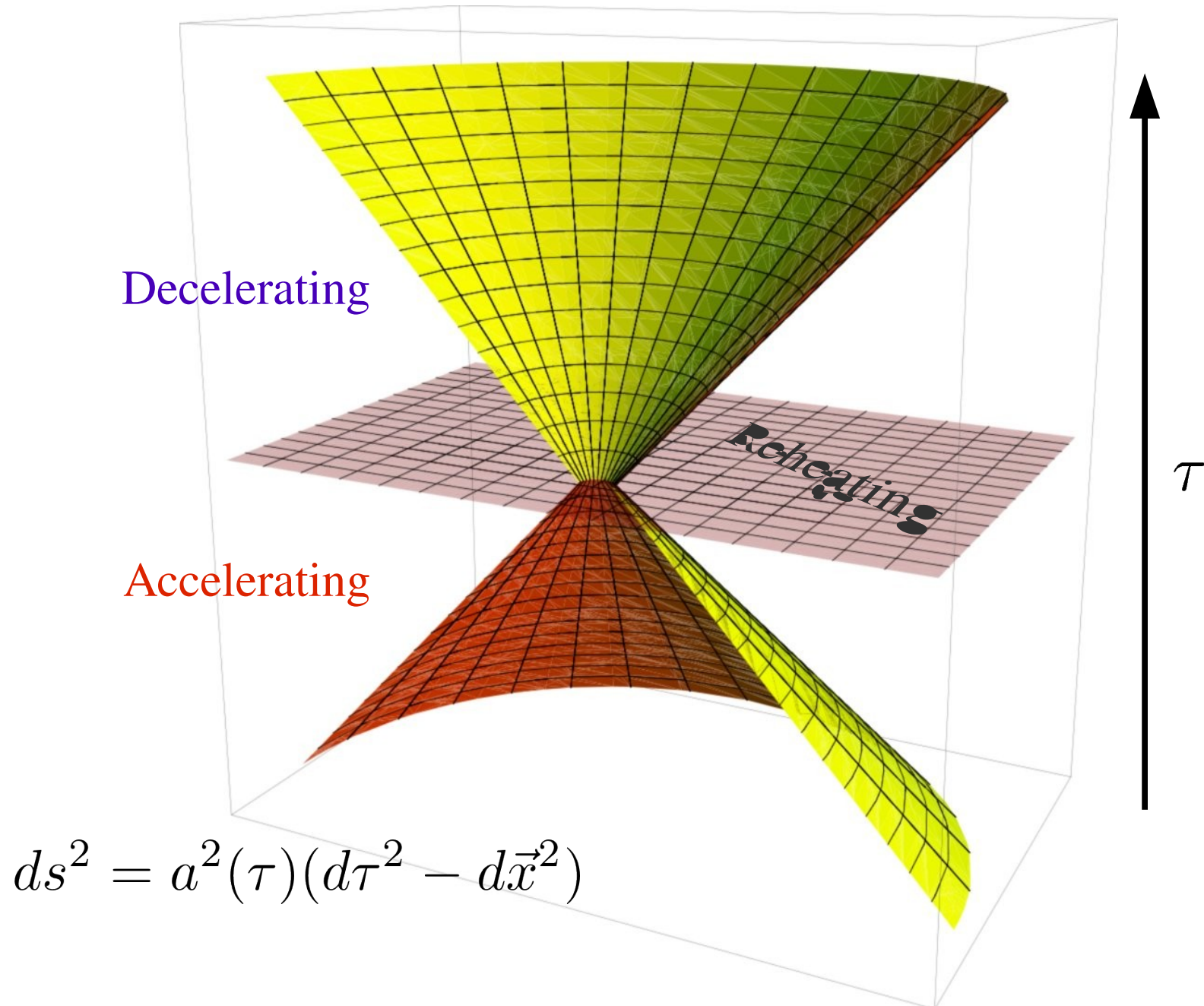


(QuaD Collaboration, arXiv:0906.1003)

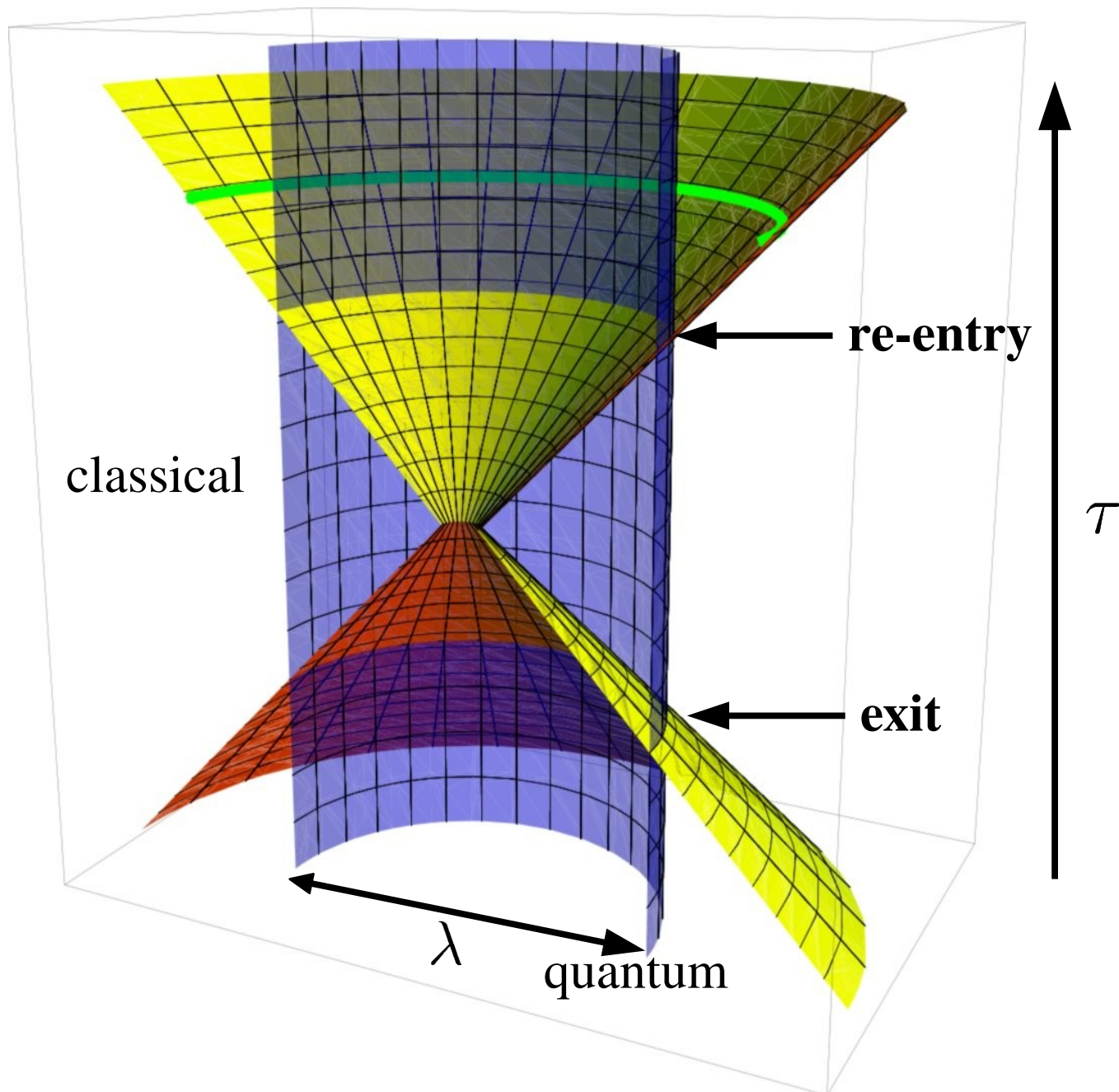
Inflation: Basic Predictions

- Adiabatic density perturbations ✓
- Superhorizon correlations
- Gaussian statistics

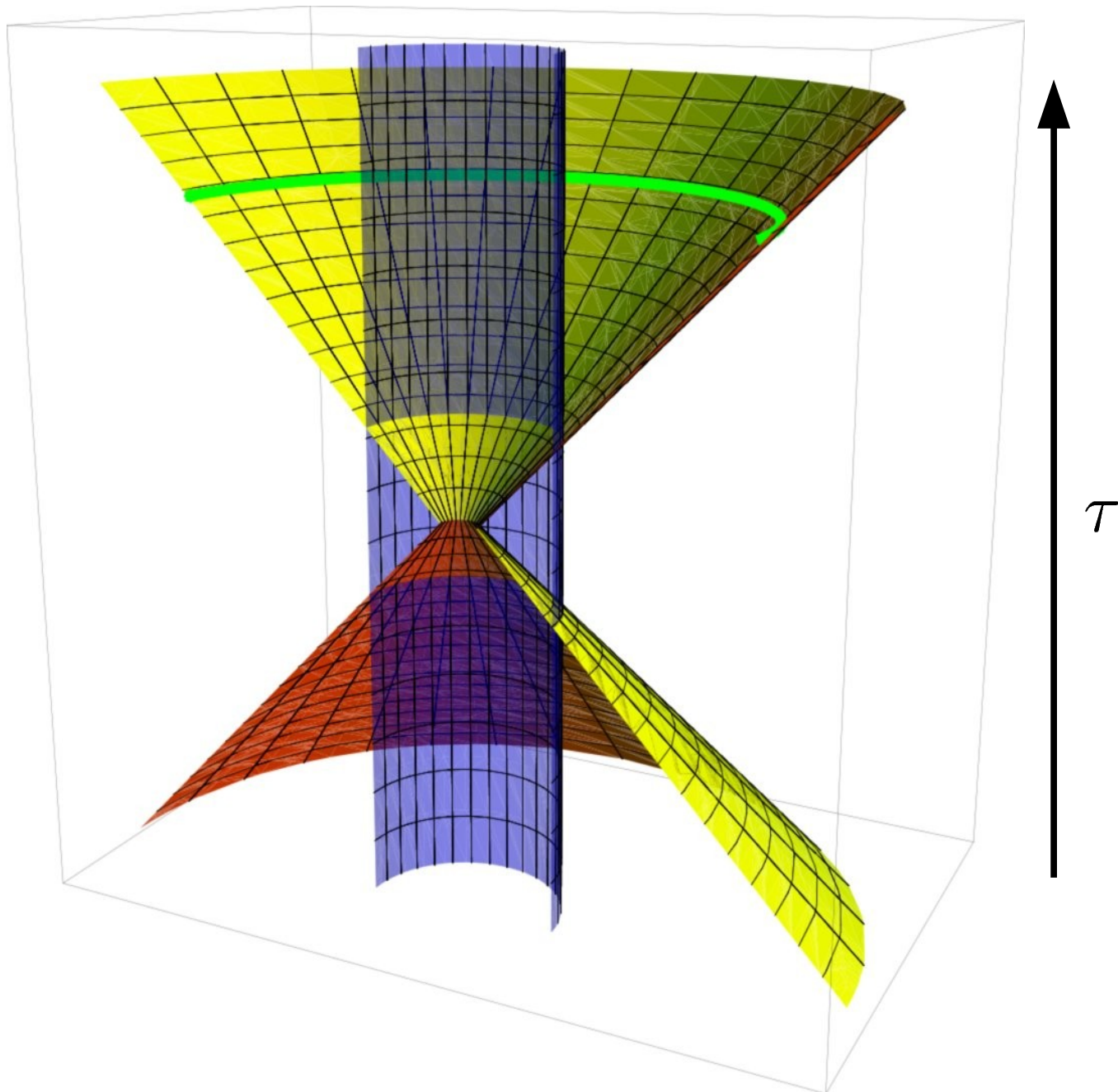
The Horizon in Inflation



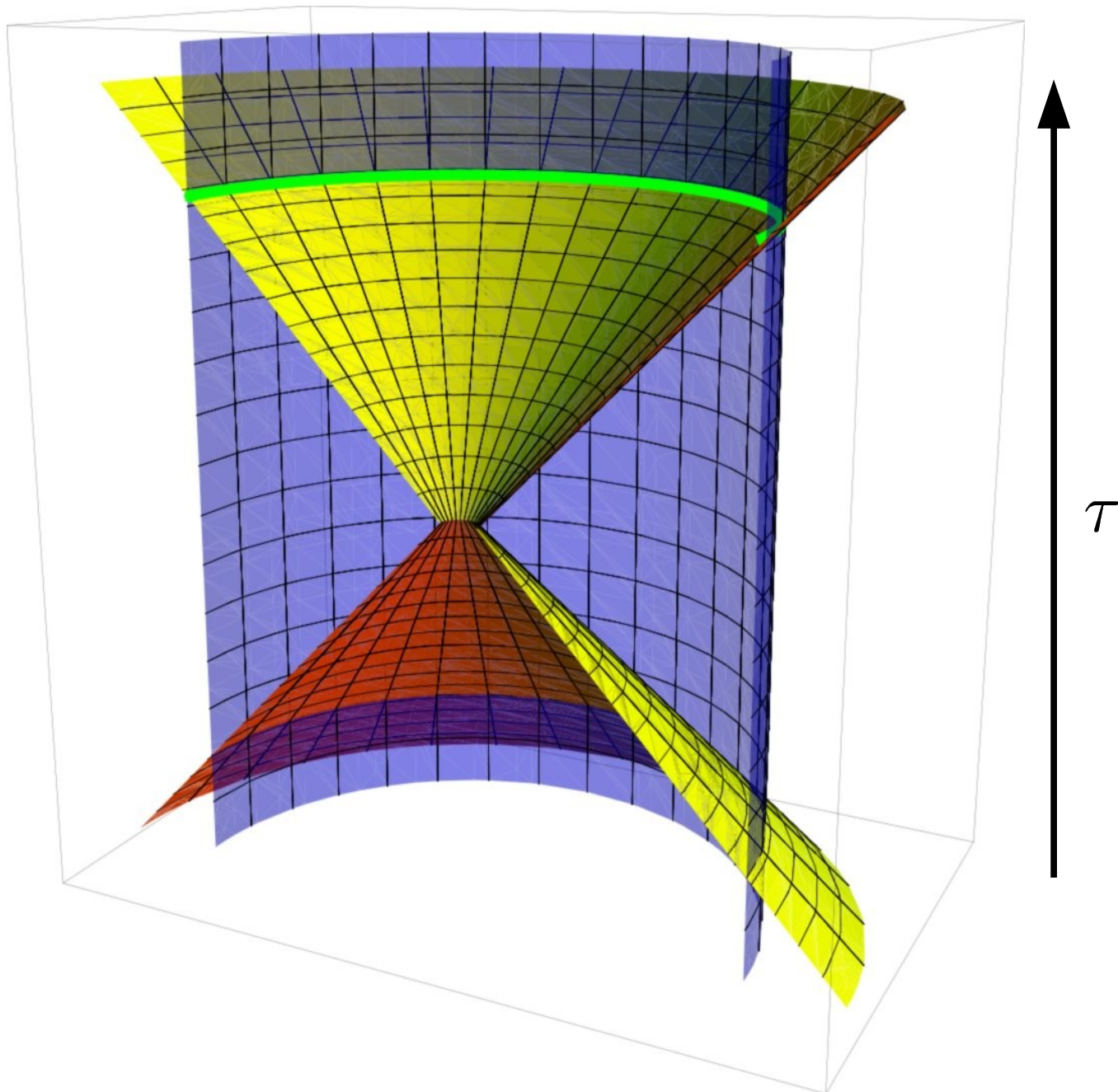
Mode Exit and Reentry



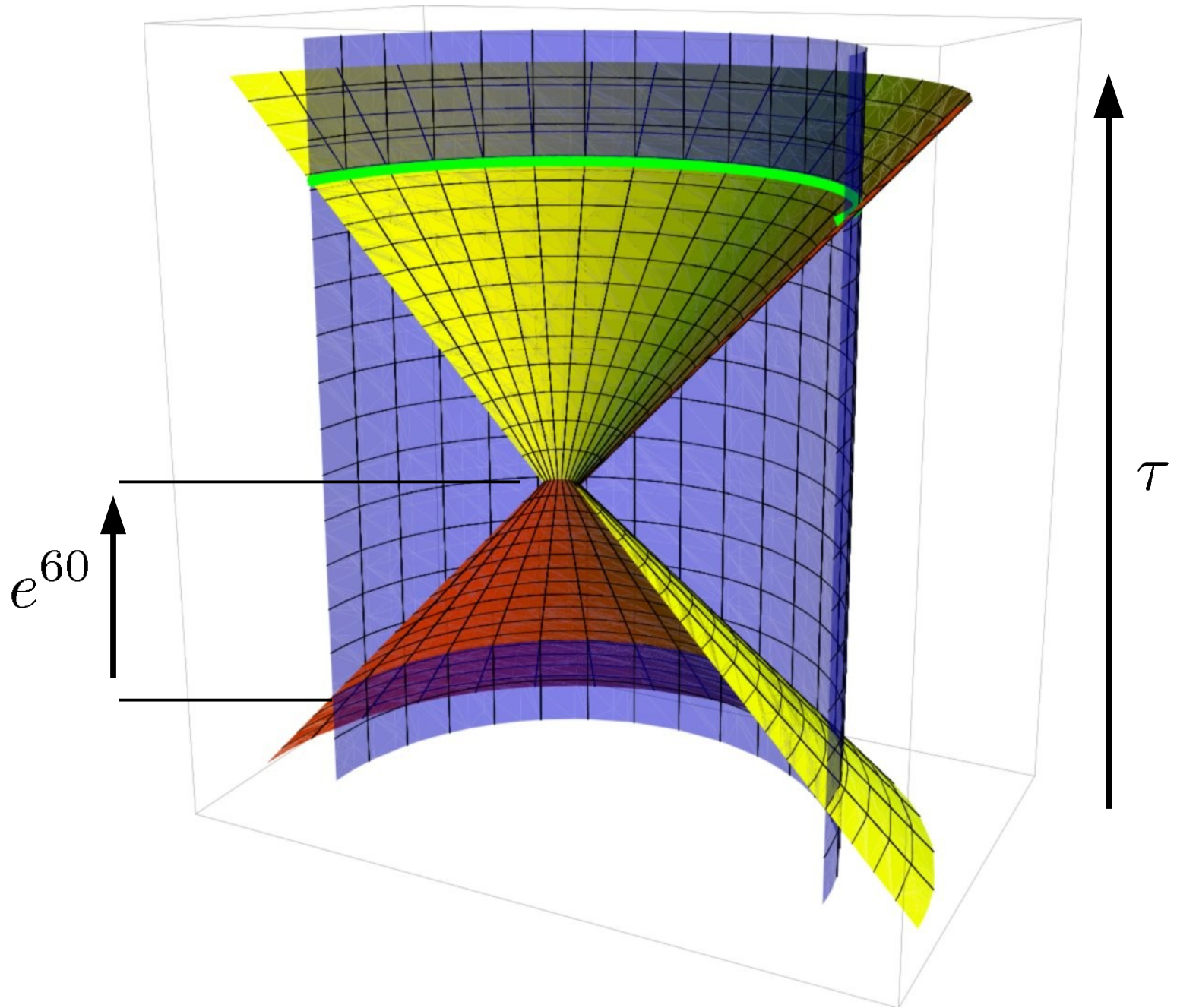
Shorter Wavelength Modes Exit Later



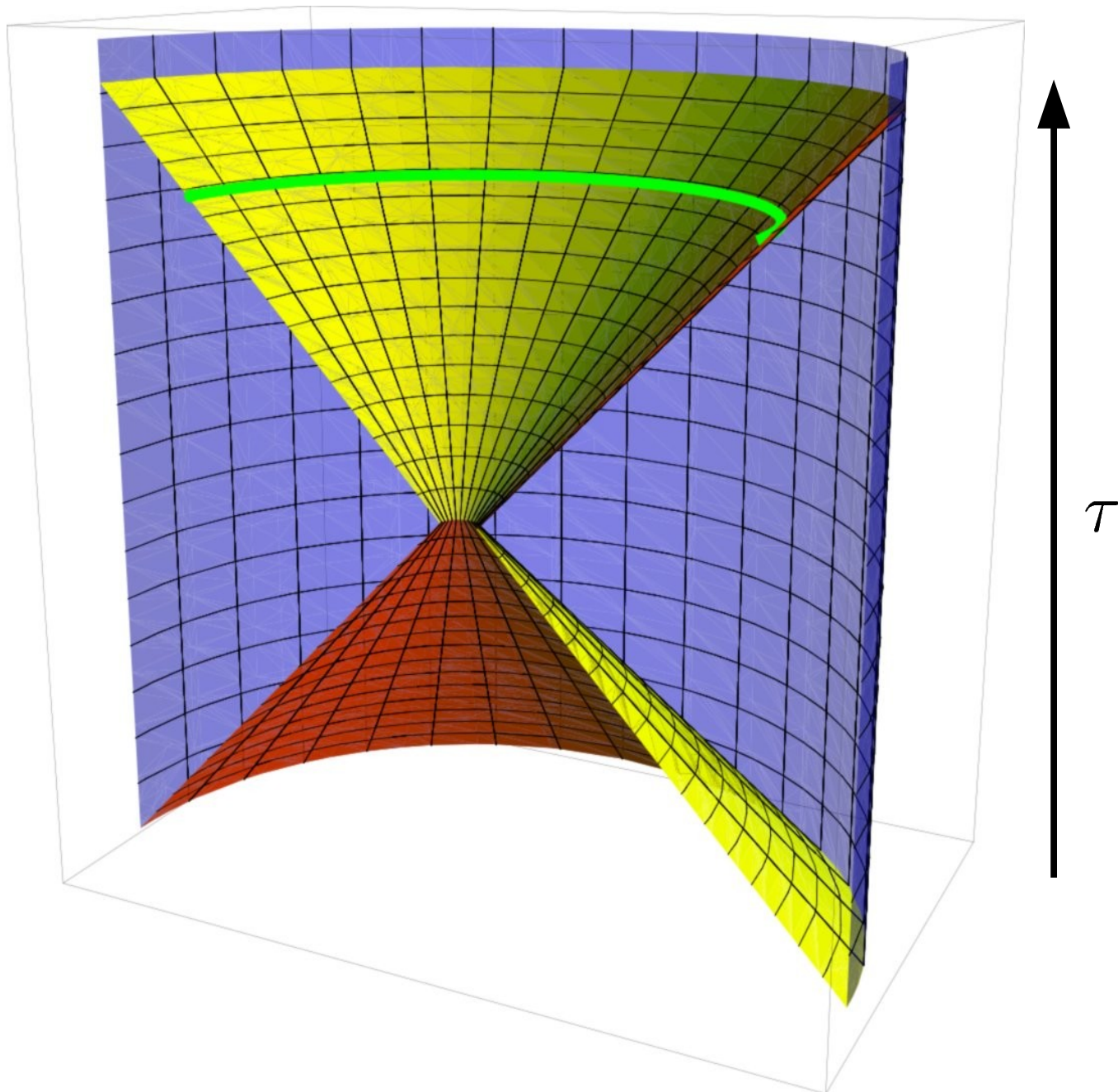
Longer Wavelength Modes Exit Earlier



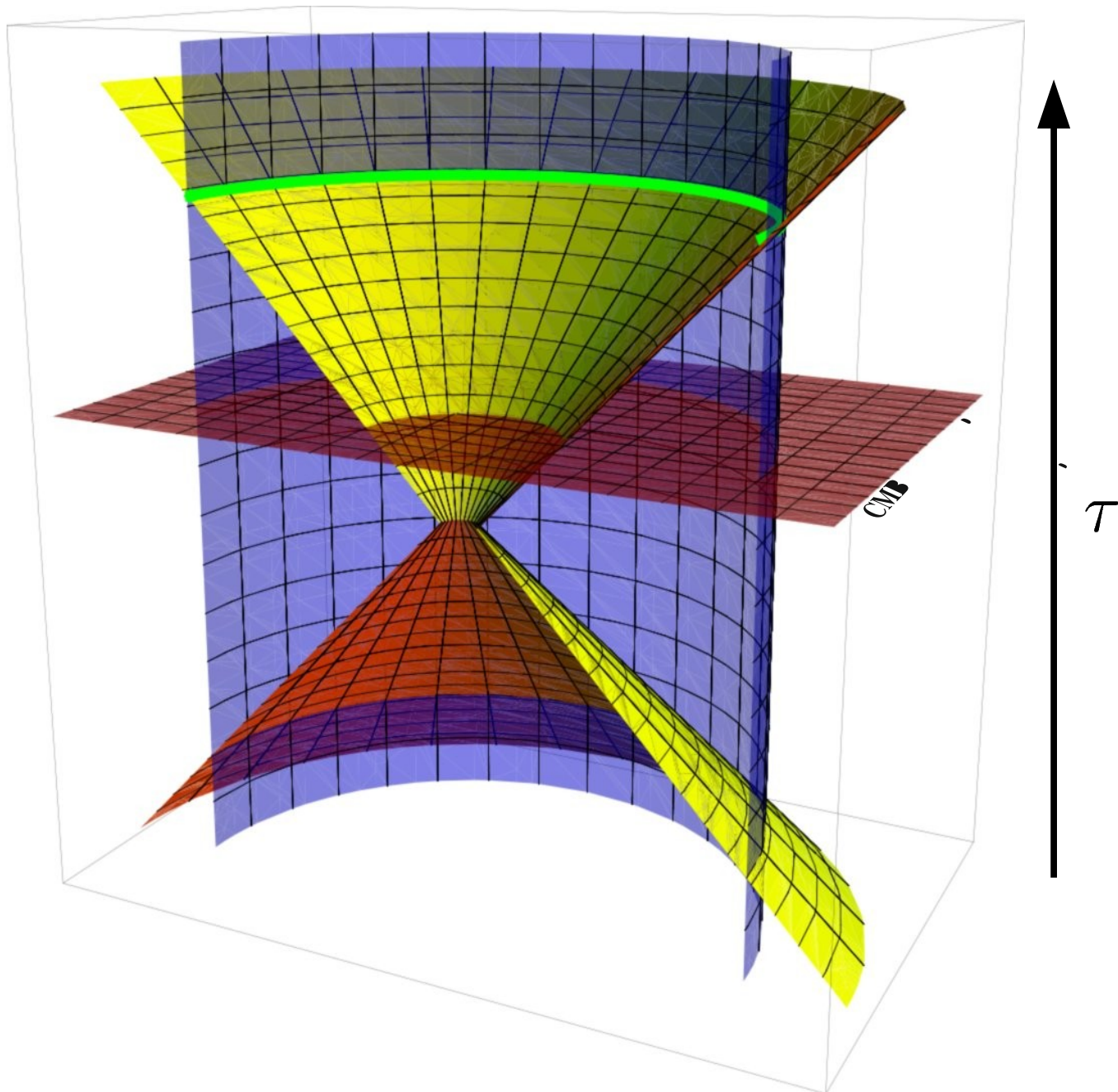
We See The *Last* 60 E-folds



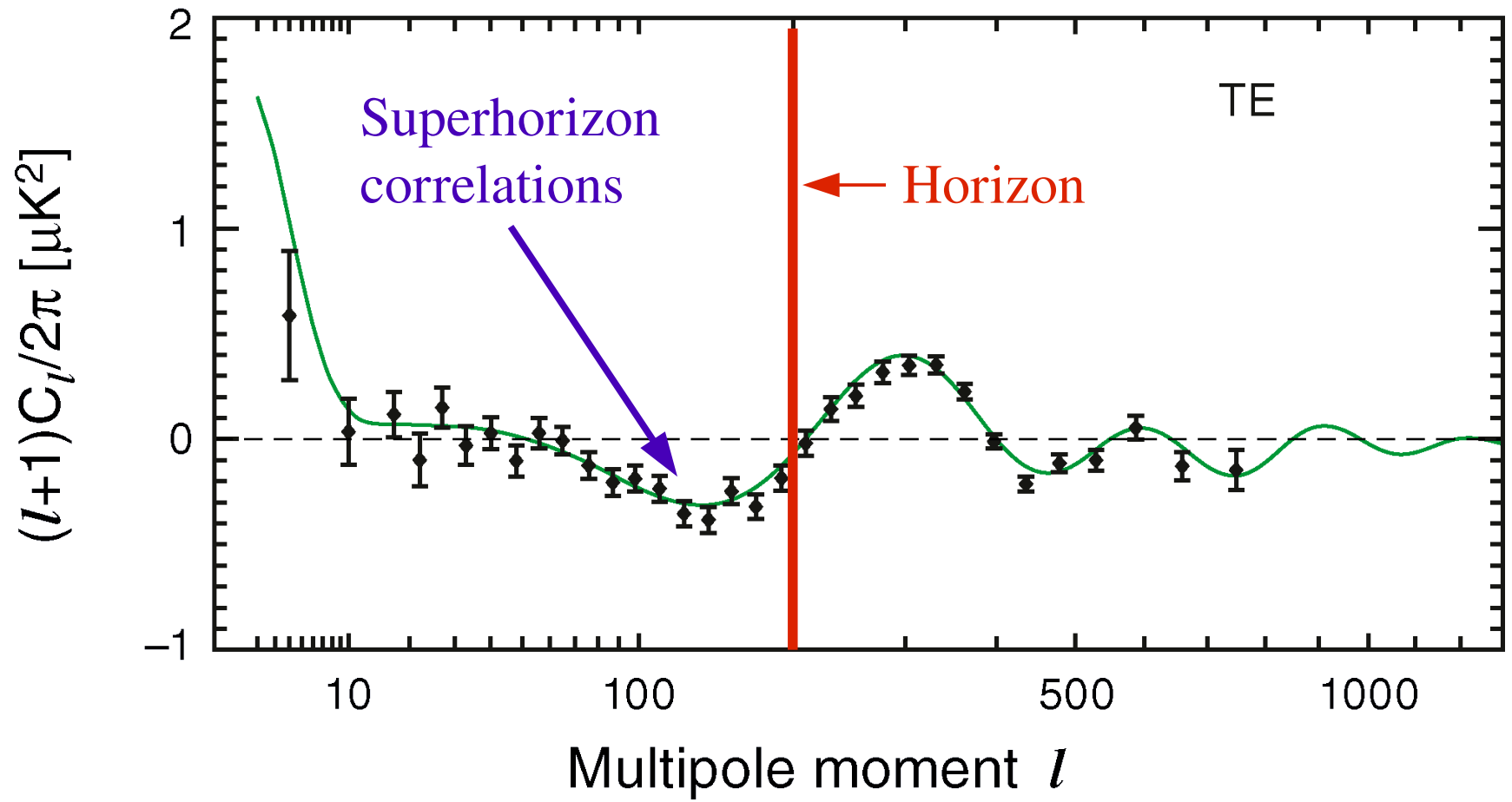
Initial Conditions: Inaccessible



Superhorizon Perturbations



Large-Scale CMB Polarization (WMAP7)



(Figure: NASA/WMAP science team)

Generating Superhorizon Perturbations

In an *expanding universe*, to generate perturbations consistent with observation, must have one of:

(1) Accelerated Expansion

(2) Superluminal Sound Speed

(3) Super-Planckian Energy Density

Inflation: Basic Predictions

- Adiabatic density perturbations ✓
- Superhorizon correlations ✓
- Gaussian statistics

Non-Canonical Lagrangians

Lagrangian with arbitrary kinetic term:

$$\mathcal{L} = F(X, \phi) - V(\phi) \quad X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$= \frac{1}{2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

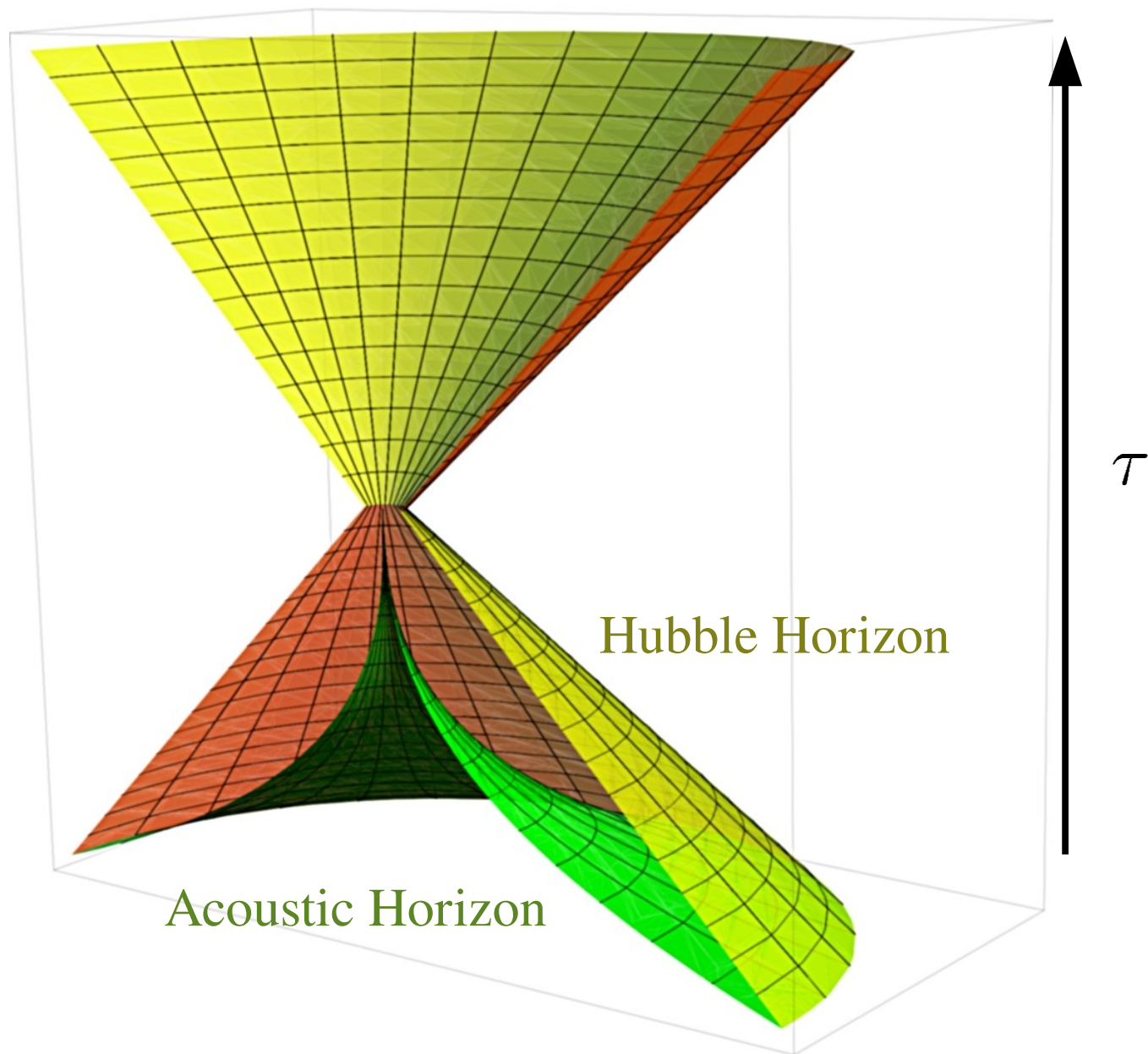
Acoustic metric

Light cone: $g^{\mu\nu} dx^\mu dx^\nu = 0$

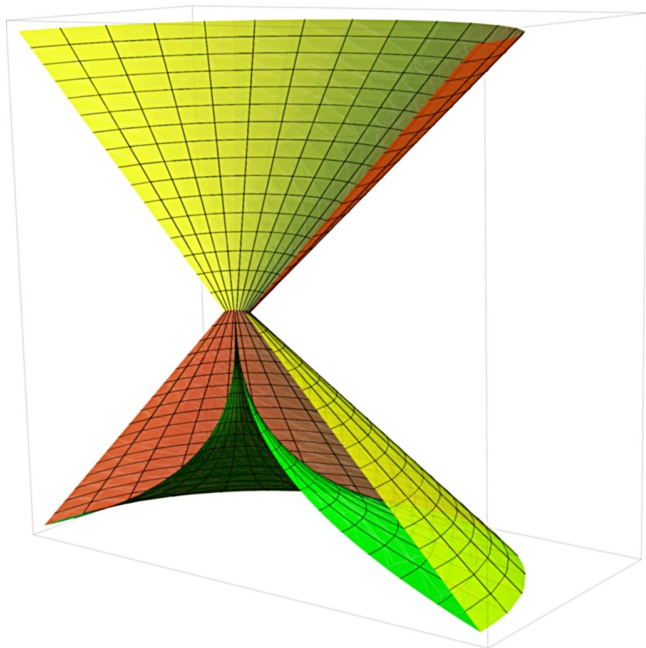
Acoustic cone: $G^{\mu\nu} dx^\mu dx^\nu = 0$

Two horizons!

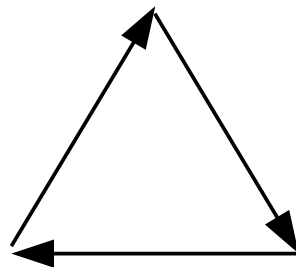
Inflation from non-Canonical Lagrangians



Inflation from non-Canonical Lagrangians

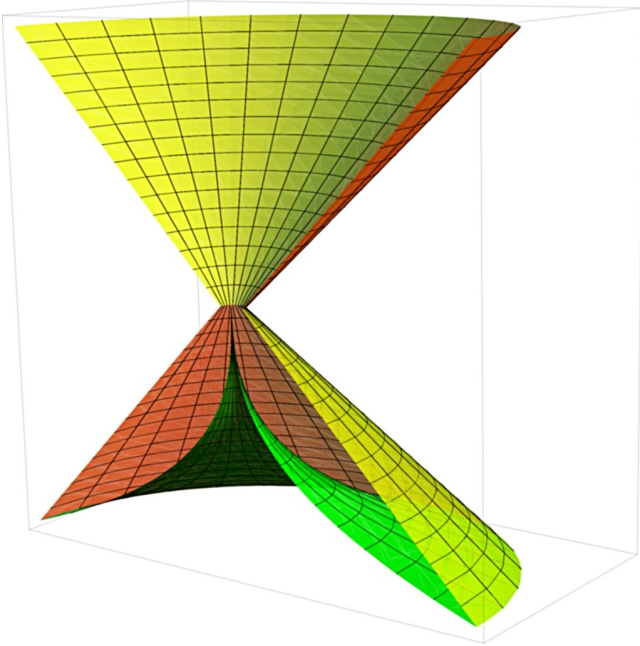


Signature: equilateral bispectrum

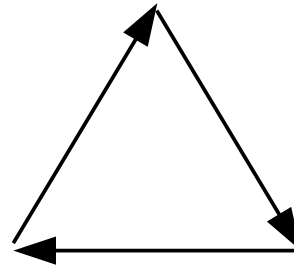


$$f_{\text{NL}} \sim c_S^{-2}$$

Inflation from non-Canonical Lagrangians



Signature: equilateral bispectrum

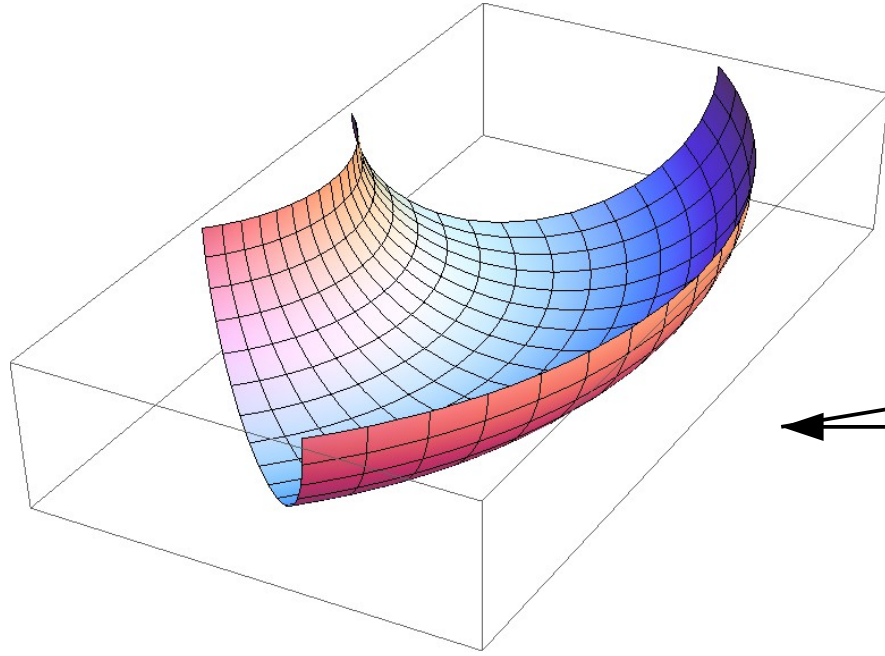


$$f_{\text{NL}} \sim c_S^{-2}$$

$$\text{Planck: } f_{NL}^{\text{equil}} = 42 \pm 75$$

No evidence for non-canonical inflation

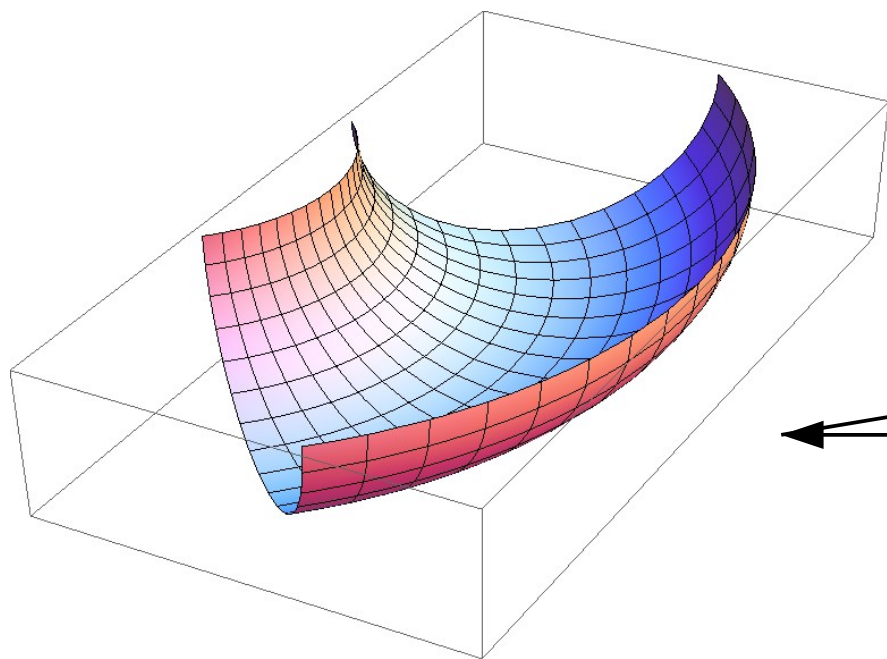
Multi-Field Inflation



Signature: local bispectrum

$$f_{\text{NL}}^{\text{local}} = \frac{\delta^2 N}{\delta\phi_i \delta\phi_j} \delta\phi_i \delta\phi_j$$
Two black arrows originate from the equation. One arrow points from the right side of the equation towards the 3D plot, and the other points from the left side of the equation towards the 3D plot.

Multi-Field Inflation



Signature: local bispectrum

$$f_{\text{NL}}^{\text{local}} = \frac{\delta^2 N}{\delta\phi_i \delta\phi_j} \delta\phi_i \delta\phi_j$$

$$\text{Planck: } f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8$$

No evidence for multi-field inflation

Inflation: Basic Predictions

- Adiabatic density perturbations ✓
- Superhorizon correlations ✓
- Gaussian statistics ✓

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

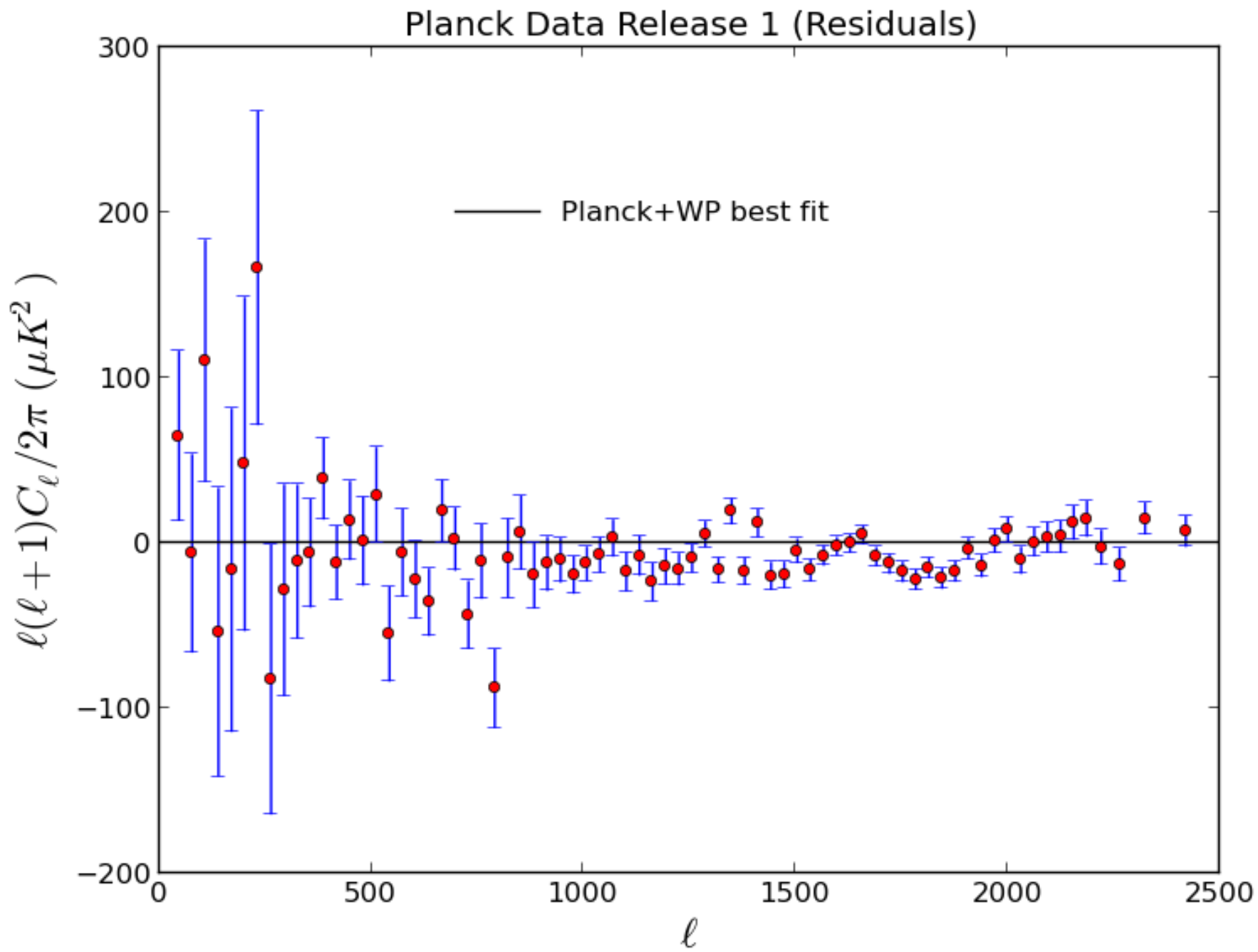
Fully consistent
with data.

Implications of Planck for Single-Field Inflation

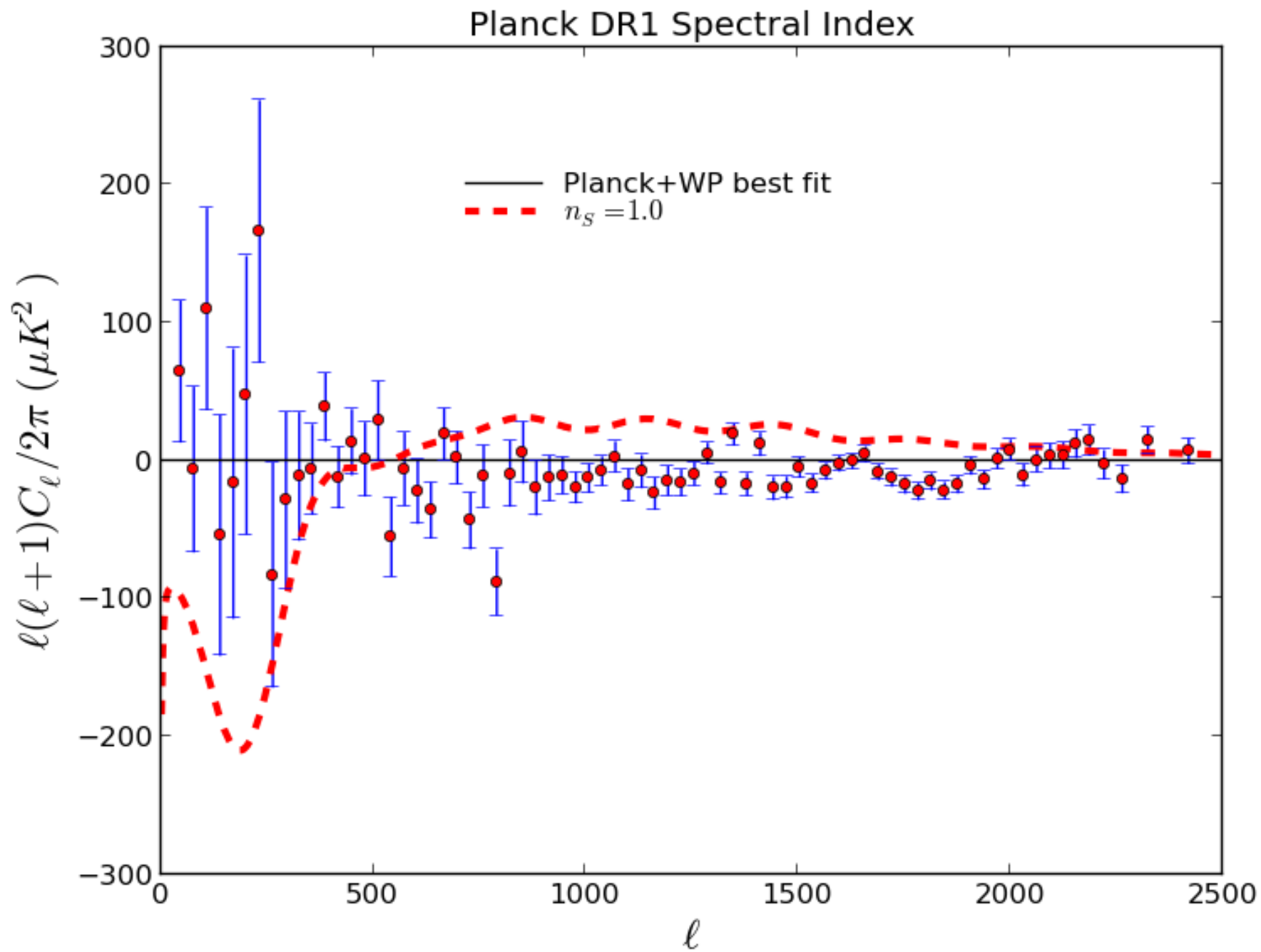
- Departure from scale invariance
- Time-dependent equation of state
- Small-field models: mass suppression

Implications of Planck for Single-Field Inflation

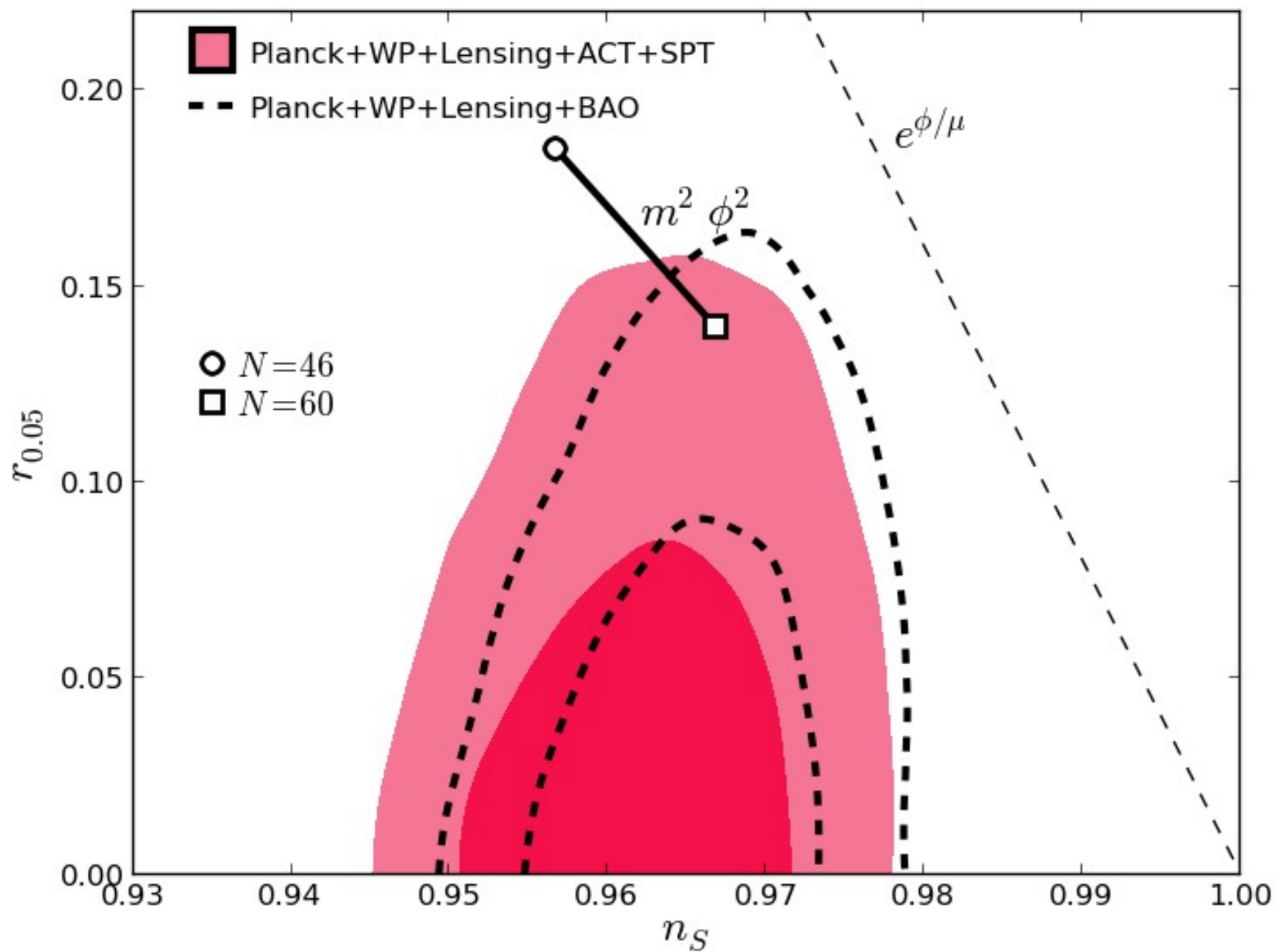
- Departure from scale invariance
- Time-dependent equation of state
- Small-field models: mass suppression



(Data: Planck Legacy Archive)



(Data: Planck Legacy Archive)



(Data: Planck Legacy Archive)

Implications of Planck for Single-Field Inflation

- Departure from scale invariance
- Time-dependent equation of state
- Small-field models: mass suppression

Power-Law Inflation

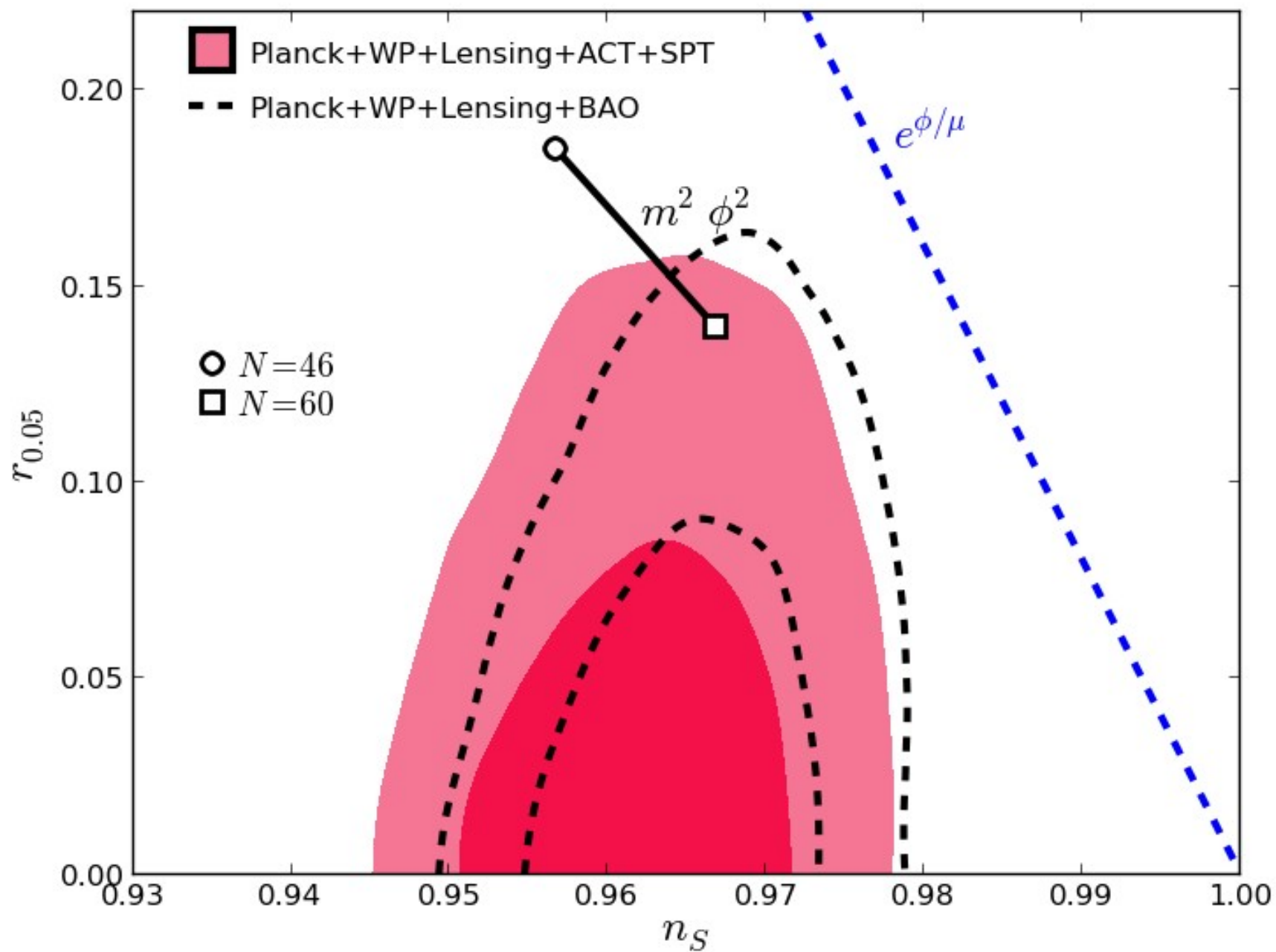
$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3(\rho + p)}{2\rho} = \text{const.} \quad a(t) \propto t^{1/\epsilon}$$

$$V(\phi) = \Lambda^4 \exp\left(\frac{\sqrt{2\epsilon}\phi}{M_{\text{P}}}\right)$$

Relation between n and r :

$$n - 1 = \frac{-2\epsilon}{1 - \epsilon} = \frac{-r}{8 - 2r}$$

Constant Equation of State Excluded



(Data: Planck Legacy Archive)

Implications of Planck for Single-Field Inflation

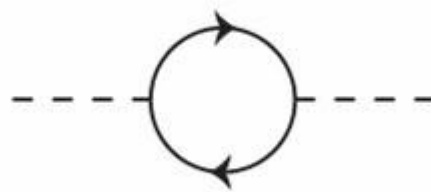
- Departure from scale invariance
- Time-dependent equation of state
- Small-field models: mass suppression

The η Problem

Second slow roll parameter:

$$\eta \sim \frac{M_P^2}{2} \frac{V''(\phi)}{V(\phi)} \sim \frac{m_\phi^2 M_P^2}{V_0} \sim 10^{-2}$$

Radiative corrections:

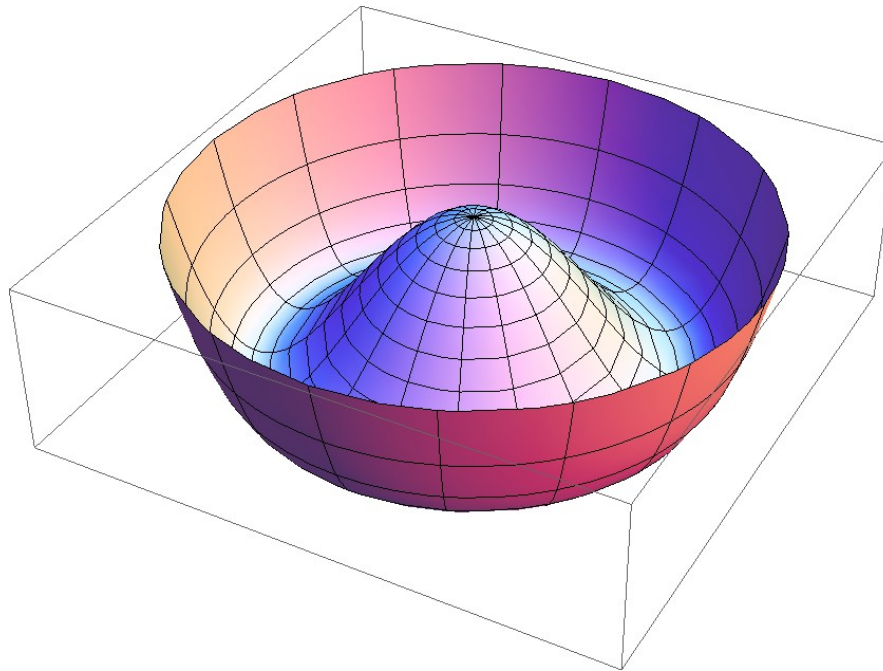
A Feynman diagram representing a radiative correction. It consists of a dashed horizontal line on the left, which enters a circular loop. The loop has two arrows indicating a clockwise direction. A dashed horizontal line exits the loop on the right. This diagram is followed by the equation $m_\phi^2 \sim H^2 \sim \frac{V_0}{M_P^2} \Rightarrow \eta \sim 1$.
$$m_\phi^2 \sim H^2 \sim \frac{V_0}{M_P^2} \Rightarrow \eta \sim 1$$

Shift Symmetries: Natural Inflation

Freese, Frieman & Olinto, hep-ph/9207245

The idea: introduce a weakly broken global symmetry which protects the mass term.

Shift symmetry: $\mathcal{L}(\phi, \partial_\mu \phi) \rightarrow \mathcal{L}(\phi + \text{const.}, \partial_\mu \phi)$



Shift Symmetries: Natural Inflation

- Non-perturbative axion: Freese *et al.*, hep-ph/9207245

$$V(\phi) \propto \left[1 + \cos \left(\frac{\phi}{\mu} \right) \right]$$

- Chiral symmetry breaking: WHK, Mahanthappa, hep-ph/9503331

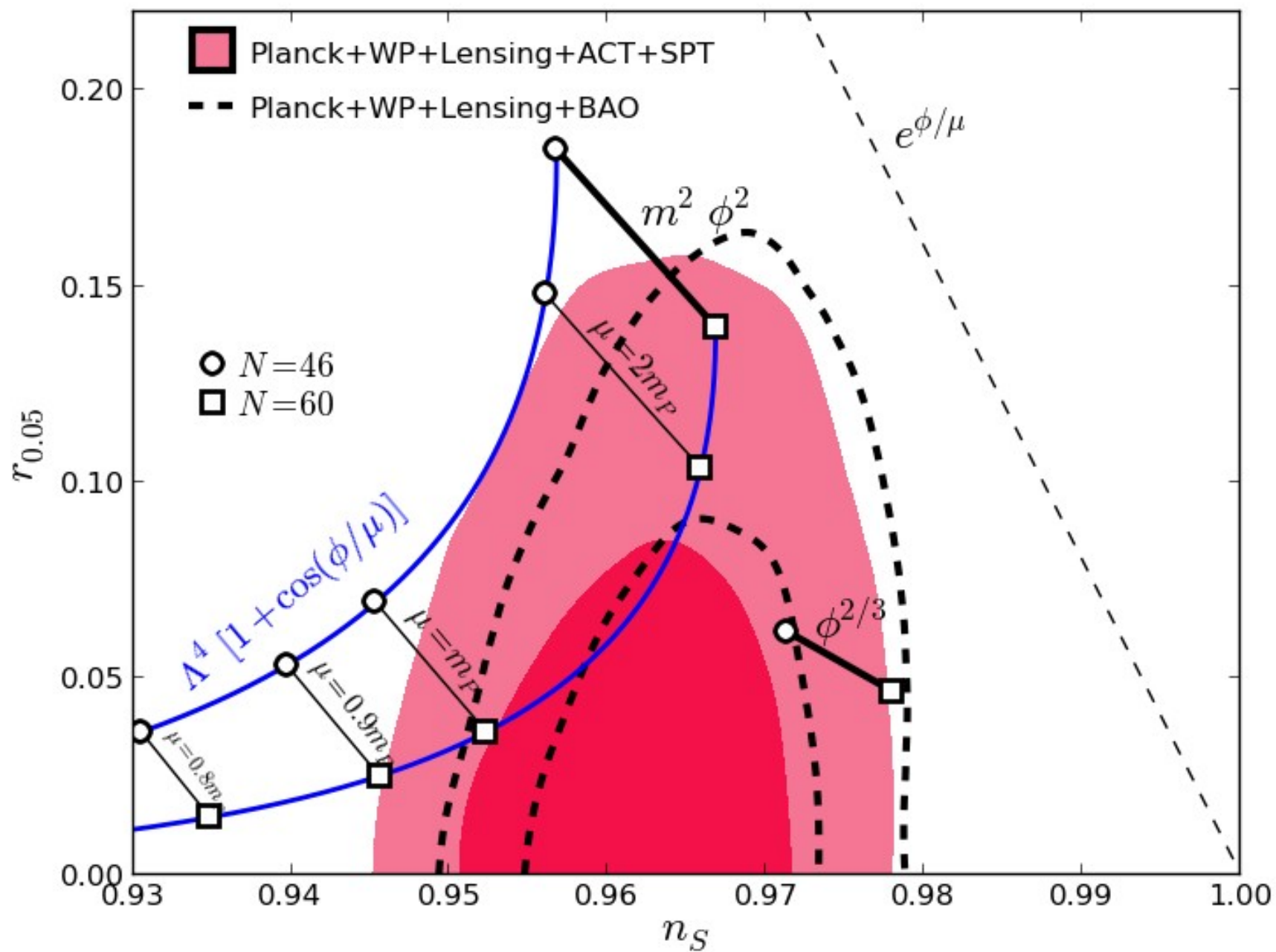
$$V(\phi) \propto [m_\psi^2(\phi)]^2 \ln [m_\psi^2(\phi)]$$

- Gauge symmetry breaking: WHK, Mahanthappa, hep-ph/9512241

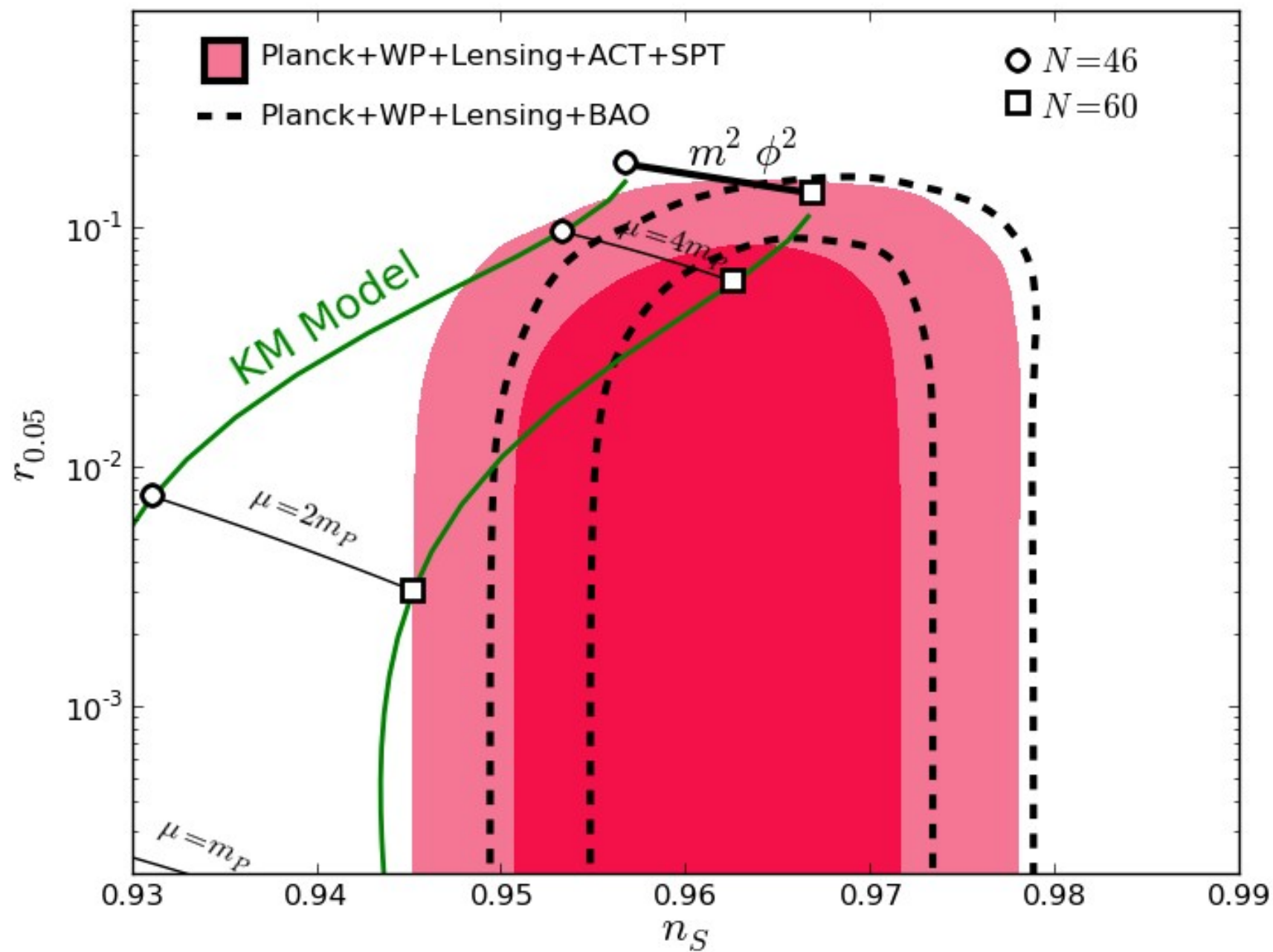
$$V(\phi) \propto \sin \left(\frac{\phi}{\mu} \right)^4 \ln \left[\sin \left(\frac{\phi}{\mu} \right)^2 \right] \quad (\text{KM Model})$$

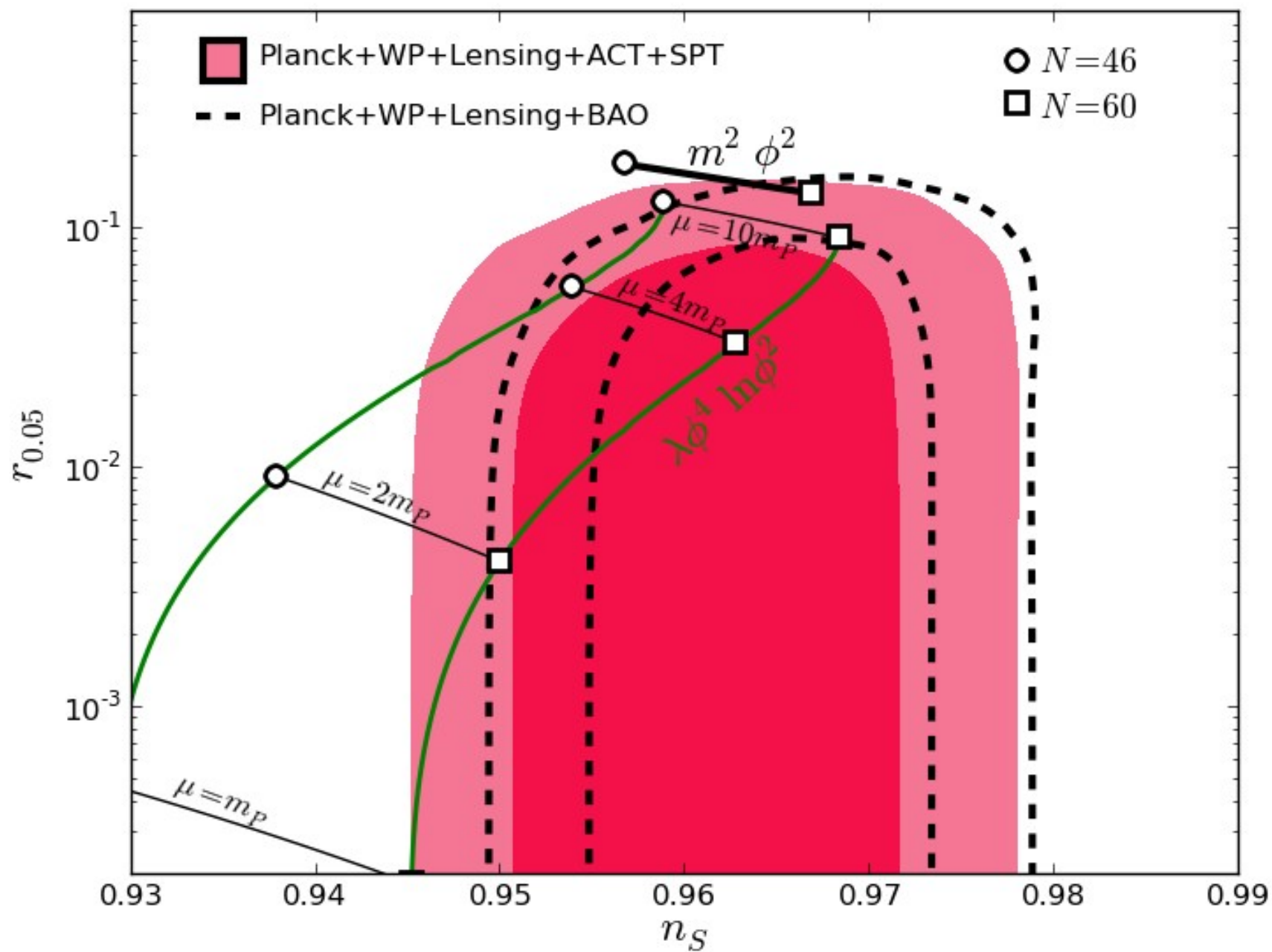
- Axion Monodromy: Silverstein, *et al.*, arXiv:0803.3085

$$V(\phi) \propto \phi^{2/3}$$



(Data: Planck Legacy Archive)





(Data: Planck Legacy Archive)

Natural Inflation and Effective Field Theory

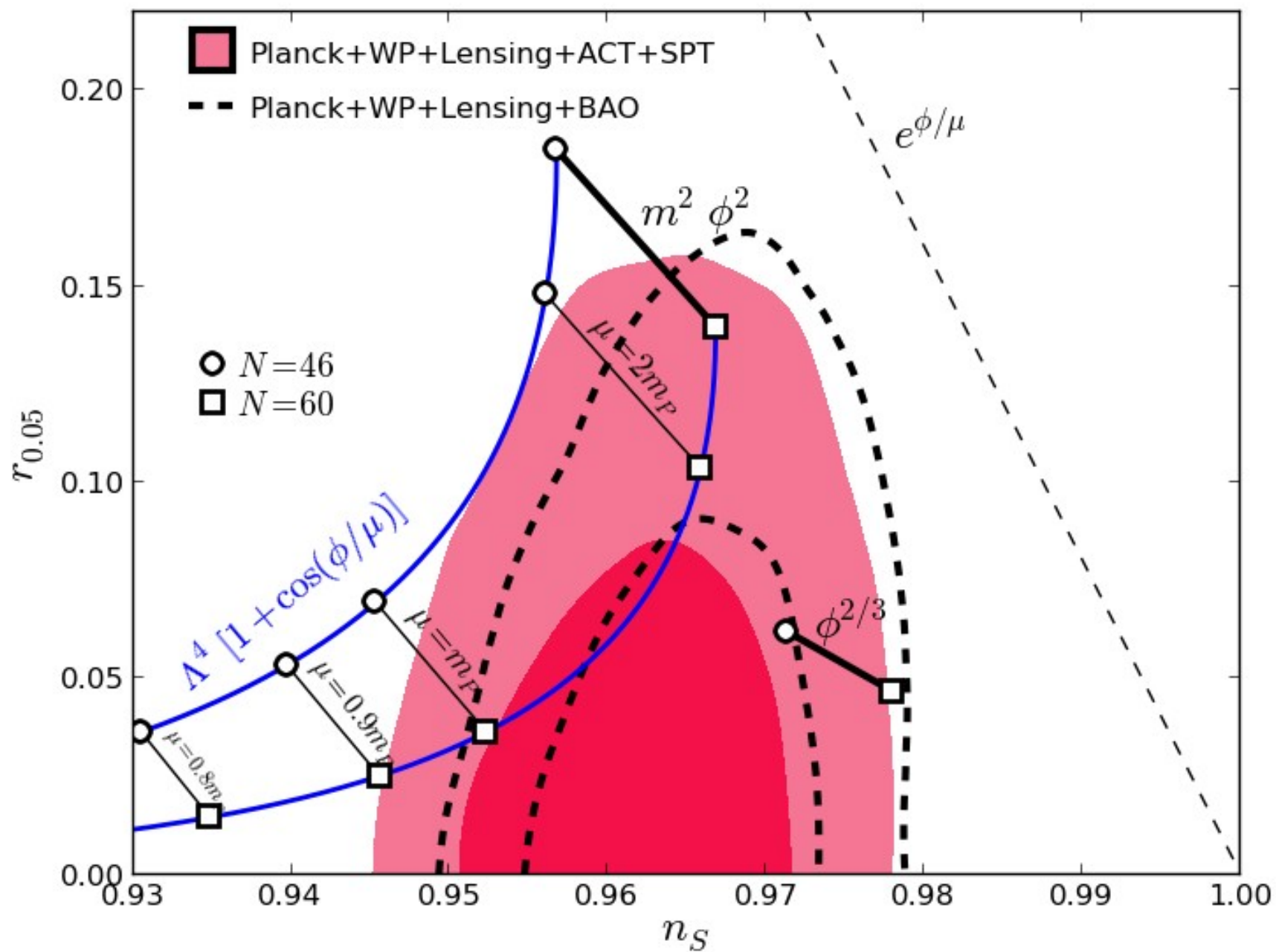
Effective potential suppressed by mass scale μ

$$V(\phi) = V_0 - \sum_p \lambda_p \left(\frac{\phi}{\mu} \right)^p$$

Lowest-order term *always* dominates near origin:

$$p = 2 : \quad V(\phi) \simeq V_0 - m^2 \phi^2 + \dots$$

$$p = 4 : \quad V(\phi) \simeq V_0 - \lambda \phi^4 + \dots$$



(Data: Planck Legacy Archive)

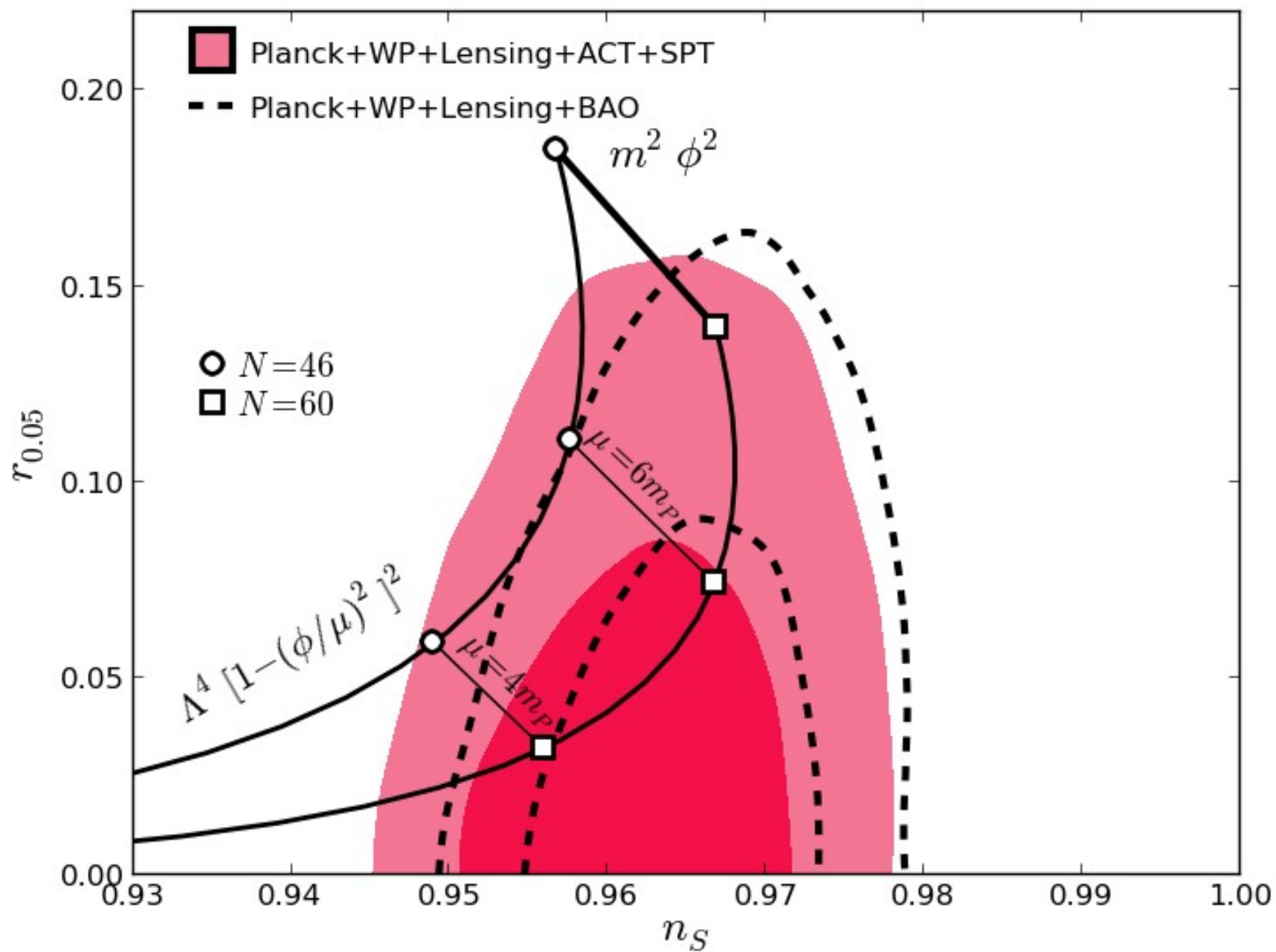
Natural Inflation and Effective Field Theory

Effective potential suppressed by mass scale μ

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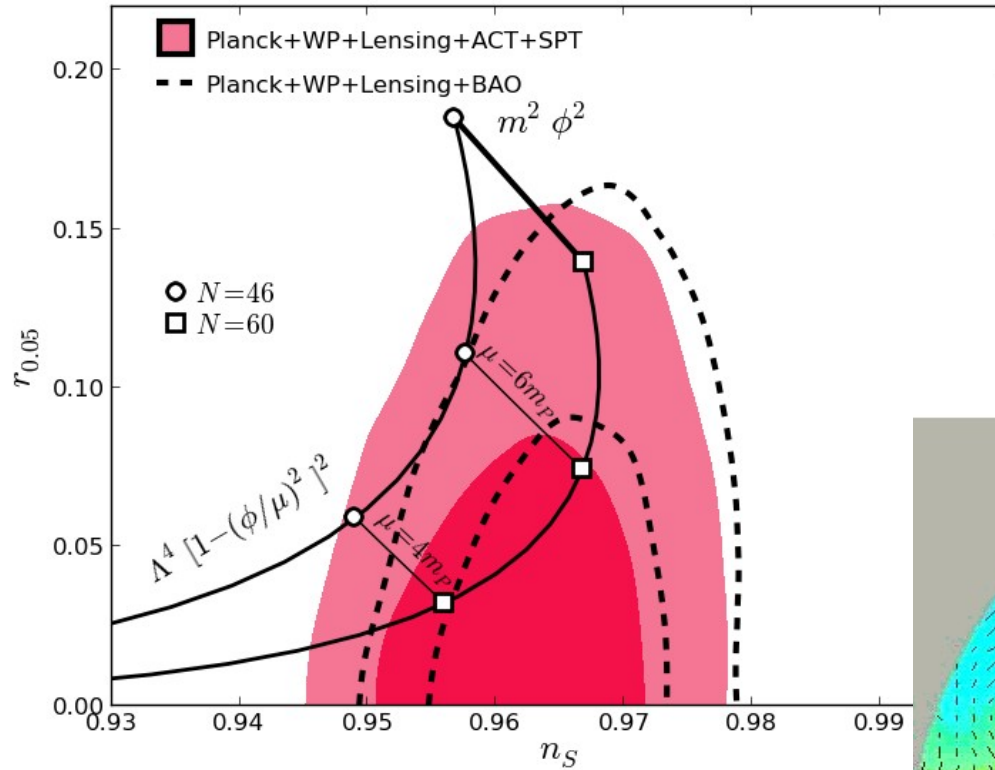
$p = 2$: Spectral index quadratic in scale

$$n = 1 - \frac{1}{4\pi} \left(\frac{m_{\text{P}}}{\mu} \right)^2$$



(Data: Planck Legacy Archive)

Tensors and CMB Polarization

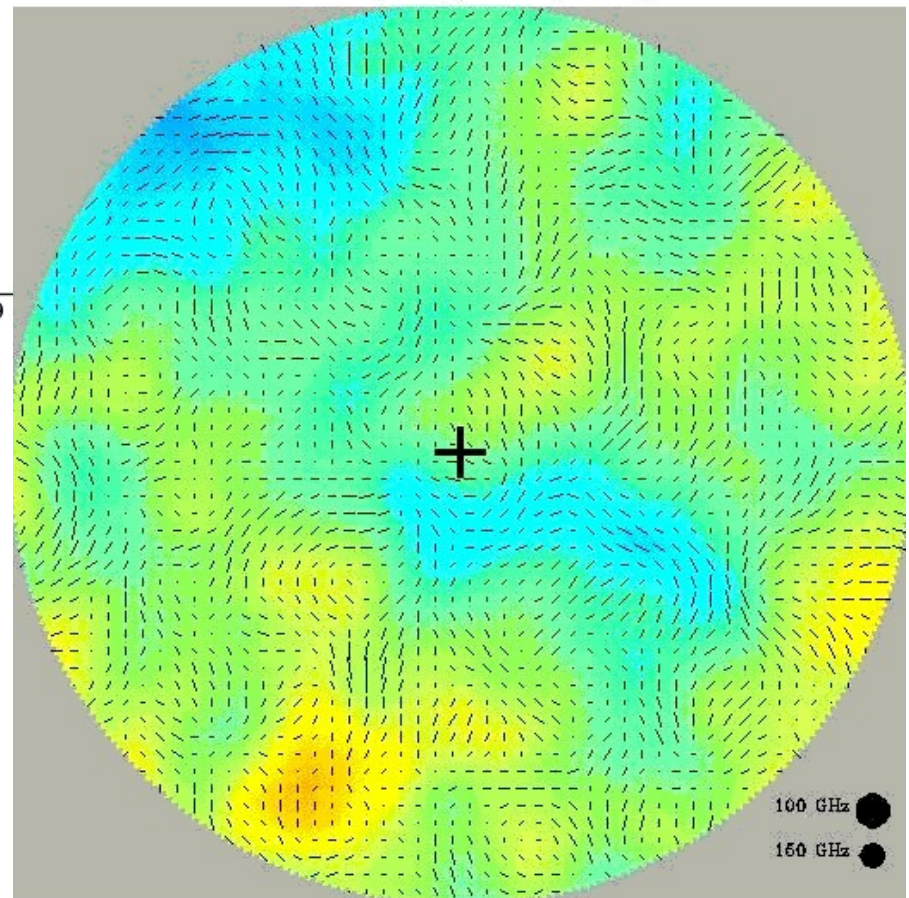


Quadratic potentials:

$$r > 0.02$$

Tensor Perturbations

42' beam, 30deg. diam. polar cap



100 GHz
 150 GHz

-200 200 μK

3.53 μK

BICEP / POLAR / ABS /
 POLARBEAR / QUIET /
 SPTpol / ACTpol / SPIDER /
 EBEX / QUIJOTE

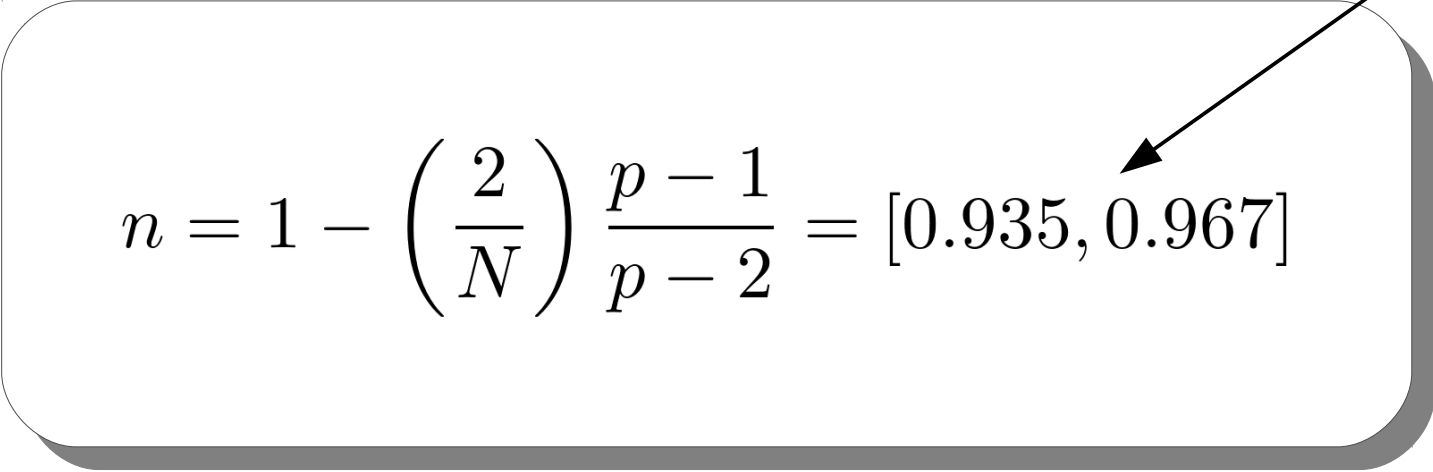
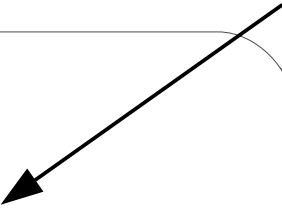
(Figure: BICEP collaboration)

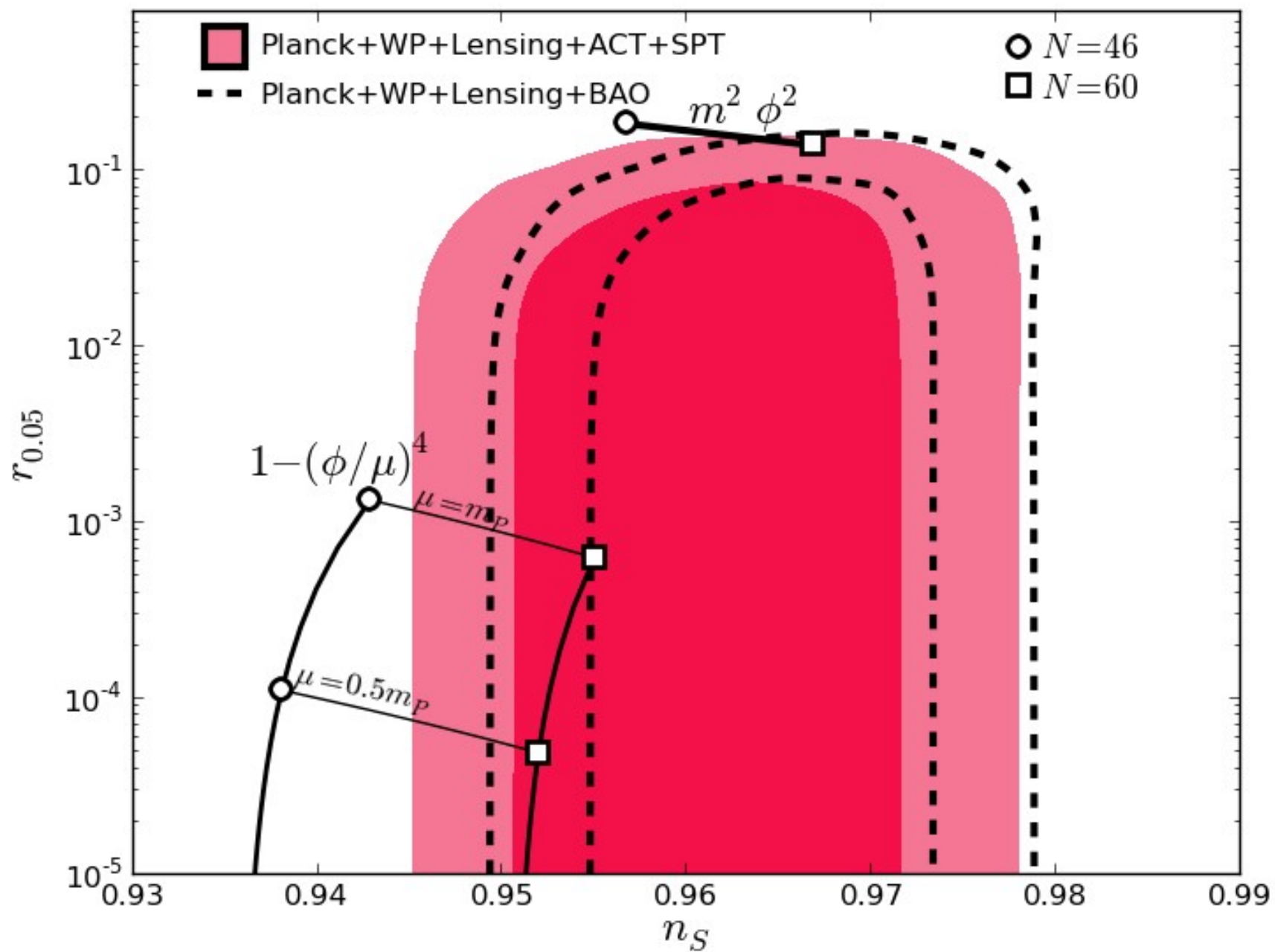
Natural Inflation and Effective Field Theory

Effective potential suppressed by mass scale μ

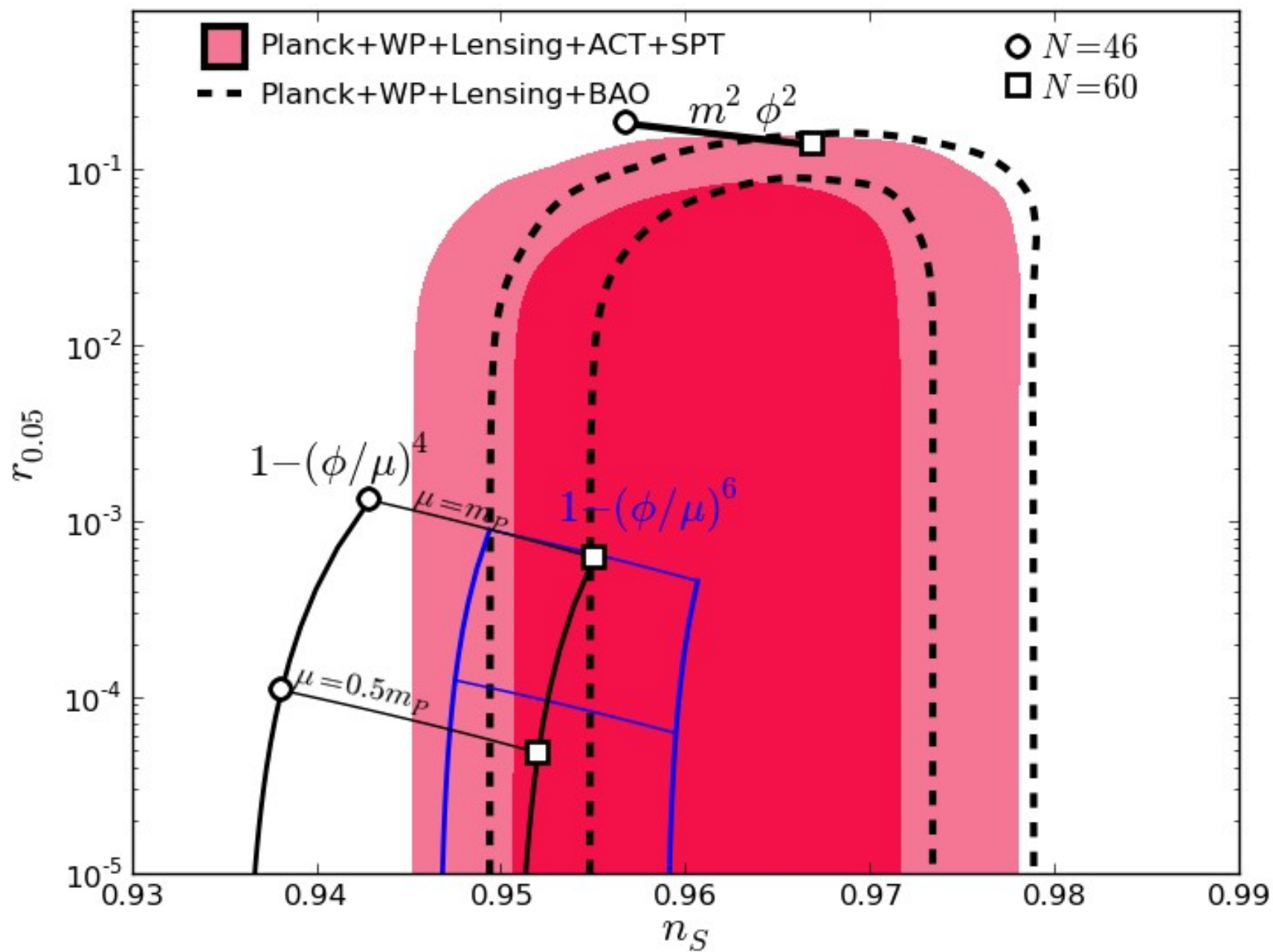
$$V(\phi) = V_0 - \sum_p \lambda_p \left(\frac{\phi}{\mu} \right)^p$$

$p > 2$: Spectral index *independent* of scale $p \geq 4$

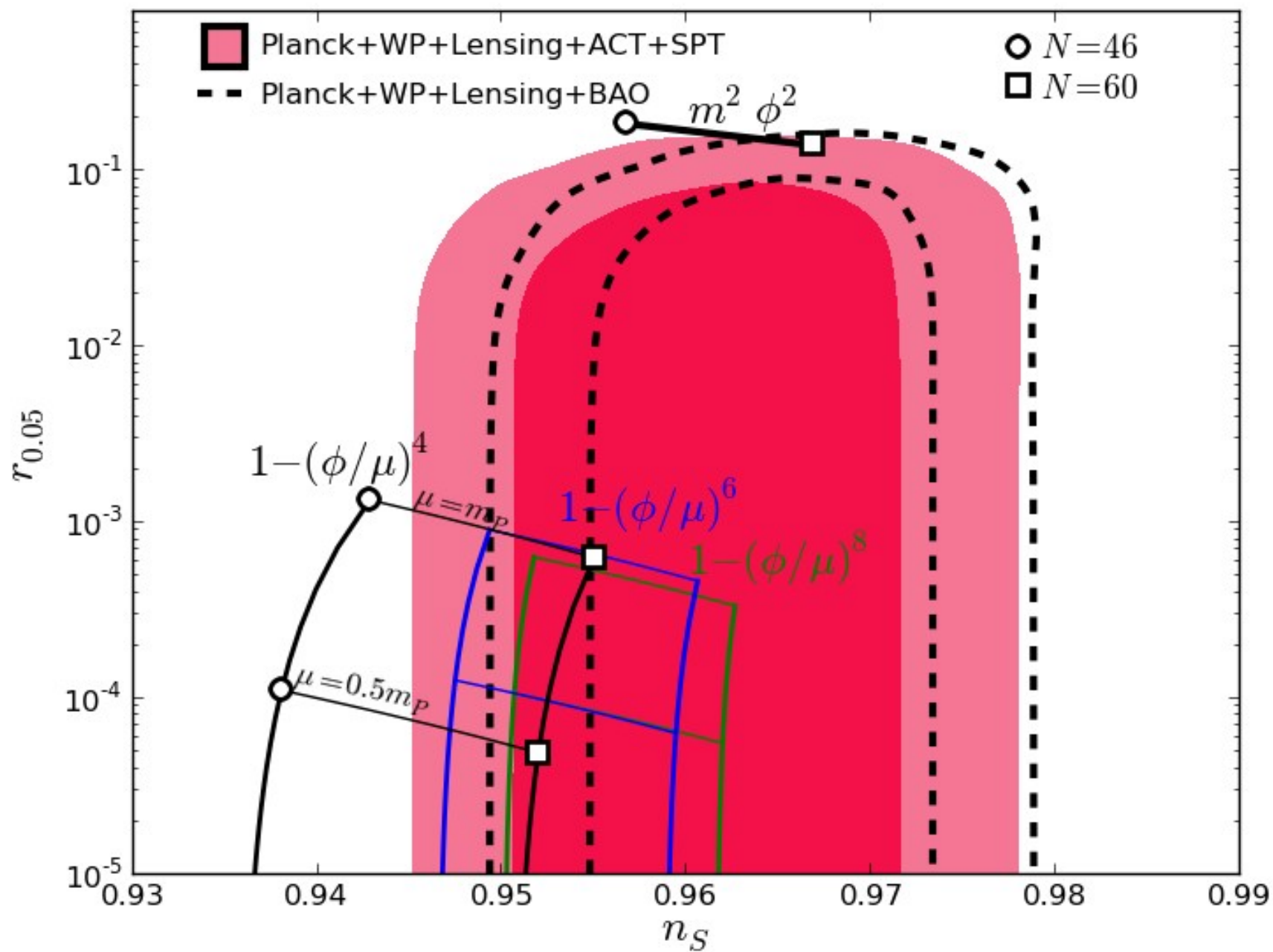

$$n = 1 - \left(\frac{2}{N} \right) \frac{p-1}{p-2} = [0.935, 0.967]$$




(Data: Planck Legacy Archive)

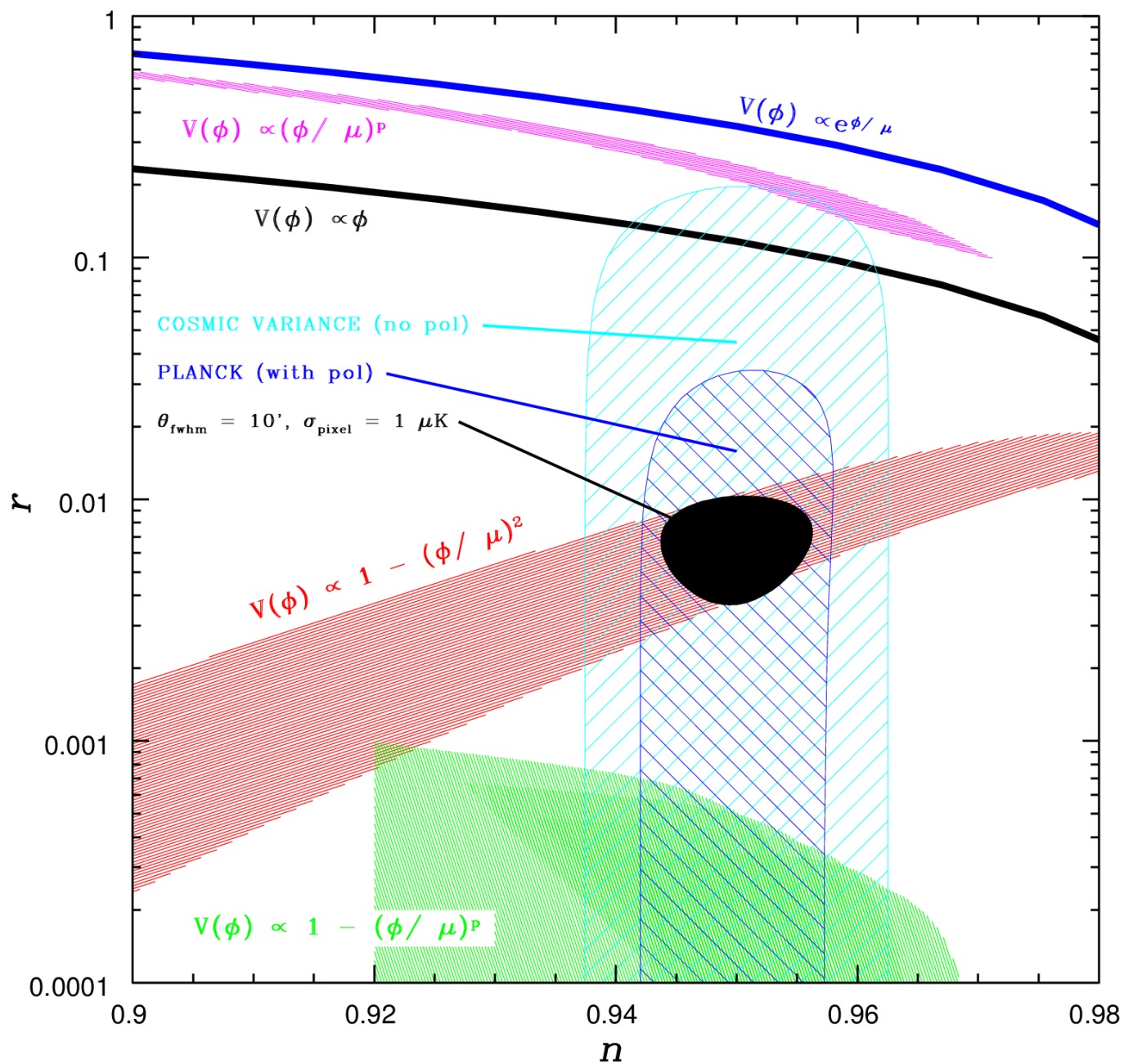


(Data: Planck Legacy Archive)



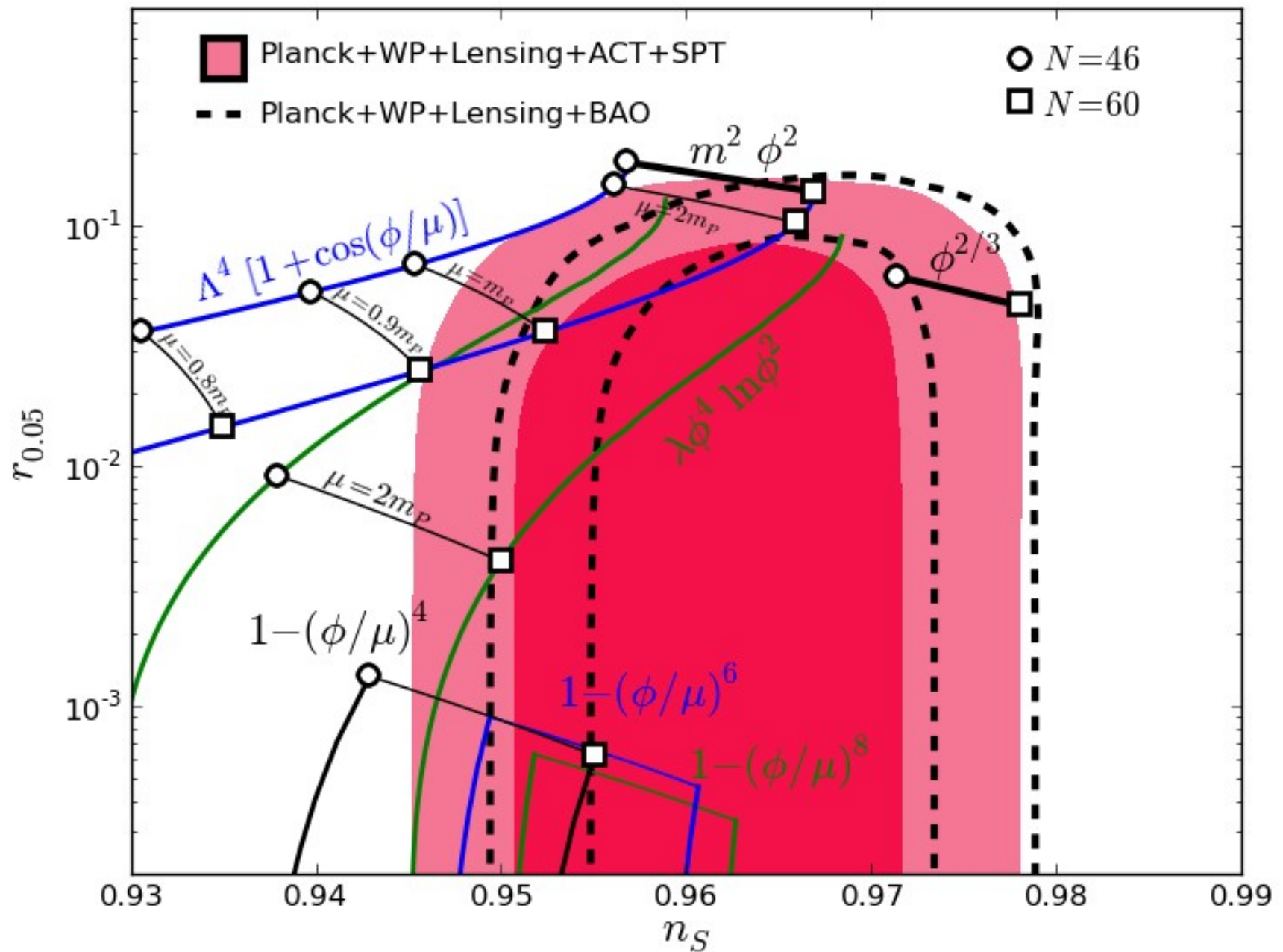
(Data: Planck Legacy Archive)

Planck: 1998 Forecast



(WHK, astro-ph/9806259)

Summary: Planck and Natural Inflation



(Data: Planck Legacy Archive)