Non-Isentropic Inflation Models and Constraints from CMBR[1]

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[1] ROR and L. A. da Silva, JCAP 03 (2013) 032; ROR, I. G. Moss, A. Berera and M. Basteiro-Gil, J. Rosa, work in progress

Cosmological Inflation

$$\ddot{a} > 0, \quad p < -\rho/3 \quad \Rightarrow \quad \text{Inflation}$$

Relevent cosmological parameters for inflation:

- Curvature perturbations:

$$\Delta_R^2 = \Delta_R^2(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}, \quad \text{amplitude} = \Delta_R^2(k_0)$$

- Spectral index:

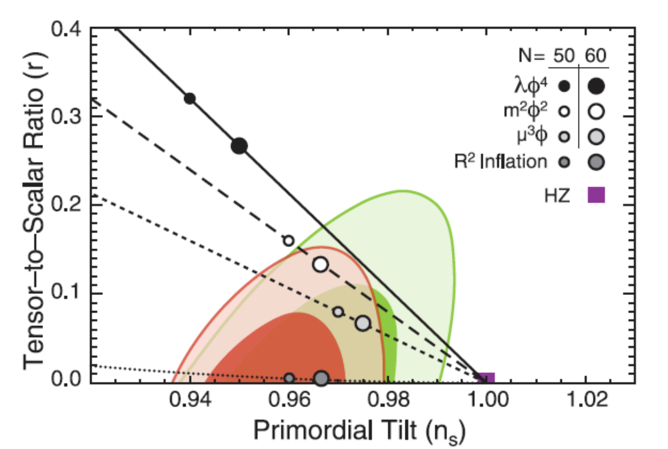
$$n_s - 1 = \frac{d\ln\Delta_R^2}{dN_e} = \frac{d\ln\Delta_R^2}{d\ln k}$$

- Tensor to scalar curvature perturbation ratio:

$$r = \frac{\Delta_T^2}{\Delta_R^2}$$

- NonGaussianity:

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$
$$B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = f_{NL}\ F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$



Tensor–to–Scalar Ratio (r)
Primordial Tilt (n_s)
0.0
0.1
0.2
0.3
0.4
0.94
0.96
0.98
1.00
1.02
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
$\mu^3\phi$
R^2 Inflation
HZ

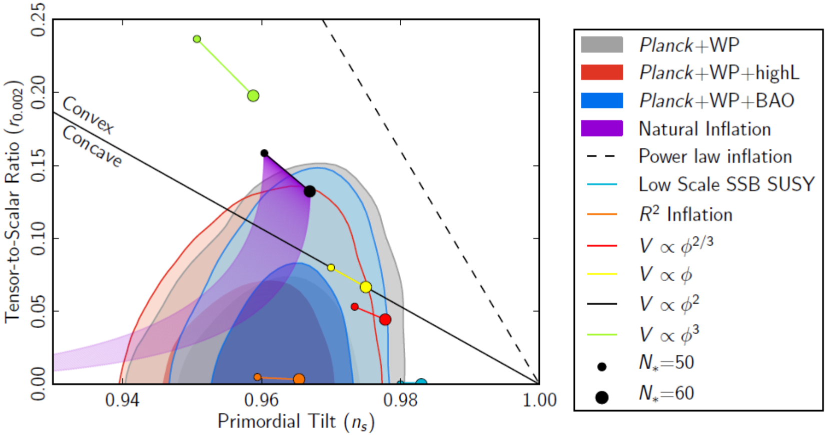


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Cold Inflation[a]:

[a] (Starobinsky '80; Guth '81; Albrecht, Steinhardt '82; Linde '82)

$$\text{Inflaton} \Longrightarrow \text{Reheating} \Longrightarrow \text{Radiation} \Longrightarrow \text{Matter}$$

Inflaton interactions with other d.o.f. only important during reheating

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (H = \dot{a}/a \sim cte)$$

$\Rightarrow$ The radiation density during inflation redshifts away: $\rho_r \sim 1/a^4$

$\Rightarrow$ density perturbations sourced by inflaton's quantum fluctuations

If the universe did not supercool, then...

Warm Inflation[a]:

[a] Ian G. Moss PLB154 1985, Yokoyama and Maeda PLB207 1988, A. Berera and L. Z. Fang PRL75 1995

$$\text{Inflaton} \Longrightarrow \text{Decay} \Longrightarrow \text{Radiation} \Longrightarrow \text{Matter}$$

Inflaton interactions with other d.o.f. is important during inflation generate dissipation/viscosity terms $\Rightarrow$ small fraction of vacuum energy density can be converted to radiation

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

$\Rightarrow$ The radiation density during inflation stabilises: $\Rightarrow \rho_r \sim \Upsilon\dot{\phi}^2/(4H)$

Warm Inflation

- The production of radiation is associated with a friction term in the inflaton equation,

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + V_\phi = 0,$$

- $\Upsilon\dot{\phi}$ describes how inflaton's interactions with other fields backreact on the inflaton dynamics.
- Effectiveness of warm inflation measured by $Q = \frac{\Upsilon}{3H}$
- Cold or warm inflation: $T \ll H$ or $T \gtrsim H$
- adiabatic density fluctuations are sourced by thermal fluctuations: amplitude[2] $\delta\phi^2 = (H\Upsilon)^{1/2} T$, density fluctuations $\zeta = H\delta\phi/\dot{\phi}$. CMB amplitude $\Delta_R^2(k_0) = (2.41 \pm 0.10) \times 10^{-9}$ (WMAP Nine-year Mean) fixes the energy scale V.

[2] for $T \gg H, Q \gg 1$, Hall, Moss, Berera 2004

Microscopic basis for warm inflation[4]

Two-stage reheating:

$$\begin{array}{ccccc} \phi & \longrightarrow & \chi, \psi_\chi & \longrightarrow & \psi_\sigma, \sigma \\ \text{inflaton} & & \text{heavy field} & & \text{light fermions or scalars} \end{array}$$

$$\text{for example}: \; \mathcal{L}_I = -\frac{1}{2} g_\chi^2 \phi^2 \chi^2 - g_\psi \phi \bar{\psi}_\chi \psi_\chi - h_\sigma M \chi \sigma^2 - h_\psi \chi \bar{\psi}_\sigma \psi_\sigma$$

- decouple the radiation from the inflaton ($m_\chi, m_{\psi_\chi} \gg T, m_{\sigma,\psi_\sigma} \ll T$)
- couplings between fields of order 0.1
- SUSY to reduce vacuum corrections, e.g., $W = g\Phi X^2 + hXY^2$

Typically[3] ($c + 2a - 2b = 1$, low-T: $c = 3$, high-T: $c = -1$ or no T: $c = 0$):

$$\Upsilon = C_\phi \frac{T^c \phi^{2a}}{m_X^{2b}} \sim C_\phi \frac{T^c}{\phi^{c-1}},$$

[3] M. Basteiro-Gil, A. Berera, ROR, JCAP 09 (2011) 033

[4] For a review: Berera, Moss and ROR, Rep. Prog. Phys. **72**, 026901 (2009)

Let $\phi \equiv \phi(x,t)$ and average out (integrate over) the other fields. This gives a stochastic (Langevin-like) system[5],

$$\ddot{\phi}(\mathbf{x},t)+3H\dot{\phi}(\mathbf{x},t)+\int d^4x'\Sigma_R(x,x')\phi(x')+V_{,\phi}-\frac{1}{a^2}\nabla^2\phi(\mathbf{x},t)=\xi(\mathbf{x},t).$$

The self-energy contribution is a dissipative term (in the adiabatic approximation, $\dot{\phi}/\phi, H, \dot{T}/T < \Gamma_\chi \approx h^2 m_\chi/(8\pi)$)[6],

$$\int d^4x'\Sigma_R(x,x')\phi(x') \approx \Upsilon\dot{\phi}(x,t)$$

$$\ddot{\phi}(x,t)+(3H+\Upsilon)\dot{\phi}(x,t)+V_\phi-\frac{1}{a^2}\nabla^2\phi(x,t)=\xi_T(x,t).$$

We have a spatial gradient term, a scale factor a, and a stochastic thermal source ξ_T. The source has a gaussian distribution with local correlation function

$$\langle\xi_T(\mathbf{x},t)\xi_T(\mathbf{x}',t')\rangle = a^{-3}\Upsilon T\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

[5] M. Gleiser and ROR, PRD50, 2441 (1994)

[6] A. Berera, I. G. Moss and ROR, PRD76, 083520 (2007)

Including quantum fluctuations (A.A. Starobinsky 1988, Stochastich Inflation approach):

$$\Phi(\vec{x}, t) \to \Phi_{>}(\vec{x}, t) + \Phi_{<}(\vec{x}, t) \; .$$

- $\Phi_{>}(\vec{x}, t) \to$ long wavelenght (super horizon) part
- $\Phi_{<}(\vec{x}, t) \equiv \phi_q(\vec{x}, t) \to$ summarizes high frequency, short-wavelength (sub horizon) quantum fluctuations ($k > k_h \approx aH$):

$$\Phi_{<}(\mathbf{x}, t) \equiv \phi_q(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} W(k, t) \left[\phi_{\mathbf{k}}(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \hat{a}_{\mathbf{k}} + \phi^*_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{a}^\dagger_{\mathbf{k}}\right]$$

$W(k, t)$ is a filter or window function that projects out the long wavelength modes.

Backreaction of $\Phi_<$ into the dynamics of local order parameter $\Phi_>$

$\Downarrow$

modelled as a (quantum) noise term

$\Downarrow$

combine with the derivation in the warm inflation context[7]

$$\left[\frac{\partial^2}{\partial t^2} + (3H + \Upsilon)\frac{\partial}{\partial t} - e^{-2Ht}\nabla^2\right]\Phi_>(\vec{x},t) + \frac{\partial V(\Phi_>)}{\partial \Phi_>} = \xi_q(\vec{x},t) + \xi_T(\vec{x},t)\;,$$

quantum noise: $\xi_q = -\left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\nabla^2 + V_{,\phi\phi}(\Phi_>)\right]\phi_q$

$\Rightarrow$ Eq. for $\Phi_>$ is similar to a Langevin equation with quantum and thermal noise terms (stochastic process w/ gaussian noises).

[7] ROR and L. A. da Silva, JCAP 03 (2013) 032

Equation for the fluctuations:

$$\delta\ddot{\varphi}(\vec{k},t) + (3H+\Upsilon)\delta\dot{\varphi}(\vec{k},t) + V_{,\phi\phi}(\phi)\delta\varphi(\vec{k},t) + a^{-2}k^2\delta\varphi(\vec{k},t) = \tilde{\xi_T}(\vec{k},t) + \tilde{\xi_q}(\vec{k},t)\ .$$

General solution can be expressed in terms of a Green function

$$\delta\varphi(\mathbf{k},z) = \int_z^\infty dz'\, G(z,z')\frac{(z')^{1-2\nu}}{z'^2H^2}\left[\tilde{\xi}_q(z') + \xi_T(z')\right],\ \ (z\equiv\frac{k}{aH}),$$

$$G(z,z') = \frac{\pi}{2}z^\nu z'^\nu\left[J_\alpha(z)Y_\alpha(z') - J_\alpha(z')Y_\alpha(z)\right]\ , z' > z$$

$$\nu = 3(1+Q)/2\ ,\ \ \alpha = \sqrt{\nu^2 + \frac{3\beta Q}{1+Q} - 3\eta}$$

$$\varepsilon = \frac{1}{16\pi G}\left[\frac{V_{,\phi}}{V}\right]^2\ ,\ \ \eta = \frac{1}{8\pi G}\frac{V_{,\phi\phi}}{V}\ ,\ \ \beta = \frac{1}{8\pi G}\frac{\Upsilon_{,\phi}V_{,\phi}}{\Upsilon V}$$

slow-roll coefficients: $\varepsilon,\eta,\beta \ll 1+Q$, $Q = \Upsilon/(3H)$

Thermal ξ_T and quantal ξ_q noises are uncorrelated (decoupled), so they give separated contributions to the power spectrum:

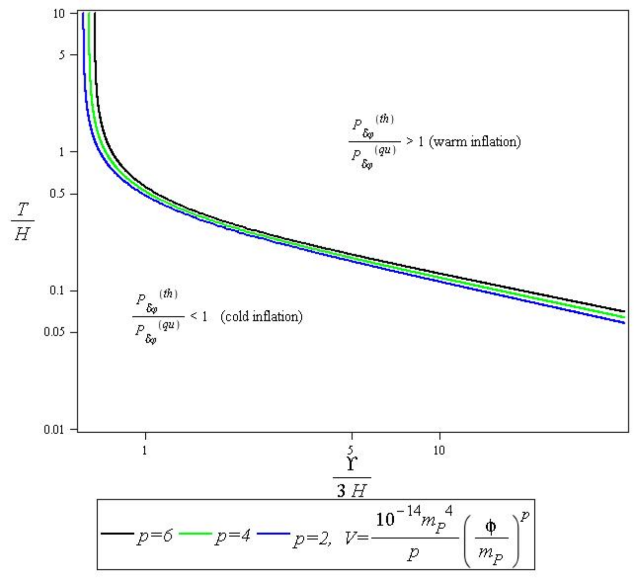
$$P_{\delta\varphi}(z) = \frac{k^3}{2\pi^2}\int \frac{d^3k'}{(2\pi)^3}\langle \delta\varphi(\mathbf{k},z)\delta\varphi(\mathbf{k}',z)\rangle = P^{(\mathrm{th})}_{\delta\varphi}(z) + P^{(\mathrm{qu})}_{\delta\varphi}(z)$$

Explicitly, we find:

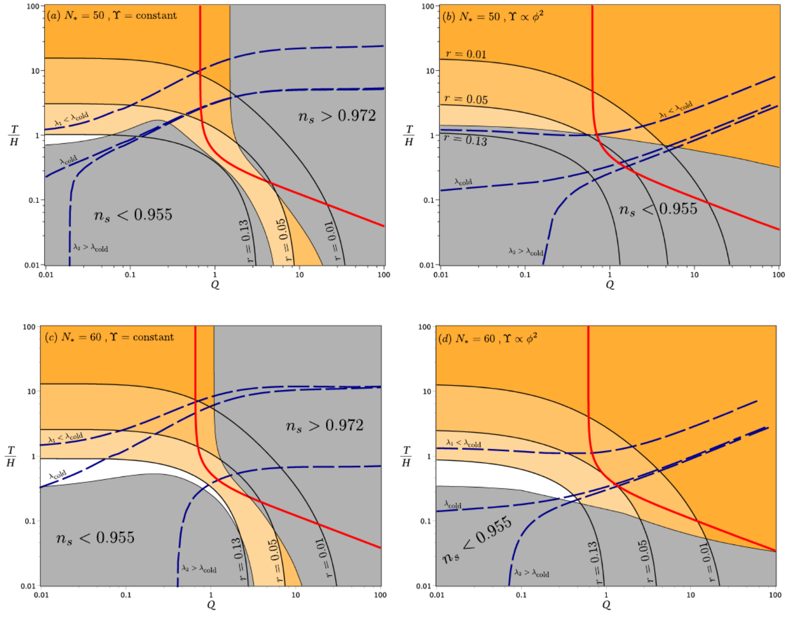
$$P^{(\mathrm{qu})}_{\delta\varphi}(z) \simeq \left[\frac{2}{\exp(H/T)-1}+1\right]\frac{z^{3-2\mu}H^2}{4\pi^2}\,, \quad \mu = \sqrt{9/4-3\eta}$$

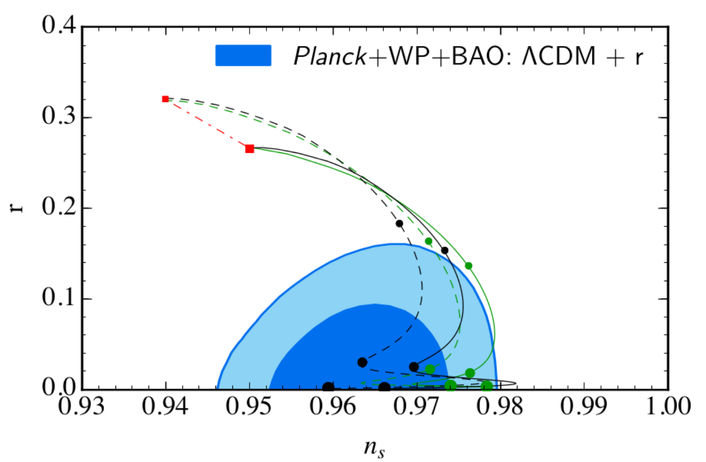
$$P^{(\mathrm{th})}_{\delta\varphi}(z) \simeq \frac{\Upsilon T}{16\pi^2} z^{2\nu-2\alpha}\frac{\left[2^{\nu}\Gamma(\nu)\right]^2\Gamma(\nu-1)}{\Gamma(2\nu-1/2)\Gamma(\nu-1/2)}\,, \quad \nu = 3(1+Q)/2$$

Conventional inflation: $P_{\delta\varphi}(T/H\to 0)\to \frac{H^2}{4\pi^2}$


10
5
1
0.5
0.1
0.05
0.01
$\frac{T}{H}$
$\frac{P_{\delta\varphi}^{(th)}}{P_{\delta\varphi}^{(qu)}} > 1$ (warm inflation)
$\frac{P_{\delta\varphi}^{(th)}}{P_{\delta\varphi}^{(qu)}} < 1$ (cold inflation)
1
5
10
$\frac{\Upsilon}{3H}$
$p=6$ $p=4$ $p=2$, $V=\frac{10^{-14} m_P^4}{p}\left(\frac{\phi}{m_P}\right)^p$

ϕ^4 potential ($Q = \Upsilon/(3H)$):



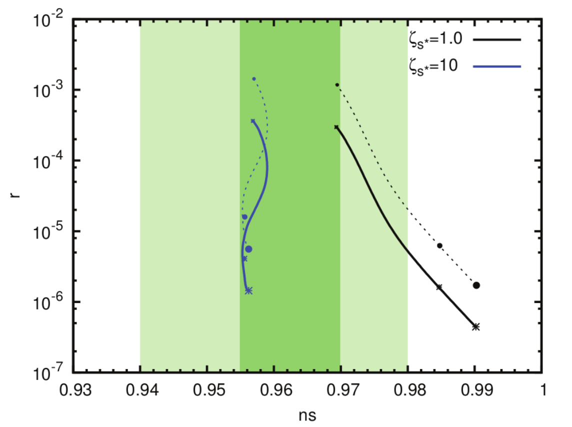


dashed lines: $N_* = 50$-efolds, solid lines: $N_* = 60$-efolds

dot: $T/H = 100, 10, 1, \quad \rho_{\rm rad} \sim g_* \pi^2 T^4/30$

$(W = g\Phi X^2 + hXY^2)$

Green: $g_* = 15/4 \ (N_Y = 1)$, Black: $g_* = 228.75 \ (MSSM)$



dots, stars: $Q = 50, 25, 1,$

solid line: $g_* = 15/4 \ (N_Y = 1)$, dashed line: $g_* = 228.75 \ (MSSM)$

Warm inflation can also provide the conditions for baryogenesis (assuming B- and CP-violation in the interactions between the heavy and the light fields) [8]

$\Rightarrow$ thermal fluctuations of the inflaton field will be imprinted on the baryon-to-entropy ratio

$\Rightarrow$ generates baryon isocurvature perturbations: $B_B = (\delta\eta_s/\eta_s)/\zeta$ (where $\eta_s = n_B/s$, is the baryon-to-entropy ratio $\zeta = -H\delta\rho/\dot{\rho}$) correlated with adiabatic perturbations:

$$B_B \simeq -0.119, \quad n_{\rm iso} \simeq 1.02$$

(Planck: $|B_B| \lesssim 0.51$)

[8] M. Basteiro-Gil, A. Berera, R.O.R., J. G. Rosa, PLB 712 (2012) 425

Conclusions and Outlook

- Warm inflation fully includes the effects of interactions in the inflaton dynamics

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- Warm inflation fully includes the effects of interactions in the inflaton dynamics
- Warm inflation $\rightarrow$ dissipation mechanisms $\rightarrow$ radiation production $\rightarrow$ backreaction effect on the power spectrum
- Thermal fluctuations bring $\lambda\phi^4$ model back to business:
  Both n_s and r restricts warm inflation to a region:
  $T/H \gtrsim 1$ and for $\Upsilon/(3H) \lesssim 1$, (high-T, low dissipation regime)

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- Warm inflation $\rightarrow$ dissipation mechanisms $\rightarrow$ radiation production $\rightarrow$ backreaction effect on the power spectrum
- Thermal fluctuations bring $\lambda\phi^4$ model back to business: Both n_s and r restricts warm inflation to a region: $T/H \gtrsim 1$ and for $\Upsilon/(3H) \lesssim 1$, (high-T, low dissipation regime)
- nongaussianities should further constrain warm inflation models[9] (expected to be true also for cold inflation models)

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Warm inflation: This model, where dissipative effects are important, predicts $f_{\rm NL}^{\rm warm} = -15 \ln(1 + r_{\rm d}/14) - 5/2$ (Moss & Xiong 2007) where the dissipation parameter $r_{\rm d} = \Gamma/(3H)$ must be large for strong dissipation. The limit from *WMAP* is $r_{\rm d} \leq 2.8 \times 10^4$ (Moss & Xiong 2007). Assuming a prior $0 \leq \log_{10} r_{\rm d} \leq 4$, our constraint $f_{\rm NL}^{\rm warmS} = 4 \pm 33$ at 68% CL (see Sect. 7.3.5) yields a limit on the dissipation parameter of $\log_{10} r_{\rm d} \leq 2.6$ (95% CL), improving the previous limit by nearly two orders

[9] (M. Basteiro-Gil, A. Berera, I. Moss, ROR, to appear)