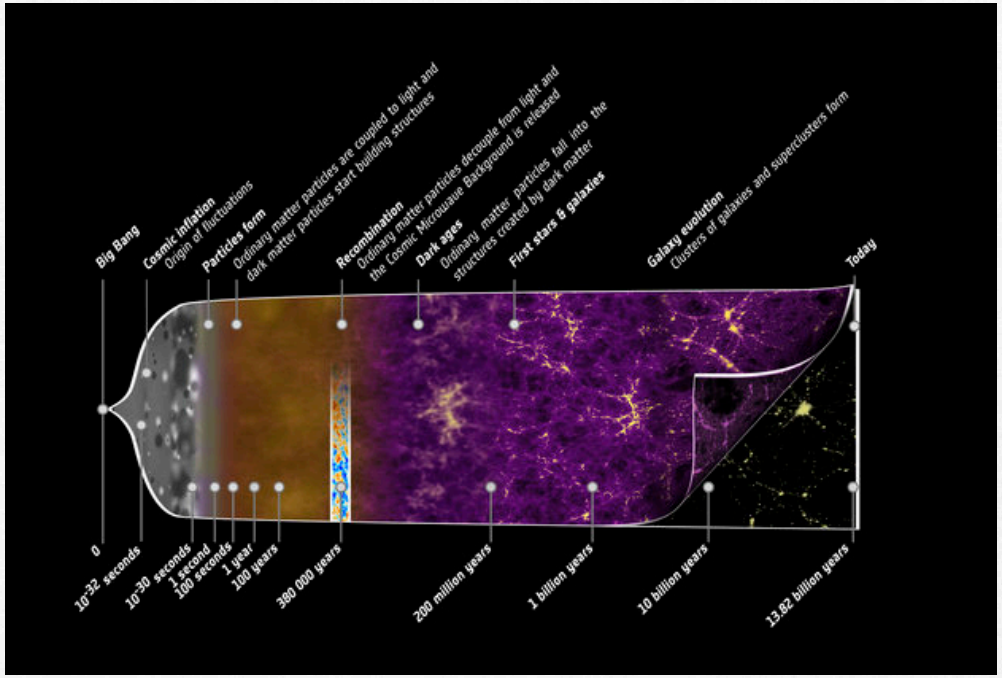
Strongly coupled phase of inflation & holography

Yuko Urakawa (U. of Barcelona)
with J. Garriga (U. of Barcelona)

Y.U. and J.G. arXiv:1303.5997, to appear in JCAP
Y.U. and J.G., ... in progress

Probing the early universe



©Planck

time →

← energy scale

- Cosmological perturbation
- Quantum field theory

$$P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{k^3}$$

AdS/CFT correspondence

Maldacena (97)

$\mathcal{N} = 4$ SU(N) super Yang-Mills theory in 4D

$(N \gg 1,\ N g_s \gg 1)$

Duality $\Updownarrow$

Classical type IIB SUGRA on $AdS_5 \times S^5$ in 10D

- SO(2,4) × SO(6) symmetry
- Correlation functions in CFT from gravity

Gubser, Klebanov, Polyakov (98), Witten (98)

$$Z_{\text{bulk}}\left[\Phi(z,\mathbf{x})|_{z=0}\right] = \left\langle e^{-\int d^4\mathbf{x}\, \Phi(\mathbf{x}) O(\mathbf{x})} \right\rangle_{\text{CFT}} \equiv Z_{\text{CFT}}$$

Gauge/Gravity correspondence

- Holographic principle

't Hooft(92), Susskind(95)

Holographic principle suggests that a gravity theory should be related to a non-gravitational theory in one fewer dimension.

d-dim gauge theory $\longleftrightarrow$ (d+1)-dim gravity theory
+ RG flow

- Non-trivial duality

Maldacena (97)

Boundary CFT		Bulk gravity
'tHooft coupling λ	$\lambda=(r_0/l_s)^4$	Curvature scale r_0
Strong coupling	$\lambda \gg 1,\ r_0 \gg l_s$	Weak coupling
Weak coupling	$\lambda \ll 1,\ r_0 \ll l_s$	Strong coupling

dS/CFT

Inflationary spacetime ~ de Sitter (dS) spacetime

Strominger(01), Witten(01)

- CFT lives on the spacelike boundary at the future infinity of dS.

Maldacena(02)

- Wave function from CFT

$$\Psi_{\mathrm{dS}}[g] = Z_{\mathrm{CFT}}$$

<u>Holography for dS</u>

- dS/CFT, Higher spin gravity — Strominger et al.(11)
- DW/Cosmology — McFadden, Skenderis, ...(09)
- dS/dS — Alishalia, Karch, Silverstein,...(04)

AdS and dS

<u>Anti de Sitter (AdS)</u>

Vacuum with $\Lambda < 0$

in $\mathrm{R}^{2,3}$ (-,-, +, +, +) SO(2,3)

$$-X_0{}^2 - X_1{}^2 + \sum_{a=2,3,4} X_a{}^2 = -A^2$$

$$ds^2 = l_{\mathrm{AdS}}^2 \left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \right)$$

<u>de Sitter (dS)</u>

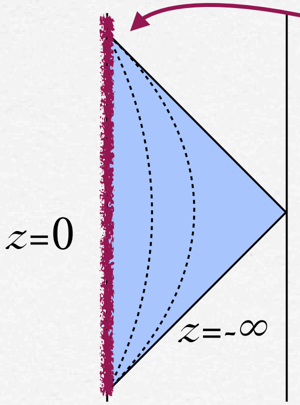
Vacuum with $\Lambda > 0$

in $\mathrm{R}^{1,4}$ (-,+, +, +, +) SO(1,4)

$$-X_0{}^2 + X_1{}^2 + \sum_{a=2,3,4} X_a{}^2 = A^2$$

$$ds^2 = l_{\mathrm{dS}}^2 \left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2} \right)$$

Boundary

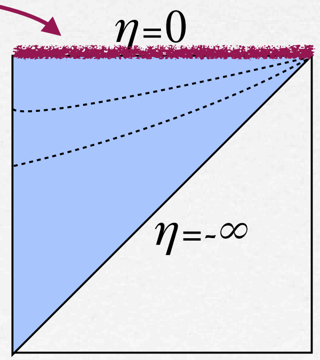


$\cdots\cdots$ z:const, R^3

$$l_{\mathrm{AdS}} \to i l_{\mathrm{dS}}$$

$$z \to i\eta$$

$$t \to -iw$$



$\cdots\cdots$ η:const, R^3

Breaking symmetry

de Sitter space

4D hyperboloid:

$$ds_4^2 = \{\eta_{\mu\nu}X^\mu X^\nu = H^{-2}\}$$

in 5D flat spacetime $R^{1,4}$

SO(1,4)

$\longleftrightarrow$

CFT on R^3

- Poincare T.
- Dilatation
- Special C.T.

$\downarrow$ Breaking dS sym.

Inflation

$\downarrow$ Breaking CS

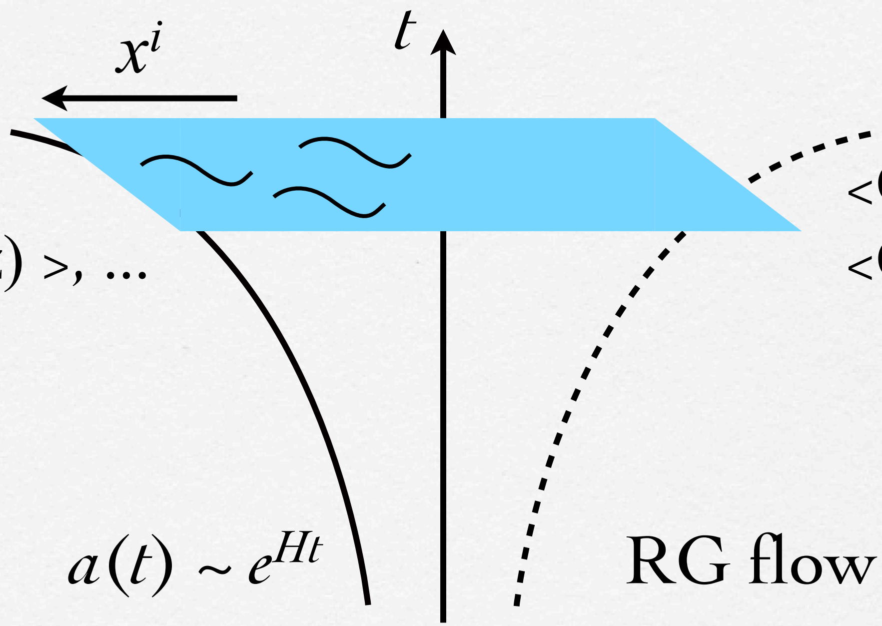
Deformed CFT

(ex)CFT+ mass

Inflation from holography

Geometry (bulk) | Field theory

$\langle\zeta(t,x)\zeta(t,y)\rangle$,
$\langle\zeta(t,x)\,\zeta(t,x)\,\zeta(t,z)\rangle$, ...



$\langle O(x)O(y)\rangle$,
$\langle O(x)O(y)O(z)\rangle$, ...
@energy scale μ

RG flow

Strongly coupled limit ⟵ Weakly coupled limit

curvature scale ≪ string scale l_s

Inflation from holography

Breaking SO(1,4)

dS → quasi dS (Inflation)   CFT → deformed CFT
(FP)   (Near FP)

Basic assumption

$$\Psi_{\mathrm{qdS}}[g,\ \Phi] = \left\langle e^{-\int d^3\mathbf{x}\,\Phi(\mathbf{x})O(\mathbf{x})} \right\rangle_{\mathrm{dCFT}}$$

Inflaton	Φ	External field
Expansion of universe	$a(t) \propto \mu \nearrow$	RG flow: IR to UV
Weakly coupled SPT	$r_0 \gg l_s,\ \lambda \gg 1$	Strongly coupled
Strongly coupled New!!	$r_0 \ll l_s,\ \lambda \ll 1$	Weakly coupled

Progresses so far

Weakly coupled limit in the bulk

- Power spectrum $$P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{k^3}$$

  Maldacena (02), Larsen et al. (02)van der Schaar(03),...

- Consistency relation $$4f_{NL} \simeq -(n_s - 1)$$

  Maldacena (02), Larsen & McNees(03),...

- Bi-spectrum

  Lidsey & Seery (06)

Strongly coupled limit in the bulk

???

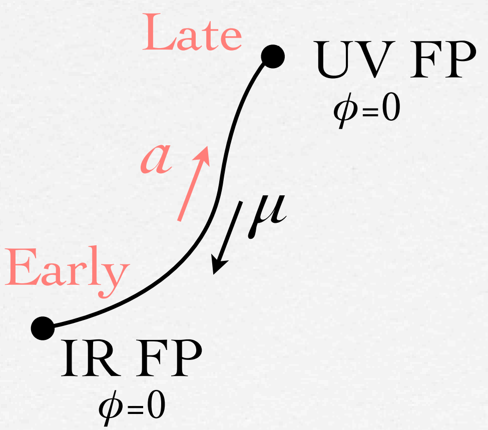
cf. DW/Cosmology McFadden, Skenderis, ...(09)

Deformed CFT

dCFT, dual to single field inflation $\qquad M_{\rm pl}=1$

$$S_{\rm dCFT}=S_{\rm CFT}+\int d^3\mathbf{x}\sqrt{g}\,\mu^{-(\Delta_c-3)}\phi O$$

RG flow $\quad \beta\equiv\dfrac{d\phi}{d\ln\mu}=(\Delta_c-3)+\beta_{\rm quant}$ $\quad \xrightarrow{\text{bulk}}$ $\quad \dfrac{d\phi}{d\ln a}=\dfrac{\dot\phi}{H}$



<u>Gauge of holographic plane</u> $\qquad$ Y.U. & Garriga (13)

$\delta\phi$=0, $\quad h_{ij}=e^{2\zeta}\delta_{ij}$

<u>Holographic description</u>

$$\Psi_{\rm qdS}\left[\phi(t),\,\zeta\right]=\left\langle e^{-\int d^3\mathbf{x}\,\mu^{-(\Delta_c-3)}\phi(\mu)O(\mathbf{x})}\right\rangle_{\rm dCFT}$$

$<\zeta\zeta>$, $<\zeta\zeta\zeta>$, ... $\qquad$ $<OO>$, $<OOO>$, ...

Holographic description

$$\langle\zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\cdots\zeta(\boldsymbol{x}_n)\rangle = \int D\zeta\, P[\zeta]\,\zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\cdots\zeta(\boldsymbol{x}_n) \qquad P[\zeta]=\left|\Psi_{\rm qdS}[\phi(t),\,\zeta]\right|^2$$

$$\longleftrightarrow \qquad \Psi_{\rm qdS}\left[\phi(t),\,\zeta\right] = \left\langle e^{-\int d^3\mathbf{x}\,\mu^{-(\Delta_c-3)}\phi(\mu)O(\mathbf{x})}\right\rangle_{\rm dCFT}$$

Remark! Applies to weakly&strongly coupled case

Power spectrum

$$P(k) = -\frac{1}{2{\rm Re}[\lambda^2\tilde{\phi}(\mu)^2 P_O(k)]}$$

$$\langle O(\mathbf{k}_1)O(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)P_O(k_1)$$

$$\tilde{\phi}(\mu)\equiv\mu^{-\lambda}\phi(\mu) \qquad \lambda\equiv\Delta_c-3$$

Bispectrum

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle_{\rm conn} = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B\left(\boldsymbol{k}_1,\,\boldsymbol{k}_2,\,\boldsymbol{k}_3\right)$$

$$B(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3) = -\prod_{i=1}^{3}P(k_i)\left[2{\rm Re}\left[(\lambda\tilde{\phi})^3\langle O(\mathbf{k}_1)O(\mathbf{k}_2)O(\mathbf{k}_3)\rangle\right] - \lambda\sum_{i=1}^{3}\frac{1}{P(k_i)}\right]$$

Power spectrum

Near conformal FP $\quad P_O(k) \sim P_O(k)$ for CFT

$$P(k) \simeq -\frac{6}{\pi^2}\frac{1}{\lambda^2\tilde{\phi}(\mu)^2 c}\frac{1}{k^3}$$

c: central charge $\sim$ # of DOFs on boundary

<u>In weakly coupled limit</u>

Friemann eqs. $\quad \beta^2 \simeq \lambda^2\tilde{\phi}(\mu)^2 \simeq 2\varepsilon$

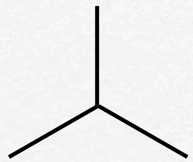
dS/CFT $\quad c \simeq -(l_{\rm dS}M_{\rm pl})^2$ $\quad$ Strominger(01)

$$\longrightarrow \quad P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2\frac{1}{k^3}$$

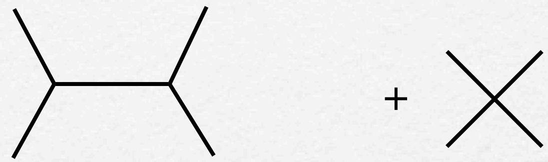
Maldacena(02)

SY inequality

$$f_{\rm NL} \equiv \frac{5}{12}\lim_{k_1\to 0}\frac{B(\boldsymbol{k}_1,\,\boldsymbol{k}_2,\,-(\boldsymbol{k}_1+\boldsymbol{k}_2))}{P(k_1)P(k_2)}$$

$B \sim$ 

$$\tau_{\rm NL} \equiv \frac{1}{4}\lim_{k_{12}\to 0}\frac{T(\boldsymbol{k}_1,\,\boldsymbol{k}_2,\,\boldsymbol{k}_3,\,\boldsymbol{k}_4)}{P(k_1)P(k_3)P(k_{12})}$$

$T \sim$ 

SY inequality

$$\tau_{\rm NL} \geq \left(\frac{6}{5}f_{\rm NL}\right)^2$$

Equality holds for single field case

If Gaussian at k~aH and δN can be applied

Suyama&Yamaguchi(08)

If f_{NL}&τ_{NL} are k independent or have the same k dependence

Assassi, Baumann & Green(12)

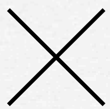
SY inequality in holography

dCFT dual to single field inflation

$$\tau_{\rm NL} - \left(\frac{6}{5}f_{\rm NL}\right)^2 = -\frac{1}{4}\lim_{k_{12}\to 0}\hat{W}^{(4)}\left(\boldsymbol{k}_1,\,\boldsymbol{k}_2,\,\boldsymbol{k}_3,\,\boldsymbol{k}_4\right)\underline{\frac{P(k_1)P(k_3)}{P(k_{12})}} \qquad \propto (k_{12})^3 \to 0$$

If $\displaystyle\lim_{k_{12}\to 0}\frac{\hat{W}^{(4)}(\mathbf{k}_1,\,\mathbf{k}_2\,,\mathbf{k}_3\,,\mathbf{k}_4)}{P(k_{12})} \to 0$, $\displaystyle\tau_{\rm NL} = \left(\frac{6}{5}f_{\rm NL}\right)^2$

Four point function $W^{(4)}$



$$S_4 \sim \prod_{i=1}^{4}\int d^3\mathbf{k}_i\hat{W}^{(4)}(\mathbf{k}_1,\,\mathbf{k}_2\,,\mathbf{k}_3\,,\mathbf{k}_4)\delta(\mathbf{k}_1+\cdots+\mathbf{k}_4)\,\zeta_{\mathbf{k}_1}\cdots\zeta_{\mathbf{k}_4}$$

$$\hat{W}^{(4)} \ni \underline{\langle OOOO\rangle},\,\langle OOO\rangle,\,\langle OO\rangle$$

For local theory, SY inequality holds.

Four point fn. depends on details of (d)CFT.

Conclusion

Holographic description of inflation seems feasible.

- In the weakly coupled limit, the results from the standard cosmological perturbation theory are reproduced.

  Power spectrum, Consistency relation, SY inequality,...

- We derived the universal expression, which can apply to both strongly and weakly coupled regime.

- An extension to multi field cases is possible.

$$S_{\mathrm{dCFT}} = S_{\mathrm{CFT}} + \sum_a \int d^3\mathbf{x}\sqrt{g}\,\tilde{\phi}_a O_a$$

Y.U., Garriga, ... in progress