Three form meets one form

The sad story of a turbulent relationship

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Meet the three form

- Three forms are creatures with three indices
- Why three forms?
  - They're fun to work on
  - They support FLRW isotropy
  - They inflate, go dark energy, and so on
- They can be coupled to other fields easily

The basic action is standard

$$\mathcal{L} = -\frac{1}{48}F^2 - V(B^2) \quad \text{where} \quad F_{\mu\nu\rho\sigma} \sim \partial_\mu B_{\nu\rho\sigma}$$

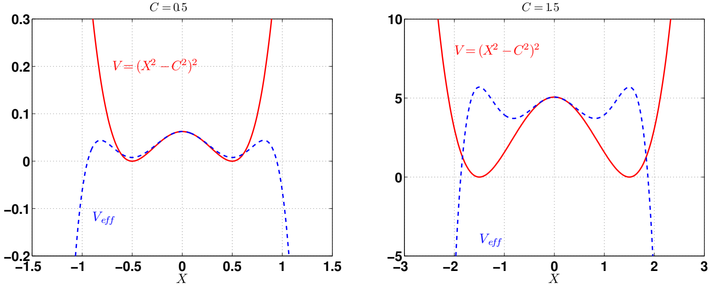
Koivisto and Nunes, arXiv:0907.3883

Inflating three forms

At the background level there is only one scalar dynamical degree of freedom: $B_{ijk} \sim \epsilon_{ijk} X(\eta)$

$$K'' - \mathcal{H}K' + a^2 V_X = 0$$

K is the *effective* dynamical field $K' = X' + 3\mathcal{H}X$: fun dynamics!



Fixed Points and Perturbations

- There are two types of distinct fixed points

$V(X)/V_0$	M: stability	E: stability
$\exp(-\beta X)$	Max_+ S for $\beta > 0$, Max_- S for $\beta < 0$	S
$\exp(-\beta X^2)$	S for $\beta > 0$	S for $\beta < 0$
X^2	U	S
X^{2n}, $n > 1$	U	S
$\left(X^2 - C^2\right)^2$	S for $C > \sqrt{2/3}$	Ext_1 S, Ext_2 U

- E(xt) are extrema of V; M(ax) are where $\mathcal{H}$ saturates (new!)
- E(xt) $\rightarrow$ red spectrum; M(ax) $\rightarrow$ blue spectrum

Coupling to light

The three form can couple to A_μ photons through

$$\mathcal{L}_{AB} = \boxed{-\frac{1}{2}\lambda_1 F_{\mu\nu}(A)F^{\mu\nu}(B)} \; ✔$$
$$+ \lambda_2(\nabla_\mu A^\mu)(\nabla_\nu B^\nu) \; ✘$$
$$+ (\lambda_3 + \lambda_4 R)A_\mu B^\mu \; ✘$$
$$+ \lambda_5 A^\mu R_{\mu\nu} B^\nu \; ✘$$
$$+ \lambda_6 A^\mu R_{\mu\nu\rho\sigma} B^{\nu\rho\sigma} \; ✘$$

but only the first term preserves the $U(1)$ gauge invariance of the photon

Koivisto and FU, arXiv:1112.1356

Photon Dynamics

- The vector metric perturbations couple to the three form perturbations
- The photon (a perturbation) also couples to them

Dynamics is governed by

$$\mathcal{A}^{(4)} + 2\frac{f'}{f}\mathcal{A}^{(3)} + \frac{1}{f}(f'' - 1 + 2k^2 f)\mathcal{A}'' + 2\frac{f'}{f}k^2\mathcal{A}'$$

$$+ k^2(\frac{f''-1}{f} + k^2)\mathcal{A} = 0$$

$$f = 2\lambda^2 \frac{X}{V_{,X}}\left(\frac{2V_{,X}X}{k^2} - 1\right)$$

Magnetise!

- Analytical solutions are possible for stationary $f' = 0$
- Growing solution:

$$\boxed{\mathcal{A} \sim \exp(-\Gamma k \eta)}$$

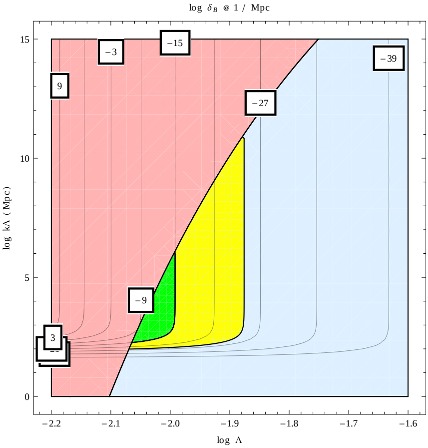
- Growth is not for everyone:

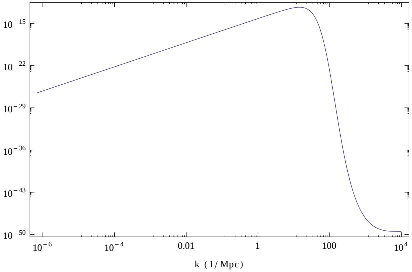
$$\boxed{\Gamma^2 = \frac{k_\Lambda^2 - k^2}{\Lambda^2 k_\Lambda^2 + k^2}}$$

- See, only the little ones are boosted: $k \leq k_\Lambda \rightarrow$ natural UV cutoff

Pretty pictures

Some results...





FU and Koivisto, arXiv:1207.7328

Why we like it

- This model does not suffer from the usual backreaction problem
- Because only a small band of modes are amplified
- Thus, it is possible to give all available power to them
- The UV cutoff is easily very low, only large scales are magnetised
- It's different: dynamics driven by metric and inflaton vector modes
- It is *not* just another way to write a scalar

Anisotropy?

- Since we're tampering with vectors, sure we don't generate any anisotropy?
- The logics:
  1. A modes when they leave the horizon classicalise
  2. From the inside observer point of view, they make a background of vectors
  3. In any given realisation this is not isotropic, but picks a preferred random direction
  4. At the end of inflation the spectrum will be anisotropic
- We need to look at curvature perturbations generated by...

$$A_i = \bar{A}_i(\eta) + \int \mathrm{d}^3 k \, \varepsilon_i^{\alpha} \, \hat{a}_{\alpha} \, \mathcal{A}_k(\eta) \, e^{ik\eta} + \text{h. c.}$$

Bartolo, Matarrese, Peloso, Ricciardone, arXiv:1210.3257

Computing the curvature – 1

EOM sourced by second order perturbations

$$\zeta''_{\bar{k}} + \frac{2}{\eta}\zeta'_{\bar{k}} + c_s^2 k^2 \zeta_{\bar{k}} = S_{\bar{k}}(\eta)$$

where $S_{\bar{k}}(\eta) = S[\bar{A}_i(\eta), \mathcal{A}_k(\eta), \delta^2 X]$

The second order three form perturbations are small as usual

Excitation ladder: $X(\eta) \to \delta X \to \mathcal{A}$

Computing the curvature – 2

The two point function is

$$\langle \zeta_{\bar{p}} \zeta_{\bar{q}} \rangle = \int \mathrm{d}y \mathrm{d}z \, \mathcal{G}(x,y) \mathcal{G}(x,z) \, \langle S_{\bar{p}}(y) S_{\bar{q}}(z) \rangle$$

where $$\mathcal{G}(x,x') \to -\frac{1}{3} x'$$

for small (x, x')

- The structure of the coupling differs from $f^2 F^2$ or $f \, F * F$
- ...there's no slow-roll enhancement here: $F(A)F(B)$ is direct

Computing the curvature – 3

Results

$$\langle \zeta^2 \rangle_{\bar{A}} \simeq \left(\frac{\rho_{\mathrm{em}}}{\epsilon \bar{\rho}} \right)^2 \frac{\mathcal{H}^8}{\Gamma^4 k^4 k_\Lambda^4} \sin^2 \vartheta \, \frac{\delta^3(\bar{k} + \bar{k}')}{k^3}$$

$$\langle \zeta^2 \rangle_{\mathcal{A}} \simeq \left(\frac{\rho_{\mathrm{em}}}{\epsilon \bar{\rho}} \right)^2 \frac{\mathcal{H}^8}{k_\Lambda^8} \left(1 + \frac{3 \Gamma k}{k_{\mathrm{UV}}} + \dots \right) \frac{\delta^3(\bar{k} + \bar{k}')}{k^3}$$

Computing the curvature – 3

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Ouch!

Needless to say, they're too big

Conclusion

- The generated curvature perturbations are too large
- That's because the $\exp(-\Gamma k \eta)$ rush is too steep
- Similar effects would appear for N-flation
  - Notice that the anisotropic part is enhanced for low momenta!
- The model can be saved, but slower growth is to be engineered
  - We lose the mathematical simplicity
  - But we may obtain a working model

⋆ ...slow progress underway...

FU, arXiv:1306.hope