Inflationary growth of coupled scalar fields and its implications

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Introduction: massless scalars + inflation

Inflation is a successful paradigm explaining difficulties of the hot big bang

Flatness, horizon initial density perturbation, unwanted relics, initial boost…

Simplest realization of inflation invokes a nearly homogeneous scalar field, which provides a negative pressure

$$\phi(x,t) \simeq \phi(t)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \cancel{\frac{(\nabla\phi)^2}{2a^2}} + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - \cancel{\frac{(\nabla\phi)^2}{6a^2}} - V(\phi)$$

Acceleration eq.: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$

Inflation requires $\dot{\phi}^2 \ll V(\phi)$

$$H^2 = \frac{8\pi G\rho}{3} - \cancel{\frac{K}{a^2}} + \cancel{\frac{\Lambda}{3}} \quad \Longrightarrow \quad H \simeq \text{cte} \quad \Longrightarrow$$

Slow-roll inflation: $a(t) \propto e^{Ht}$

Introduction: quantum particle production

The inflaton field undergoes quantum particle production during inflation

$$\phi(x,t) = \phi(t) + \delta\phi(x,t)$$

$$\delta\phi(x,t) = \int \frac{d^3k}{(2\pi)^3} \left[\hat{a}_k\, \delta\phi_k(t)\, e^{ikx} + h.c.\right]$$

Evolution of the Fourier modes of the field

$$\delta\ddot{\phi}_{\boldsymbol{k}} + 3H\delta\dot{\phi}_{\boldsymbol{k}} + \left(\frac{k^2}{a^2} + m^2\right)\delta\phi_{\boldsymbol{k}} = 0$$

On superhorizon scales the field obtains a non-zero VEV if $m \ll H$

Particle production occurs for light scalars **other than the inflaton** too

$$m \ll H \quad \Longrightarrow \quad |\delta\phi_k|^2 \simeq \frac{H^2}{2k^3}$$

Any massive, light field is produced during inflation

Introduction: condensate development

On superhorizon scales the field fluctuations accumulate

$$\phi(x,t) = \phi(t) + \int \frac{d^3k}{(2\pi)^3} \left[\hat{a}_k \, \delta\phi_k(t) \, e^{ikx} + h.c.\right] \Rightarrow$$

Development of condensates

The uses of scalar condensates

Enqvist and Mazumdar'02

① **Baryogenesis**

Afleck-Dine baryogenesis: decay of a condensate with baryon charge

② **Cosmological fluctuations**

Curvature perturbation: curvaton fields

Perturbations in light scalars: Curvature pert. at the end of inflation

③ **Reheating:**

Inhomogeneous reheating:

Parametric resonance

Non-perturbative decay of flat directions

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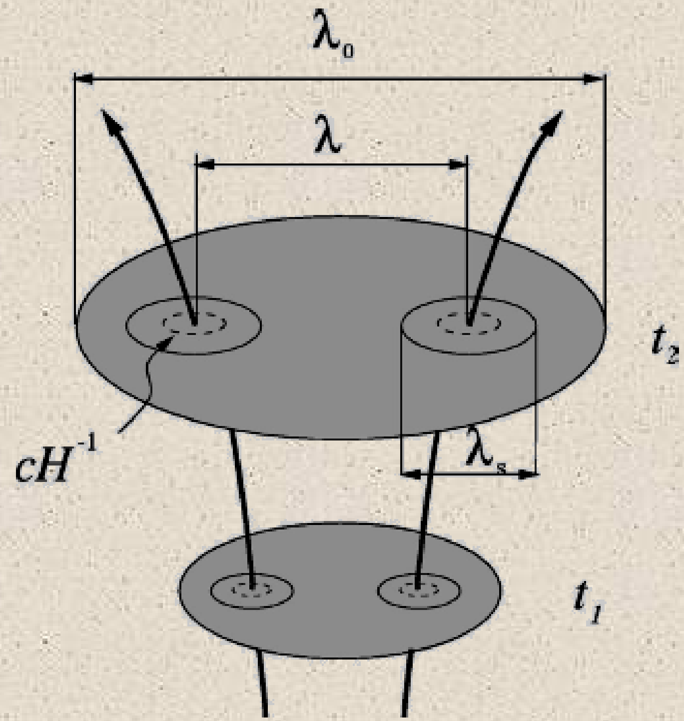
Development of condensates

Separate Universes: nearly homogeneous **superhorizon patches evolve independently** of each other Wands et al.'00



Stochastic formalism (self-interacting field)

Field equation in de Sitter space:

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) - a^{-2}\nabla^2\Phi = 0$$

Long/short wavelength decomposition

$$\boxed{\Phi(t,x) = \phi(t) + \phi_q(t,x)}$$

Starobinsky' 86

Short-distance field: scales smaller than k_s^{-1}

$$\phi_q(t,x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\theta(k-k_s)\left[\hat{a}_{\mathbf{k}}\varphi_k(t)\,e^{ikx} + \hat{a}^\dagger_{\mathbf{k}}\varphi^*_k(t)e^{-ikx}\right]$$

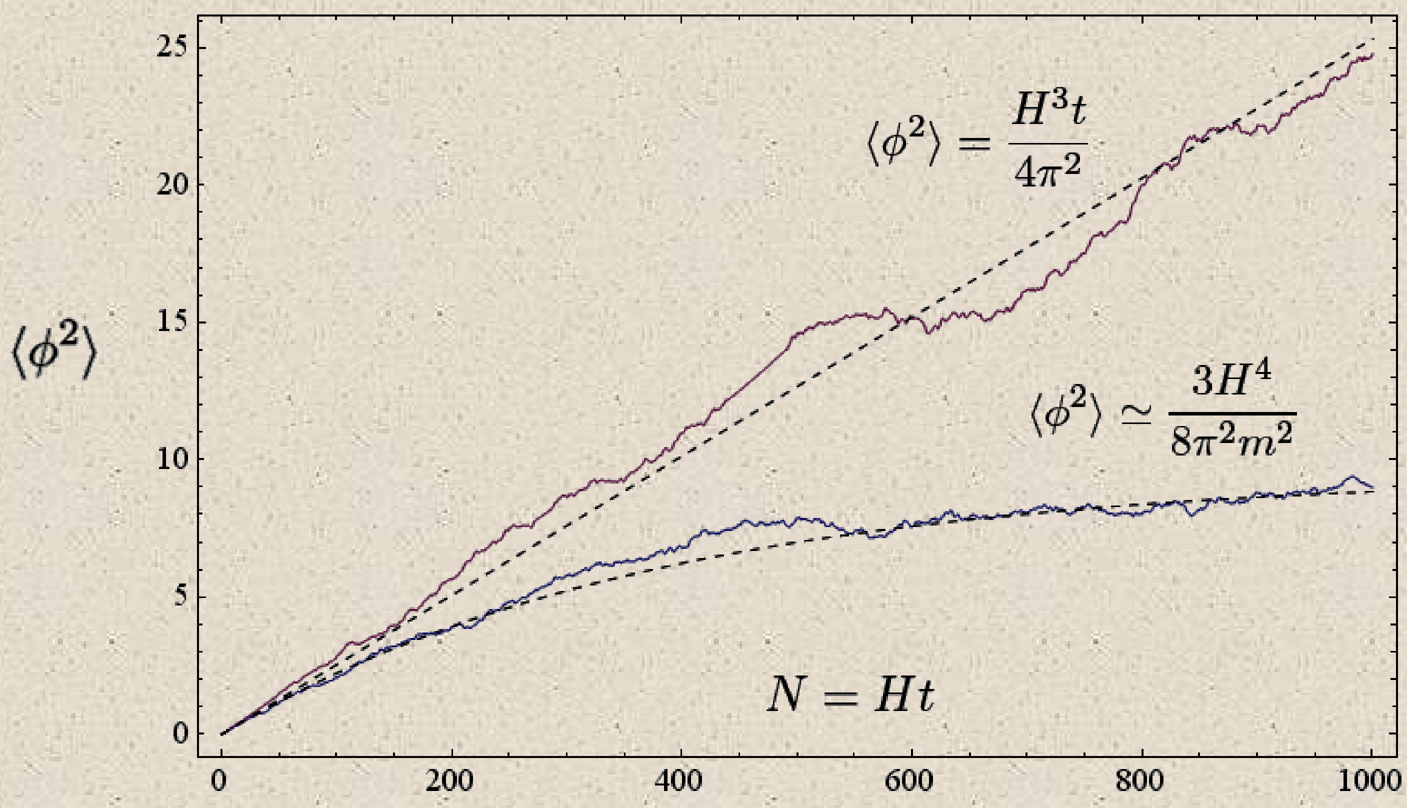
Coarse-graining scale:

$$\boxed{k_s = \epsilon a(t) H \quad , \quad \epsilon \ll 1}$$

Stochastic formalism (massive, non-int. field)

Effective EOM for coarse-grained field

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\nabla^2\phi + V'(\phi) = \xi(t, \boldsymbol{x})$$



Probabilistic evolution determined by Fokker-Planck equation:

$$\partial_t P(\phi, t) = \partial_\phi \left(\frac{V'(\phi)}{3H} P(\phi, t) \right) + \frac{H^3}{8\pi^2} \partial^2_{\phi,\phi} P(\phi, t)$$

The question

Massless fields (MSSM flat directions) have their fluctuations growing

$$\langle \phi^2 \rangle = \frac{H^3 t}{4\pi^2}$$

But they are coupled to other fields

$$V_{\text{int}}(\phi, \chi) = g^2 \phi^2 \chi^2$$

The coupling provides an effective mass for the massless field

$$m_{\text{eff}}^2 = g^2 \chi^2$$

Coupling leads to bounded fluctuations **Enqvist et al.' 11**

Is it possible to avoid this natural blocking?

Lagrangian for two massless fields, ϕ and χ

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 - g^2 \phi^2 \chi^2$$

Equations of motion:

$$\ddot{\Phi} + 3H\dot{\Phi} - a^{-2}\nabla^2\Phi + g^2\chi^2\Phi = 0$$

$$\ddot{\chi} + 3H\dot{\chi} - a^{-2}\nabla^2\chi + (g^2\Phi^2 + m_\chi^2)\chi = 0$$

Simplifying assumptions

① Massless χ: $m_\chi^2 = 0$

② Initial non-vanishing vev $\langle \Phi(x) \rangle = \Phi_0$, $m_\chi^2 = g^2 \Phi_0^2 > H^2$

Integrate χ out $\quad \chi^2 \simeq \langle \chi^2 \rangle_{\mathrm{IR}} \simeq \frac{1}{3} \left(\frac{H}{2\pi} \right)^2 \frac{H}{m_\chi} \epsilon^3$ **Enqvist et al. '88**

Effective Fokker-Planck

$$\partial_t P = \frac{g\alpha H^2 \epsilon^3}{12\pi^2} \partial_\phi P + \frac{H^3}{8\pi^2} \partial_\phi^2 P$$

The physical system

Lagrangian for two massless fields, ϕ and χ

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 - g^2 \phi^2 \chi^2$$

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Effective Fokker-Planck

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Scale dependence: $k_s = \epsilon a(t) H$

Probability density:

$$\boxed{P(\phi,t) = \frac{1}{\sqrt{2\pi\sigma^2(t)}} \exp\left[-\frac{(\phi-\mu)^2}{2\sigma^2(t)}\right]}$$

$$\sigma^2(t) \equiv \sigma_0^2 + \frac{H^3 t}{4\pi^2} \quad , \quad \mu \equiv \phi_0 - \boxed{\kappa t} \quad , \quad \kappa \equiv \frac{gH^2\epsilon^3}{36\pi^2}$$

Timescales:

Diffusion: $\tau_\sigma \equiv \frac{\sigma^2}{\dot{\sigma}^2} = \left(1 + \frac{4\pi^2\sigma_0^2}{H^2 N}\right) H^{-1}N \simeq H^{-1}N$

$\boxed{\text{Drift: } \tau_\mu \equiv \frac{\mu}{\dot{\mu}} = \left(\frac{36\pi^2}{g\epsilon^3}\frac{\phi_0}{H}\right) H^{-1}}$

$\tau_\sigma \ll \tau_\mu \Longrightarrow$ Diffusive motion dominates $\Longrightarrow$ free field fluctuations

$\tau_\mu \ll \tau_\sigma \Longrightarrow$ Diffusion freeze-out, drift dominates $\Longrightarrow$ equilibrium fluct.

On boundary conditions

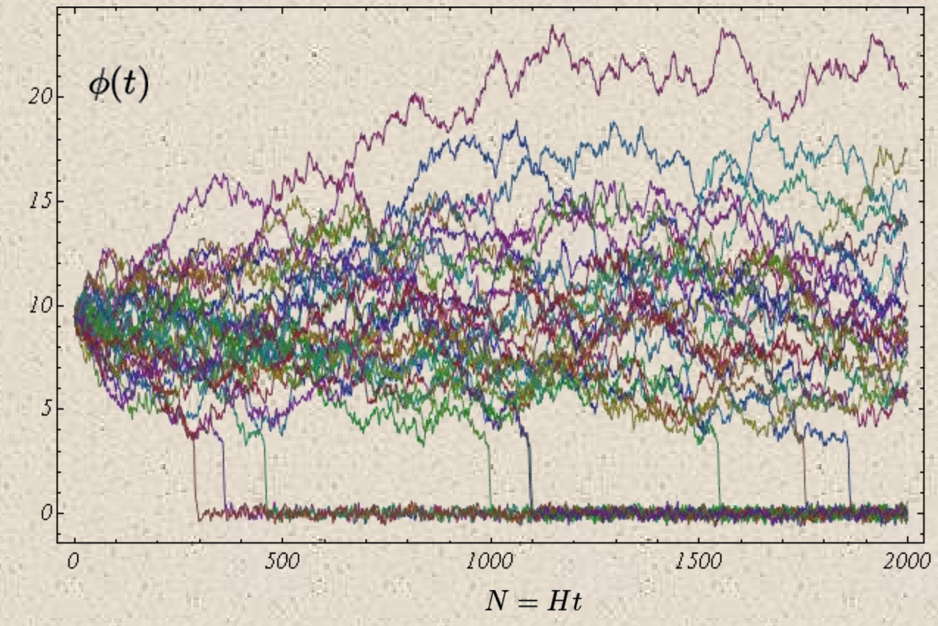
The picture is incomplete:

Initially $m_\chi^2 = g^2\Phi_0^2 > H^2$ but eventually $m_\chi^2 \sim H^2$

Production of superhorizon fluctuations of χ is no longer suppressed

Moduli trapping at symmetry points: $V_{\rm int}(\phi,\chi) = g^2\phi^2\chi^2$

$\Rightarrow$



FP equation: $\partial_t P = \kappa\,\partial_\phi P + \frac{H^3}{8\pi^2}\partial_\phi^2 P$

Boundary conditions: $P(\phi_c, t) = 0$

Chandrasekhar ‘43

The ensemble, in detail

Probability density:

$$\boxed{P(\phi,t) = \frac{1}{\sqrt{2\pi\sigma^2}}\left\{\exp\left[-\frac{(\phi-\varphi_+)^2}{2\sigma^2}\right] - C\exp\left[-\frac{(\phi-\varphi_-)^2}{2\sigma^2}\right]\right\}}$$

$$\varphi_+ \equiv \phi_0 - \kappa t \quad , \quad \varphi_- \equiv -\phi_0 + 2\phi_c - \kappa t - \frac{8\pi^2\sigma_0^2\kappa}{H^3}$$

Fraction with fluctuating field

$$\boxed{\mathcal{F}(t) = \frac{I_1(\varphi_+,t) - CI_1(\varphi_-,t)}{\sqrt{2\pi\sigma^2}}} \quad , \; I_1(\varphi) = \sqrt{\frac{\pi\sigma^2}{2}}\left[1 + \mathrm{Erf}\left(\frac{\varphi-\phi_c}{\sqrt{2\sigma^2}}\right)\right]$$

Mean square from diffusion

$$\boxed{\langle\phi^2\rangle_{\mathrm{fl}} = \frac{1}{\sqrt{2\pi\sigma^2}}\left[I_2(\varphi_+) - CI_2(\varphi_-)\right]} \quad , \; I_2(\varphi) = \int_{\phi_c}^{\infty} \phi^2 \exp\left[-\frac{(\phi-\varphi)^2}{2\sigma^2}\right] d\phi$$

The ensemble, in detail

Not too much inflation: $N \ll N_{\text{drift}} \implies \kappa \simeq 0$ quasi-free fluctuations

$$\boxed{\langle\phi^2\rangle_{\text{fl}} = \exp\left(-\frac{\delta^2}{2\sigma^2}\right)\sqrt{\frac{2}{\pi}}\,\sigma\delta + 2\phi_c\delta + (\sigma^2+\delta^2+\phi_c^2)\,\text{Erf}\left[\frac{\delta}{\sqrt{2\sigma^2}}\right]}$$

Timescale to arrive at the barrier: $\dfrac{H^3 t}{4\pi^2} + \sigma_0^2 \ll (\phi_0 - \phi_c)^2$

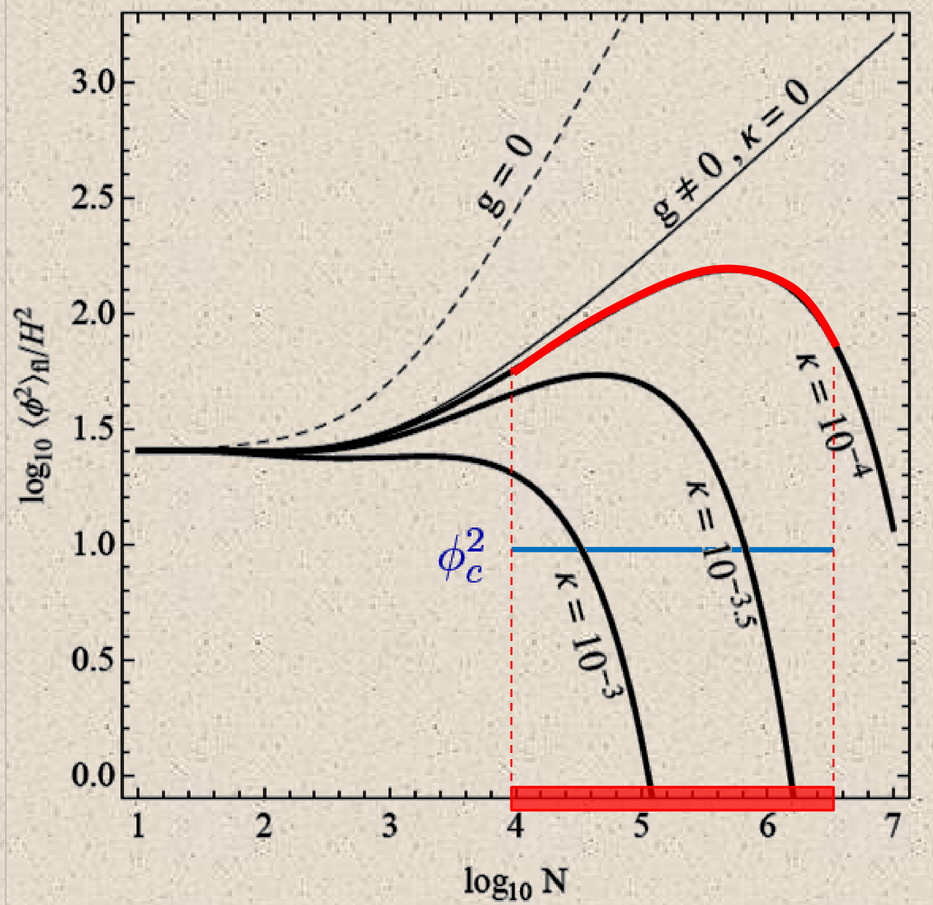
$$\boxed{\langle\phi^2(t)\rangle_{\text{fl}} \simeq 2\phi_c\delta + (\sigma^2+\delta^2+\phi_c^2) = \langle\phi^2(0)\rangle + \frac{H^3 t}{4\pi^2}}$$

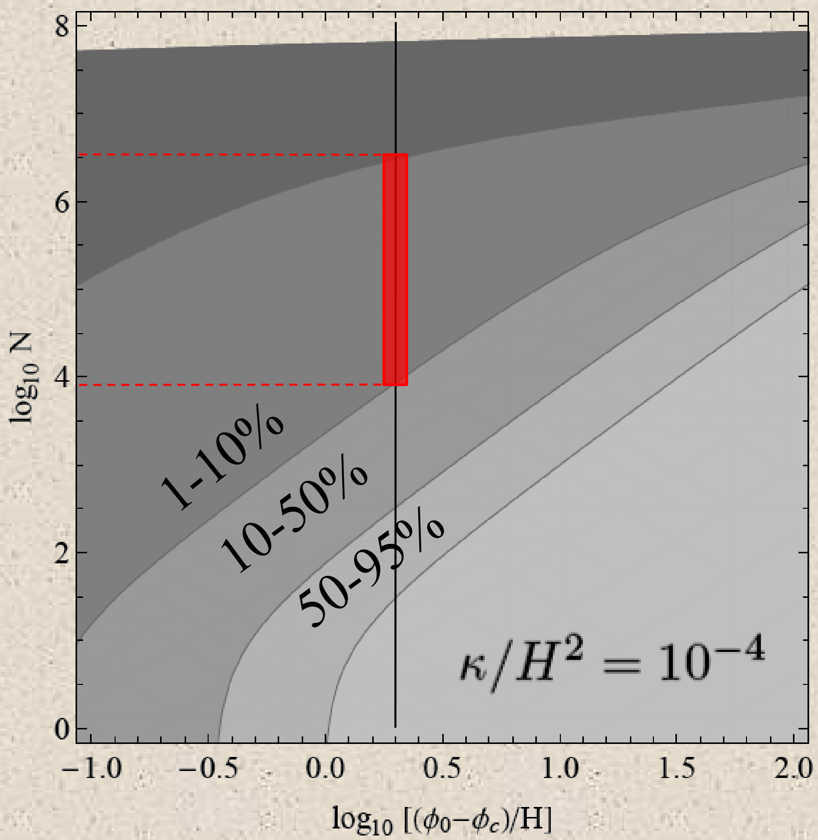
After reaching the barrier: $\dfrac{H^3 t}{4\pi^2} + \sigma_0^2 \gg (\phi_0 - \phi_c)^2$

$$\boxed{\langle\phi^2(t)\rangle_{\text{fl}} \simeq 2\delta\sqrt{\frac{2}{\pi}\left(\frac{H^3 t}{4\pi^2} + \sigma_0^2\right)}}$$

$\langle\phi^2\rangle_{\rm fl}$ from out-of-equilibrium fluctuations

ϕ begins "away" from the barrier: $\phi_c = 3H$, $\phi_0 = 5.0H$, $\sigma_0^2 = 0.25H^2$





In 1-10% of the ensemble $\langle\phi^2\rangle_{\rm fl}$ is ~10 times larger than in the equilibrium

Conclusions

Massive scalar fields have their fluctuations growing upto an upper bound

$$\langle \phi^2 \rangle \simeq \frac{3H^4}{8\pi^2 m^2}$$

A massless field with an **effective mass** determined by its coupling

$$V_{\text{int}}(\phi, \chi) = g^2 \phi^2 \chi^2$$

is able to circumvent the above blocking, if χ behaves as a heavy field. This can be arranged through the initial condition

$$\langle \Phi(x) \rangle = \Phi_0 \quad , \quad g^2 \Phi_0^2 > H^2$$

New **analytic formula** to estimate amplitude of fluctuations at any time.

$$\langle \phi^2 \rangle_{\text{fl}} = \frac{1}{\sqrt{2\pi\sigma^2}} \left[I_2(\varphi_+) - C I_2(\varphi_-) \right]$$

The coupling gives rise to parts of the Universe where the field obtains a large non-gaussian fluctuation. **Under certain conditions this non-gaussianity could show up in the CMB**.