Cold and hot spots at large scales

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Meeting on Fundamental Cosmology, Santander

Outline

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We define a spot in the field ψ through its derivatives up to second order:

$$\psi(\hat{n}) = \psi + \partial_i \psi \ \theta^i + \frac{1}{2} \partial_i \partial_j \psi \ \theta^i \theta^j \dots$$

A field in the sphere is given in terms of the spherical harmonics:

$$\psi(\hat{n}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{\ell m} Y_{\ell m}(\hat{n})$$

How to express these derivatives in terms of the spherical harmonics coefficients?

Peak height ($m = 0$)

$$\psi = \sum_{\ell=0}^{\infty} \sqrt{\frac{2\ell+1}{4\pi}} a_{\ell 0}$$

First derivatives ($m = 1$)

$$D^{\dagger}\psi = \sum_{\ell=0}^{\infty} \sqrt{\frac{2\ell+1}{4\pi}} \sqrt{\frac{(\ell+1)!}{(\ell-1)!}} a_{\ell 1}$$

Second derivatives ($m = 0$ and $m = 2$)

$$\nabla^2\psi = -\sum_{\ell=0}^{\infty} \sqrt{\frac{2\ell+1}{4\pi}} \frac{(\ell+1)!}{(\ell-1)!} a_{\ell 0}$$

Scalar ($m = 0$)

$$D^2\psi = -\sum_{\ell=0}^{\infty} \sqrt{\frac{2\ell+1}{4\pi}} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} a_{\ell 2}$$

Spinor ($m = 2$)

Quantities with different spin are uncorrelated due to rotational invariance.

- Scalar: ψ, $\nabla^2\psi$
- Vector: $D^\dagger\psi$
- Tensor: $D^2\psi$

$$\kappa = \frac{1}{a^2} + \frac{1}{b^2}$$

$$\epsilon = \frac{1}{b^2}\left(1 - \frac{b^2}{a^2}\right) e^{i2\alpha}$$

Peak variables

- Peak height $\nu = \frac{\psi}{\sigma(\psi)}$
- Mean curvature $\kappa = \frac{\nabla^2\psi}{\sigma(\nabla^2\psi)}$
- Ellipticity $\epsilon = \frac{D^2\psi}{\sigma(D^2\psi)}$

$$P(\nu, \kappa, \epsilon) = P(\nu)P(\kappa|\nu)P(\epsilon)$$

We separate the peak degrees of freedom and the rest of the field.

$a_{\ell m},\ e_{\ell m}$ $\Longrightarrow$

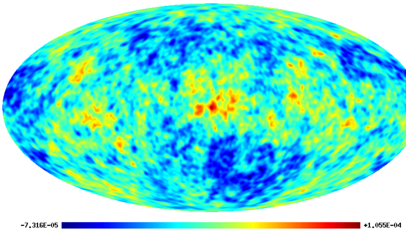
- ν, κ, ϵ: peak degrees of freedom.
- $\hat{a}_{\ell m}$, $\hat{e}_{\ell m}$: degrees of freedom uncorrelated with the peak.

We orthogonalize the covarianve matrix:

$$C = \left(\begin{array}{c|c} \nu, \kappa, \epsilon & 0 \\ \hline 0 & \hat{a}_{\ell m}, \hat{e}_{\ell m} \end{array} \right)$$

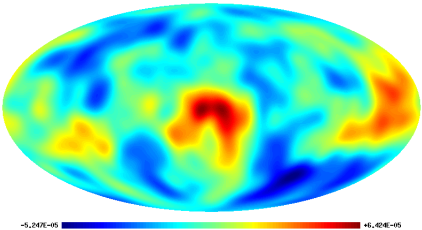
Once the peak variables are uncorrelated is very easy to simulate maps with the spots and calculate expectation values.

Simulations



Maximum with $\nu = 5$

Elliptical spot



Outline

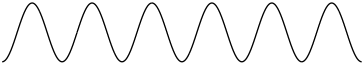
1. Simulating spots on the sphere
2. Theoretical profiles
3. Covariance of the Profiles
4. Cold Spot Analysis

In order to get the profile we calculate the expectation value of the field with some peak contraints.

There are two ways of averaging the peak degrees of freedom:

- With the density of peaks $\longrightarrow$ Stacking
- With the probability density $\longrightarrow$ Single spot

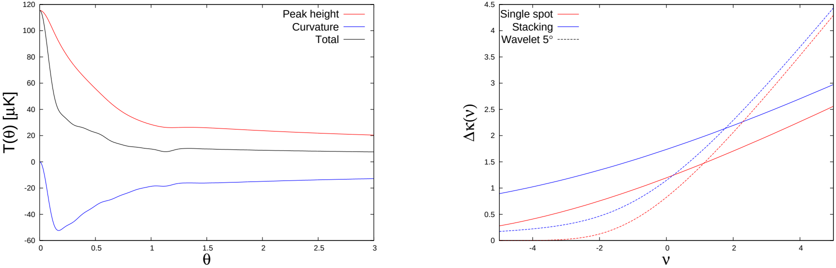
$$n(\nu, \kappa, \epsilon) \propto |H(\kappa, \epsilon)| P(\nu, \kappa, \epsilon)$$



- ν fixed and min/max selection

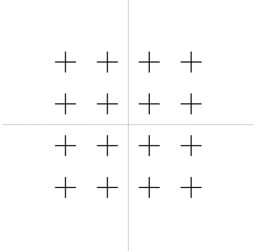
$$p(\theta) = \frac{\nu}{\sigma} C(\theta) + \frac{\Delta\kappa(\nu)}{(1-\rho^2)\sigma_2} \left(\nabla^2 - \rho \frac{\sigma_2}{\sigma} \right) C(\theta)$$

where $\Delta\kappa(\nu) = \langle\kappa\rangle_\nu - \rho\nu$ is the shift of the curvature mean due to the min/max selection.



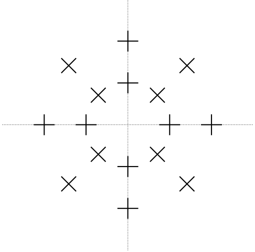
Polarization

Cartesian coordinates



Q, U

Polar coordinates

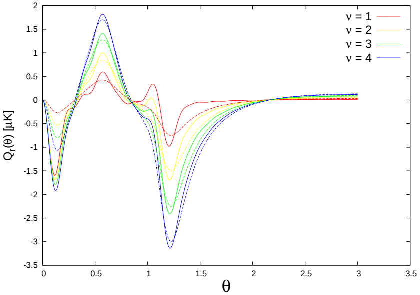


Q_r, U_r

$$T(\theta) = \mathbf{\Psi}^\dagger \mathbf{C}^{TT}(\theta)$$

$$E(\theta) = \mathbf{\Psi}^\dagger \mathbf{C}^{TE}(\theta)$$

$$B(\theta) = \mathbf{\Psi}^\dagger \mathbf{C}^{TB}(\theta)$$



$$\mathbf{\Psi}(\theta) = \begin{pmatrix} \psi_0 \\ \nabla^2\psi_0 \end{pmatrix}$$

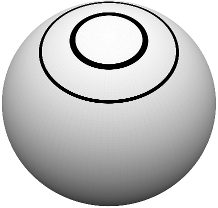
$$\mathbf{C}^{ij}(\theta) = \Sigma^{-1} \begin{pmatrix} C^{ij}(\theta') \\ \nabla^2 C^{ij}(\theta') \end{pmatrix}$$

$$E(\theta) \longleftrightarrow Q_r(\theta)$$

$$B(\theta) \longleftrightarrow U_r(\theta)$$

Outline

Profiles covariance



$$C(\theta,\theta') = \frac{1}{2\pi}\delta(\cos\theta - \cos\theta') \otimes C(\theta)$$

$$C(\theta,\theta') = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} C_\ell \; P_\ell(\cos\theta) P_\ell(\cos\theta')$$

$$C_\ell = \text{const.} \Longrightarrow C(\theta,\theta') = \frac{1}{2\pi}\delta(\cos\theta - \cos\theta')$$

Covariance bias due to peak selection

The covariance of ψ and $\nabla^2\psi$ is modified when peak constraints are imposed

$$\Sigma \longrightarrow \Sigma + \Delta\Sigma$$

$$C^{TT}(\theta,\theta') = \underbrace{C^{TT}_{\text{rings}}(\theta,\theta')}_{\text{Intrinsic}} + \underbrace{\mathbf{C}^{TT}(\theta)^{\dagger}\Delta\Sigma\mathbf{C}^{TT}(\theta')}_{\text{Peak selection}}$$

$$C^{EE}(\theta,\theta') = C^{EE}_{\text{rings}}(\theta,\theta') + \mathbf{C}^{TE}(\theta)^{\dagger}\Delta\Sigma\mathbf{C}^{TE}(\theta')$$

$$C^{TE}(\theta,\theta') = C^{TE}_{\text{rings}}(\theta,\theta') + \mathbf{C}^{TT}(\theta)^{\dagger}\Delta\Sigma\mathbf{C}^{TE}(\theta')$$

$$\mathbf{C}(\theta) = \Sigma^{-1}\begin{pmatrix} C(\theta') \\ \nabla^2 C(\theta') \end{pmatrix}$$

Outline

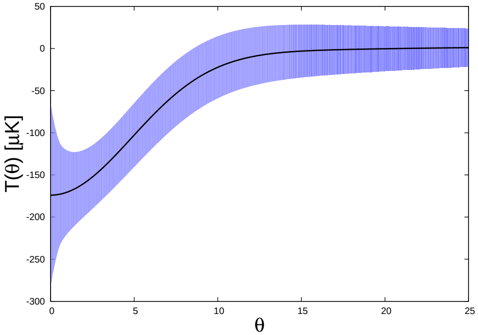
Peak selection on Wavelet space

We select the CS in terms of the wavelet coefficient.

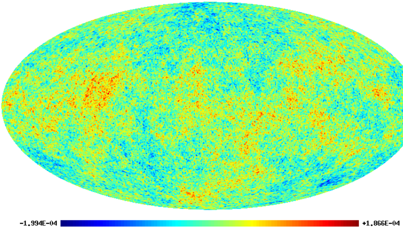
$$C(\theta) \longrightarrow w(\theta) \otimes C(\theta)$$

$$C_\ell \longrightarrow w_\ell C_\ell$$

$$p(\theta) \longrightarrow w(\theta) \otimes p(\theta)$$

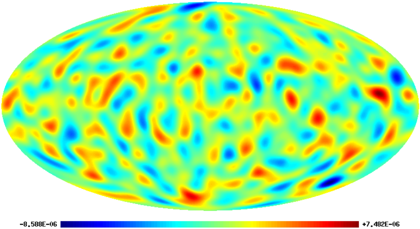


Cold Spot simulations



Real space

Wavelet space ($R = 5^\circ$)



Conclusions

- We have developed the methodology to simulate spots on the sphere (without the flat approximation).
- Previous results on peaks are recovered.
- We differenciate between density (stacking) and probability (single spot) average.
- Theoretical calculations of the covariance matrix between the profiles T and Q_r.
- Higher order moments of the profile can be calculated using this scheme.
- It is possible to generalized this procedure to spots with ellipticity (oriented stacking).
- Applications to the Cold Spot.

Thank you

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