The distribution function of Einstein radii from MUSIC clusters

Jesús Vega[1,2], Gustavo Yepes[1] and David Valls-Gabaud[2]

[1]Universidad Autónoma de Madrid (UAM)
[2]LERMA - Observatoire de Paris

Meeting on Fundamental Cosmology
Santander - Junio 2015

UNIVERSIDAD AUTONOMA DE MADRID

MUltidark SImulations of galaxy Clusters

l'Observatoire de Paris — LERMA

Laboratoire d'Étude du Rayonnement et de la Matière en Astrophysique

Summary

1. Motivations

Clusters of galaxies are expected to act as the most powerful lenses in the universe.

Massive and/or compact clusters at $0.2 \lesssim z_{lens} \lesssim 0.4$ ($z_s \gtrsim 1.0$)

The abundance of strong lensing events depends on cosmology through:

- the angular-diameter distances of the lens and the source
- the structure formation (given that the mass function of dark matter halos and the internal properties of the lenses are related to the cosmological parameters)

Strong gravitational lensing by clusters of galaxies is one of the most important test of the cosmological model:

- it is extremely sensitive to the properties of the clusters cores
- it probes the rarest high density peaks in the universe

1. Motivations

Previous attempts of using strong lensing statistics as a cosmological tool have produced controversial results.

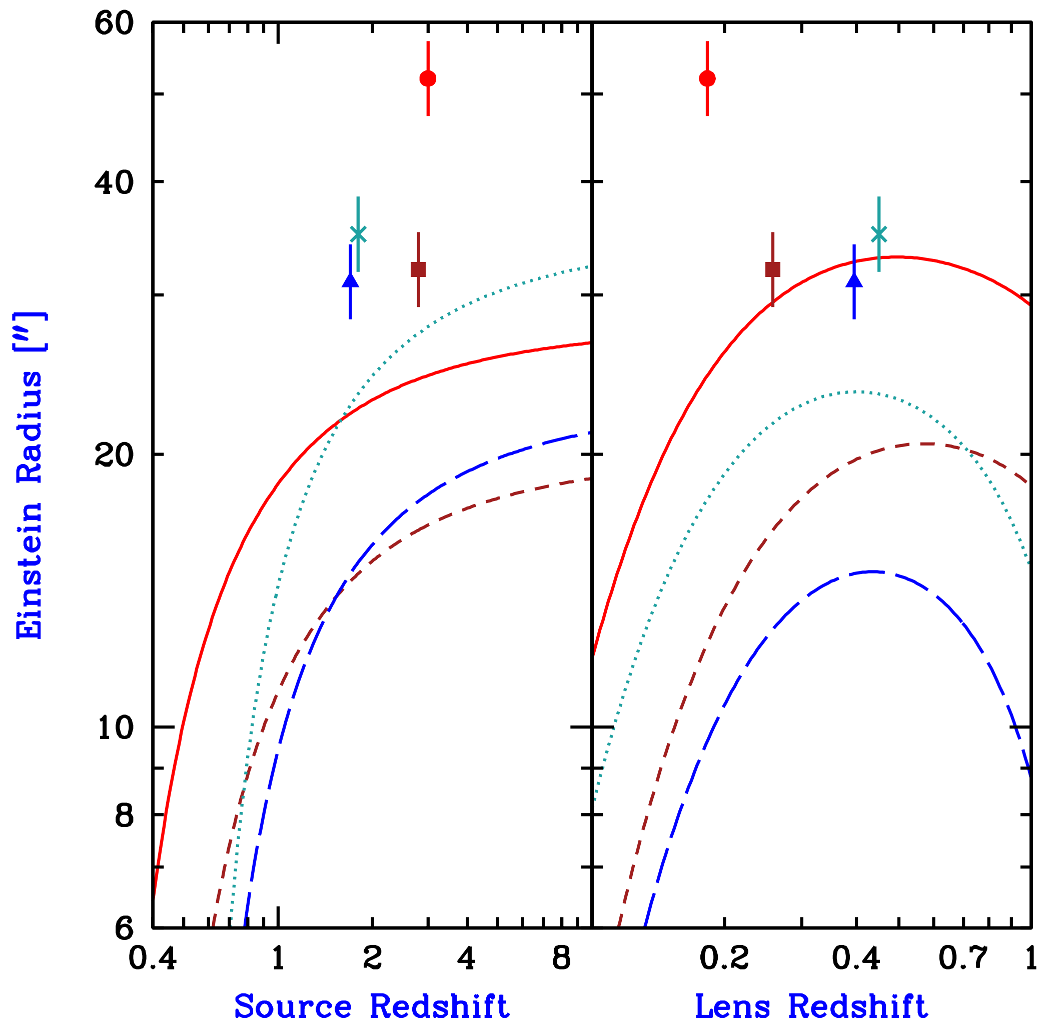
The observed distribution of Einstein radii is much larger than the one predicted by analytical models within the ΛCDM model (*Bartelmann et al. 1998*).

Arc statistics problem

I. Motivations

Arc statistics problem

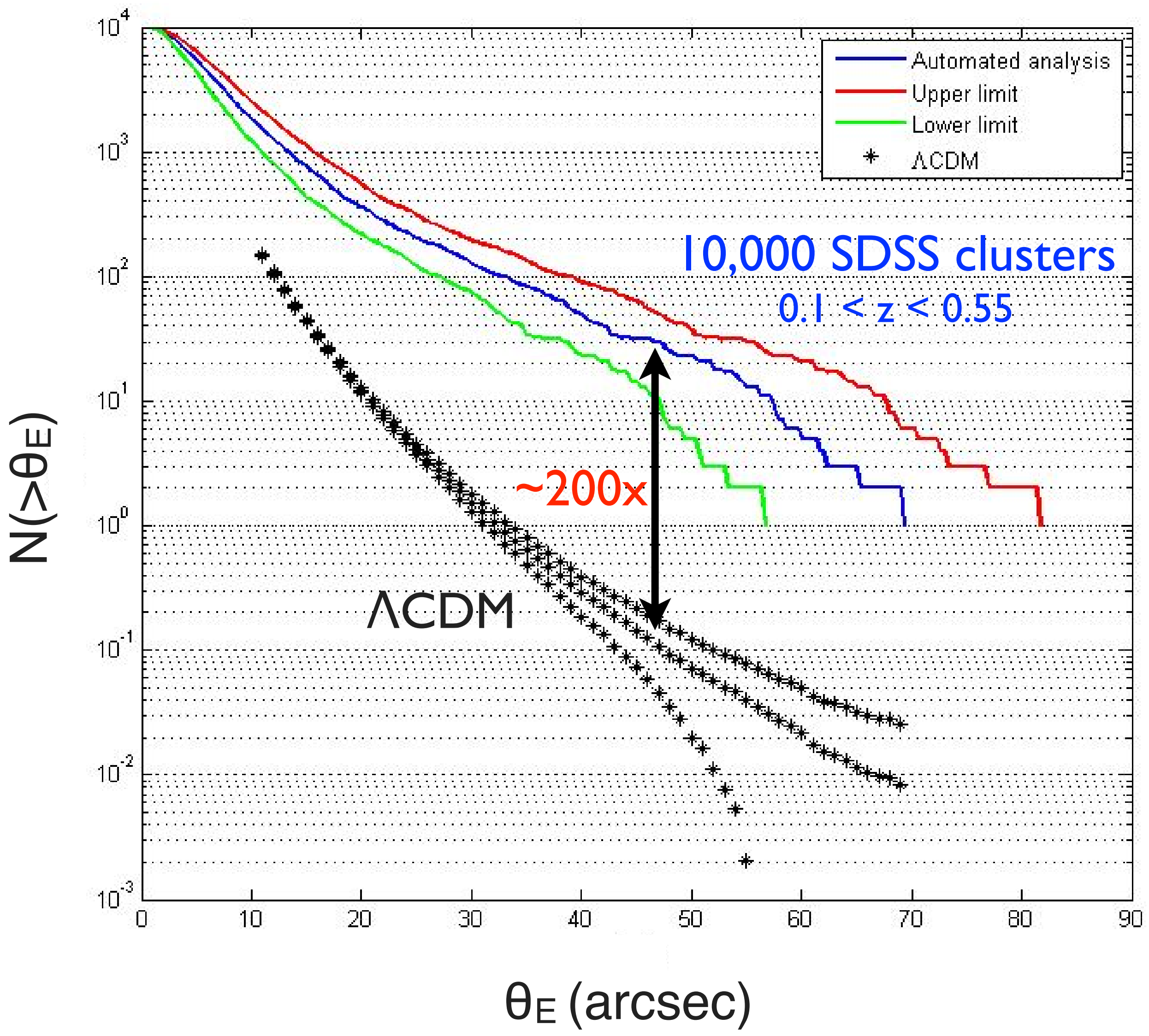
- Some galaxy clusters have **very extended critical lines** whose abundances can hardly be reproduced by cluster models in the framework of a ΛCDM cosmology (***Broadhurst & Barkana, 2008***; *Tasitsiomi et al., 2004*)



I. Motivations

Arc statistics problem

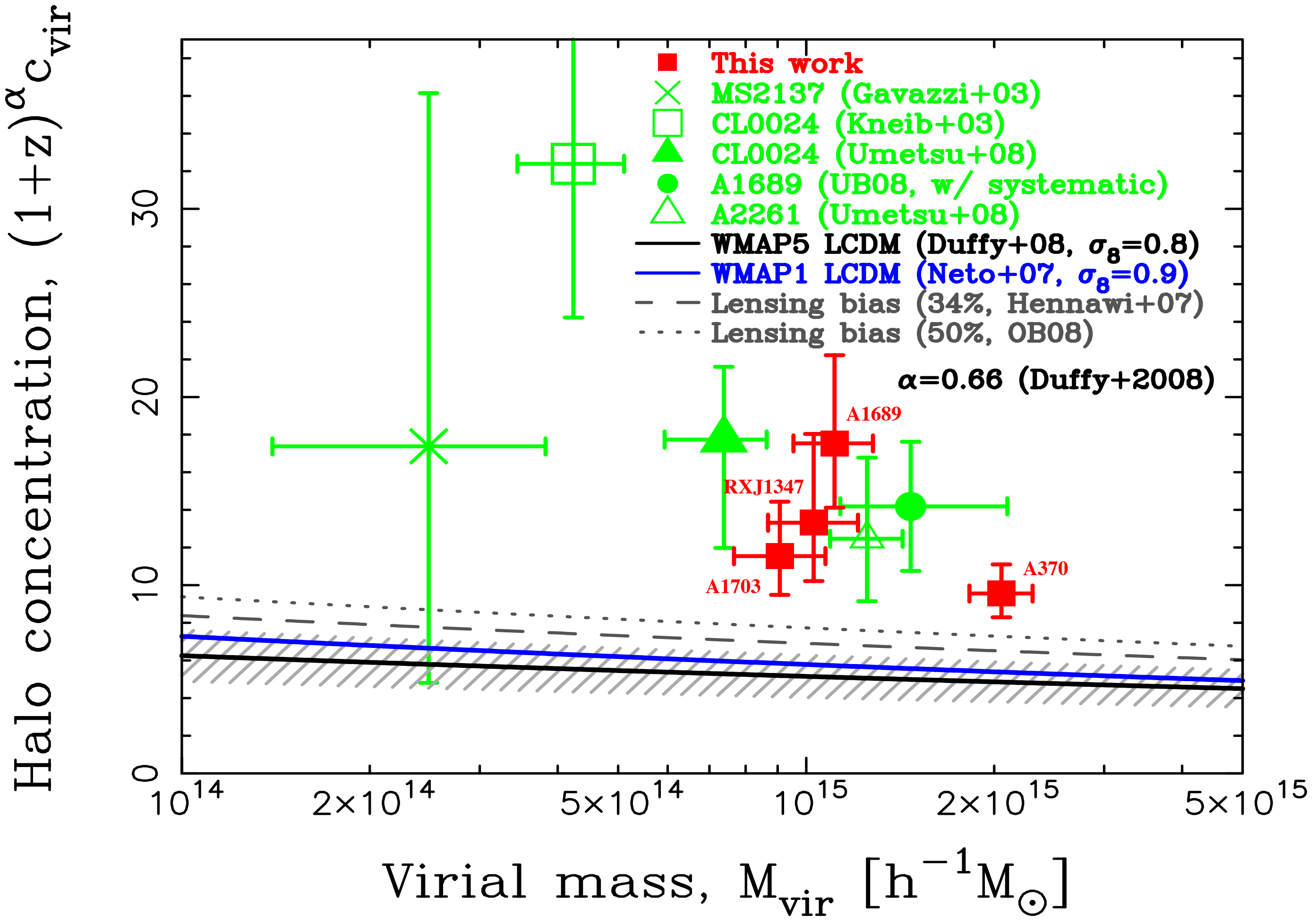
- The observed distribution of Einstein radii is much larger than the one predicted by analytical models within the ΛCDM model (***Zitrin et al., 2012***)



1. Motivations

Over-concentration problem

- Few clusters, for which high quality strong and weak lensing data became available, have concentrations which are way too large compared to numerical expectations (***Broadhurst et al., 2008; Zitrin et al., 2009***)



1. Motivations

The origin of these discrepancies is unclear and may be produced by a variety of effects:

- Cosmological abundance of clusters
- Shapes of dark matter potential wells
- Physics within the clusters in simulations
- Limitations of semi-analytical approaches
  - projection effects: cluster triaxiality, substructures,…
  - cluster mergers (increasing lensing efficiency in ~*10%*, ***Redlich et al. 2014***)

2. Objetives

While the abundance of massive clusters appear to be consistent with other observables...

Using the state-of-the-art **MUSIC dataset** of resimulated clusters:

- We address the issues of numerical simulations and dark matter potential on cluster-mass scales (triaxiality and substructures)
- We build up a semi-analytical model to estimate the **distribution of Einstein radii in the Universe** within the ΛCDM cosmological model

MAPLENS (**MA**drid-**P**aris-**LEN**sing-**S**emianalytics)

- We present predictions obtained with MAPLENS on the distribution of Einstein radii and the *c*-*M* relation, considering **full-sky** coverage
- We compare our results with **recent observational data** (SDSS, SGAS and CLASH)

3. MUSIC-MultiDark resimulated clusters

- MUSIC-MD clusters are **free from contamination** of low resolution particles
- MUSIC-MD clusters are ***distinct* objects** (i.e. halos which are not subhalos of more massive halos)
- We analyzed the **non-radiative** run of MUSIC-MD
- We included **1419** cluster-size halos with mass above $M_{min} = 2 \times 10^{14}\ h^{-1} M_\odot$
- We selected four snapshots at ***z = (0.250, 0.333, 0.429, 0.667)***
- In order to increase the statistics and to take into account projections effects we studied each halo under **500 random line-of-sight**
- We investigated a total of 1419x500 (***~7x10^5***) projections
- MUSIC-MD allow us to characterize mass distributions down to ***~15 kpc***

3. MUSIC-MultiDark resimulated clusters

Redshift	0.250	0.333	0.429	0.667
$N(\geqslant M_{min})$	403	393	365	258
f_{un}	0.39	0.41	0.47	0.48
f_{rel}	0.46	0.47	0.42	0.42
f_{sup}	0.15	0.12	0.10	0.09

1419 galaxy clusters

~44% unrelaxed clusters

Redshift	N_{halos}	N_{proj}	$N(\theta_E \geqslant 3'')$	$N(\theta_E \geqslant 10'')$	$N(\theta_E \geqslant 20'')$	$N(\theta_E \geqslant 30'')$
0.250	403	201500	42983 (21%)	22292 (11%)	5297 (3%)	993 (< 1%)
0.333	393	196500	43614 (22%)	25992 (13%)	7904 (4%)	1577 (< 1%)
0.429	365	182500	39857 (22%)	24089 (13%)	5520 (3%)	667 (< 1%)
0.667	258	129000	38429 (30%)	13223 (10%)	779 (1%)	0 (0%)

>7x10[5] lens planes

~24% strong lenses

4. Lensing properties of the MUSIC-MD clusters

- We examined the shapes of both their **density and surface density profiles** by fitting them with a NFW model:

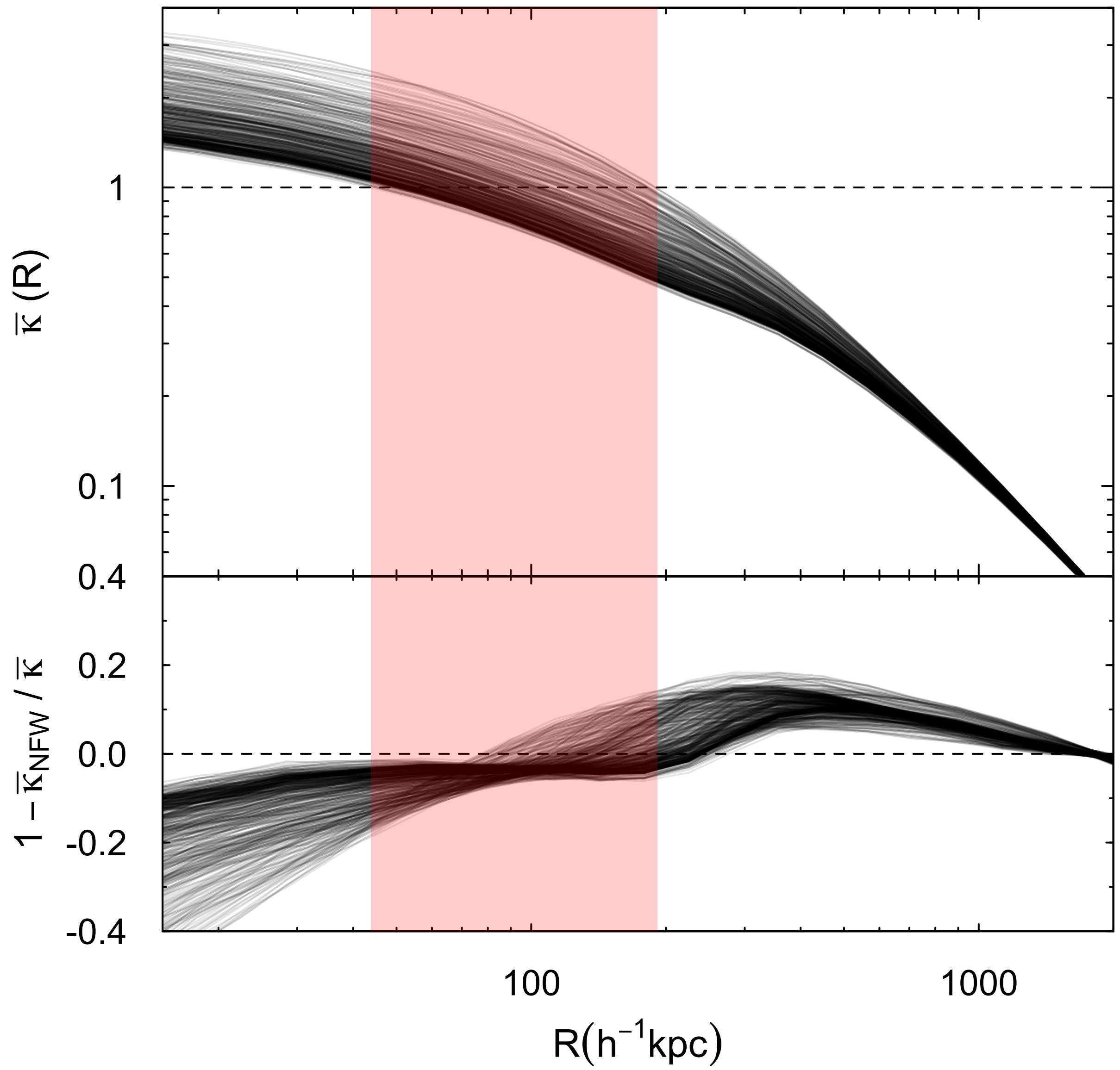
$$R^2_{3D}\,(\mathrm{M_{3D}, c_{3D}}) = \frac{1}{N_{\mathrm{dof}}} \sum_i \left[\log \rho_i - \log \rho\,(\mathrm{r}_i \mid \mathrm{M_{3D}, c_{3D}})\right]^2$$

$$R^2_{2D}(\mathrm{M_{2D}, c_{2D}}) = \frac{1}{\mathrm{N_{dof}}} \sum_{\mathrm{i}} \left[\log \Sigma_{\mathrm{i}} - \log \Sigma(\mathrm{R_i} \mid \mathrm{M_{2D}, c_{2D}})\right]^2$$

- We derived the ***c-M* relations** from the NFW fits and investigated their evolution with redshift and halo relaxation
- We also produce **two-dimensional convergence and shear maps** by means of **ray-tracing** techniques (**Skylens, *Meneghetti et al. 2010***)
- We derived the properties characterizing the **tangential critical lines**, such as its ellipticity and the *effective* Einstein radius
- The fitting procedure is based on the assumption that mass is spherically distributed in clusters, while the ray-tracing accounts for the two-dimensional mass distribution of the clusters, so the difference between the two independent procedures could give us hints on the **projection effects**

4. Lensing properties of the MUSIC-MD clusters

- ***Equivalent* Einstein radius** from the fitting procedure



$$\bar{\Sigma}(R) = \frac{1}{\pi R^2} \int_0^R 2\pi r \Sigma(r) dr$$

$$R_E = \theta_E D_L$$

$$\bar{\kappa}(R_{E,\ eqv}) = \frac{\bar{\Sigma}(R_{E,\ eqv})}{\Sigma_{cr}} = 1$$

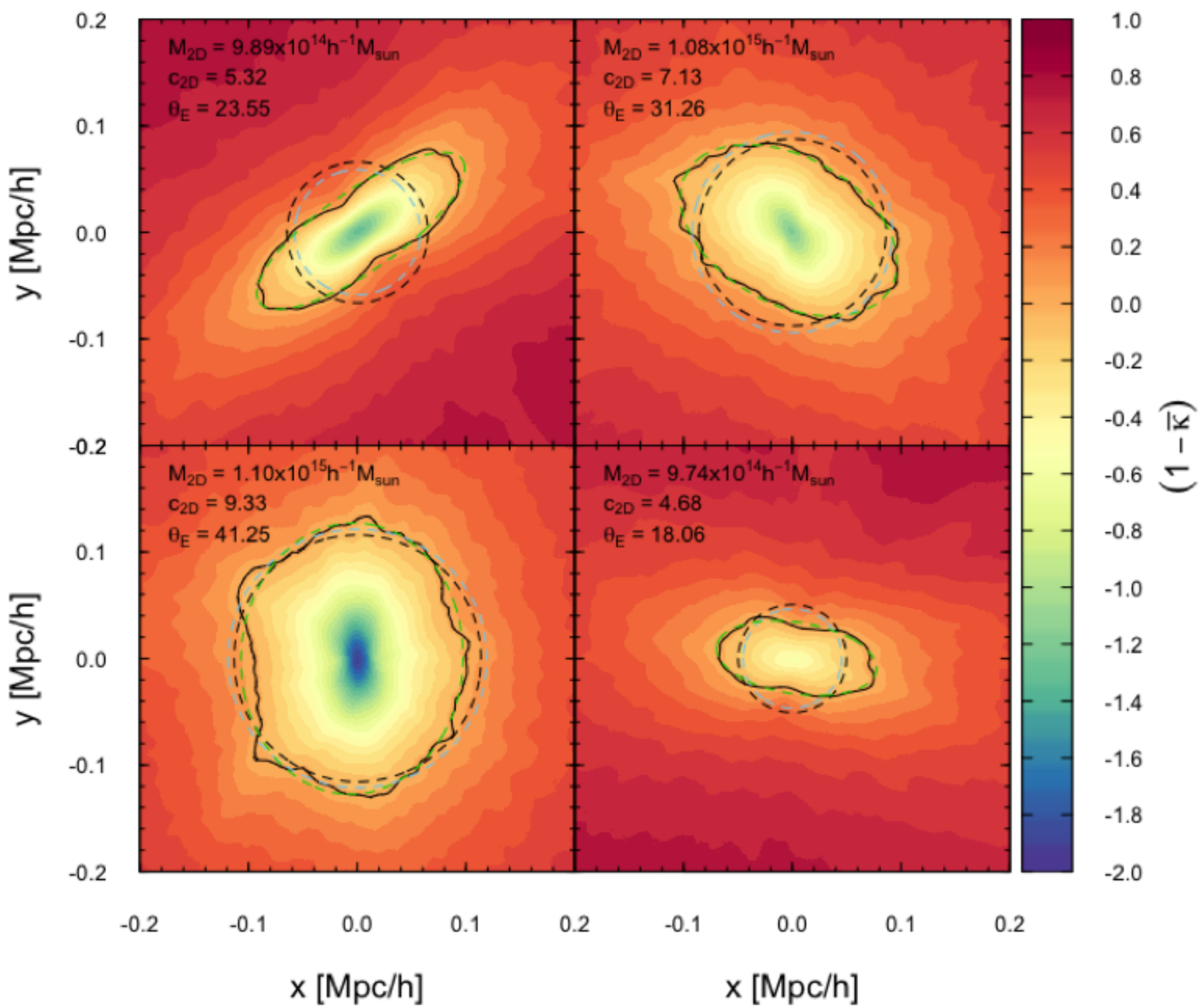
$$\theta_E \equiv \theta_{NFW}$$

Critical surface mass density

~10,000x ρ_{crit}

4. Lensing properties of the MUSIC-MD clusters

- ***Effective*** **Einstein radius** from the ray-tracing procedure



$$\theta_{E,\,eff} \equiv \sqrt{\frac{A}{\pi}}$$

Area enclosed by the tangential critical line

θ_E

θ_{NFW}

Ellipse fit

5. MAPLENS

- MAPLENS is based on the analysis of the **MUSIC-MD** simulated clusters
- Semi-analytic model to **infer the distribution of Einstein radii** from a sample of dark matter halos
- By comparing the *equivalent* Einstein radii and the *effective* Einstein radius, we **incorporate the projection effects** (triaxiality and presence of substructures) to MAPLENS with the aim to recover more realistic estimates of the Einstein radii distribution
- MAPLENS derives the lensing properties of cluster-size halos in terms of ***kernel density estimates***, once the mass and the redshift of each halo are known
- Source redshift is fixed at $z_s = 2.0$

5. MAPLENS

- **Non-parametric** way to estimate the probability density function of a random variable directly from the data
- It **do not** assumes a particular form for the underlying distribution
- For a *d*-variate random sample $(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n)$ drawn from a density *f*, the kernel density estimate is defined by:

$$\hat{f}(\mathbf{x};\mathbf{H}) = n^{-1}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x}-\mathbf{X}_i)$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_d)^t$ and $\mathbf{X}_i = (X_{i1}, X_{i2}, \ldots, X_{id})^t$ with $i = 1,2,\ldots,n$

- $K(\mathbf{x})$ is the kernel which is a symmetric probability density function and H is the bandwidth matrix

5. MAPLENS

- We evaluated the **3D-kernel density estimates** using a Gaussian kernel
- We computed two different 3D-kernel density estimates from the analysis of the 500 projections of each cluster in the MUSIC-MD dataset
  - the **first kernel** density estimate contains the information derived from the fits to the **surface density profiles** with variates:

$$\boldsymbol{x} = (M_{200}, M_{2D}, c_{2D})$$

  - the **second kernel** includes the **lensing properties** with variates:

$$\boldsymbol{x} = (\theta_{NFW}, \theta_E, \varepsilon_\theta)$$

5. MAPLENS

- Given a hypothetical halo with redshift *z* and mass M_{200}, we estimate the **projected mass and concentration (M_{2D} and c_{2D})** from the first kernel density estimate by means of *Monte Carlo* (MC) sampling with the conditional probabilities:

$$p\,(M_{2D}\,|\,M_{200})$$

$$p\,(c_{2D}\,|\,M_{200},\,M_{2D})$$

- We compute the equivalent Einstein radius from the sampled M_{2D} and c_{2D} as follows:

$$\bar{\kappa}_{NFW}\,(R_E, \mathrm{M}_{2\mathrm{D}}, \mathrm{c}_{2\mathrm{D}}) = 1 \longrightarrow R_E = \theta_E D_L \longrightarrow \theta_E \equiv \theta_{NFW}$$

- We infer the ***effective* Einstein radius** and the **ellipticity of the critical line (ε_θ and θ_E)** from the second kernel density estimate with the conditional probabilities:

$$p\,(\varepsilon_\theta\,|\,\theta_{NFW})$$

$$p\,(\theta_E\,|\,\theta_{NFW},\,\varepsilon_\theta)$$

6. All-sky distribution function of Einstein radii

- We extend our predictions on the Einstein radii distribution over a realization of dark matter halos generated with the *Tinker et al. 2008* mass function, considering *full-sky* coverage
- We run **1000 *all-sky* realizations** of dark matter haloes with $M_{200} \geq 2 \times 10^{14}\, h^{-1} M_{\odot}$ within $0.1 < z < 1.0$
- We infer the ***all-sky* Einstein radii distribution with MAPLENS**
- We propose as simple fitting formula to the ***all-sky* Einstein radii distribution** and compare the results with **10,000 SDSS clusters**
- We also derive ***c-M* relations** for galaxy clusters and compare them with recent observational data from **SGAS** and **CLASH**.

6. All-sky distribution function of Einstein radii

- *Universal* distribution function of Einstein radii

$$N(z+\Delta z, \theta_E + \Delta\theta_E) = 10^{\,n(z,\theta_E)}\, \Delta z\, \Delta\theta_E$$

$$n(z,\theta_E) = a(z) + b(z)\log\theta_E + c(z)\log^2\theta_E$$

$$a(z) = a_0 + a_1\, z + a_2\, z^2$$

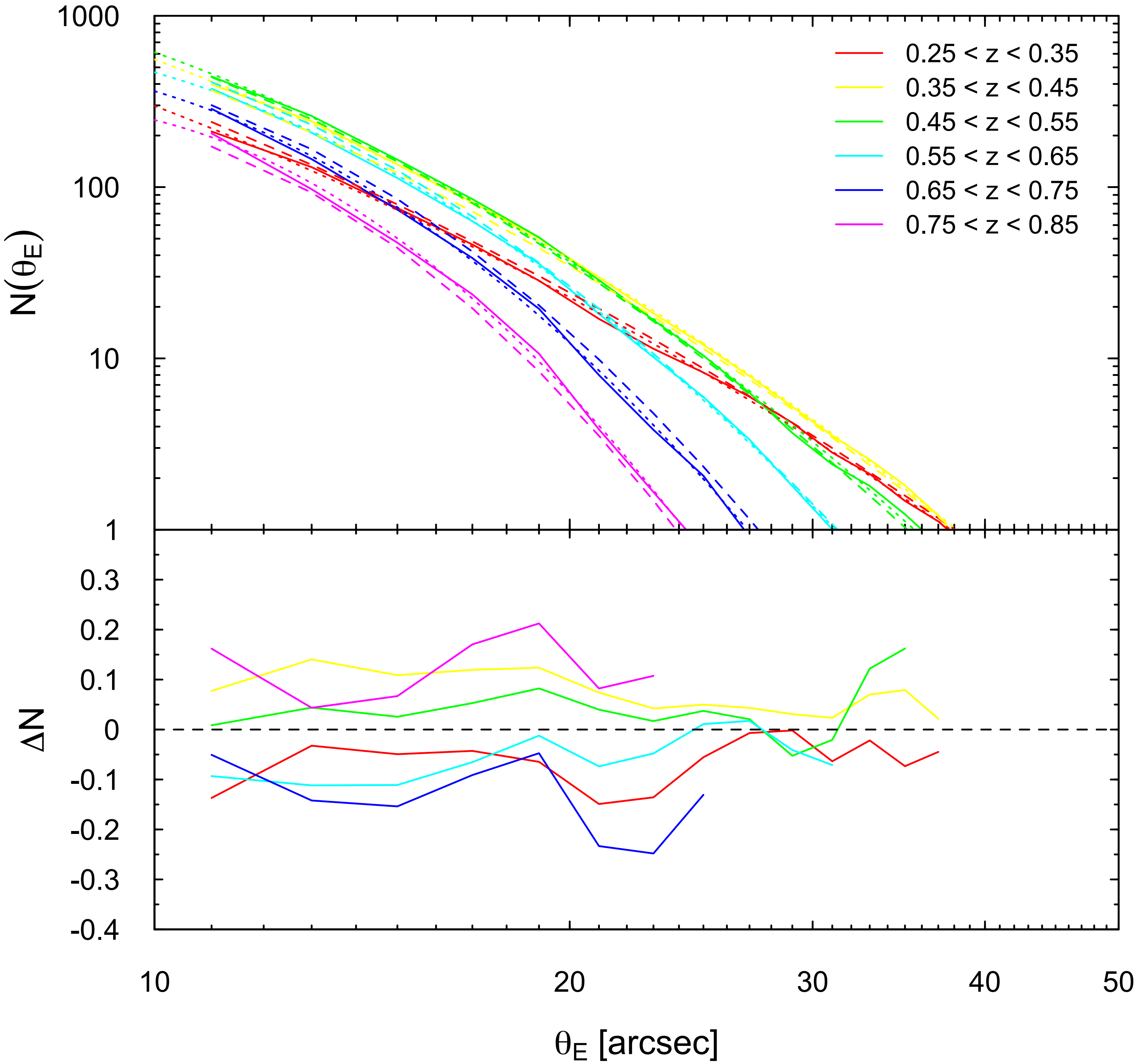
$$b(z) = b_0 + b_1\, z + b_2\, z^2$$

$$c(z) = c_0 + c_1\, z + c_2\, z^2$$

	$i=1$	$i=2$	$i=3$
a_i	3.26 ± 1.74	12.33 ± 6.99	-30.00 ± 6.54
b_i	1.12 ± 2.53	-14.75 ± 10.15	48.28 ± 9.49
c_i	-2.33 ± 0.96	7.93 ± 3.86	-23.60 ± 3.61

- The relative error in the number of lenses is constrain within $\lesssim$*20%* for lenses with $\theta_E \gtrsim$ *10* arcsec, and it is reduced to $\lesssim$ *10%* for lenses with $\theta_E \gtrsim$ *25* arcsec
- The measured dispersion in the number of Einstein radii is $\sigma_N = \sqrt{N}$

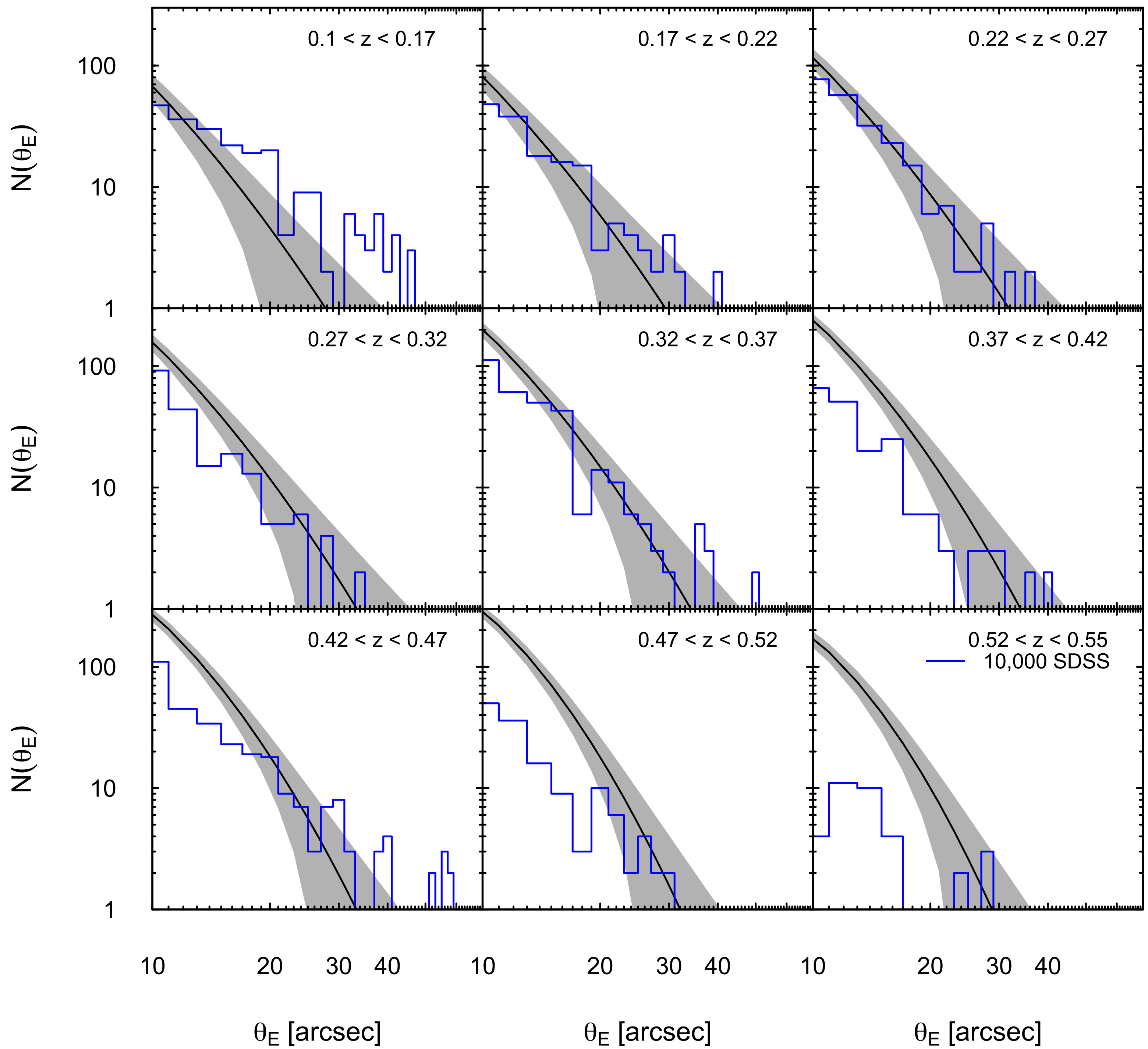
6. All-sky distribution function of Einstein radii



7. Comparison with observations: the SDSS clusters

- **Zitrin et al. 2012** presented the results from the strong lensing modeling of **10,000 SDSS clusters** in the range $\boldsymbol{0.11 < z < 0.55}$
- Light distribution observed in galaxy clusters generally traces their mass distribution
- Calibration of the **mass-to-light ratio** is based on a subsample of ten well-studied SDSS galaxy clusters that were covered by high-quality HST images
- For a direct comparison, we compute the distribution of Einstein radii produced by clusters within $0.10 \leq z \leq 0.55$

7. Comparison with observations: the SDSS clusters



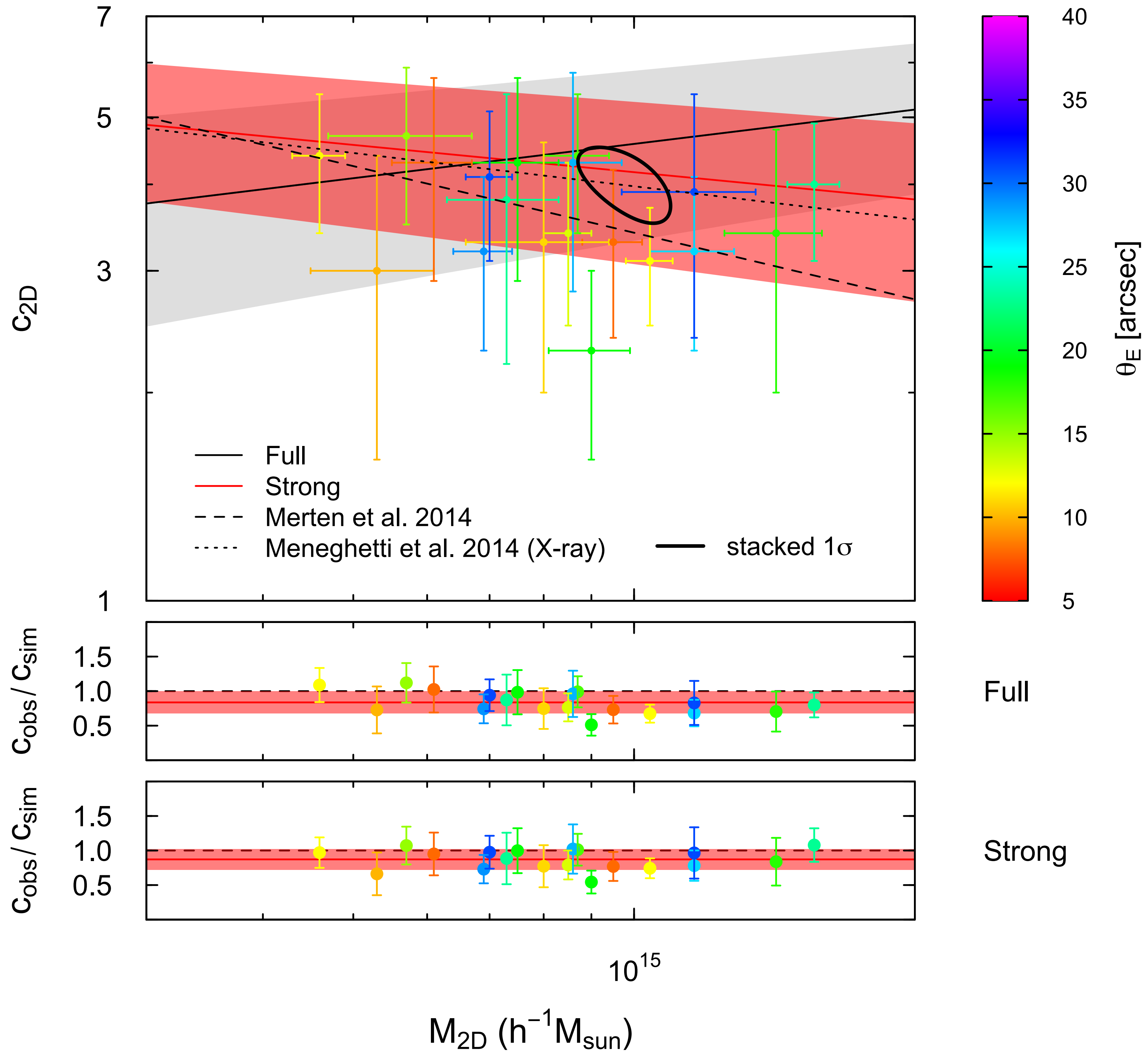
7. Comparison with observations: the CLASH sample

- ***Merten et al. 2015*** performed a lensing analysis of **19 X-ray selected galaxy clusters** from the **CLASH** cluster sample
- Mass and concentrations derived by best-fitting the surface mass profiles of the clusters to a NFW model
- Our theoretical ***c–M–z* relations** are derived by means of nonlinear least-square fitting of:

$$c(M,z) = A\left(\frac{1.34}{1+z}\right)^{B}\left(\frac{M}{8\times 10^{14}h^{-1}M_{\odot}}\right)^{C}$$

- In ***Meneghetti et al 2014*** we found that **X-ray selected** CLASH clusters are frequently **efficient strong lenses**

7. Comparison with observations: the CLASH sample



$\langle c_{obs}/c_{sim} \rangle = 0.87 \pm 0.15$

7. Comparison with observations: the SGAS clusters

- ***Oguri et al. 2012*** combined strong and weak lensing analysis for a subsample of **28 clusters** from the Sloan Giant Arcs Survey (**SGAS**) in the redshift range $\boldsymbol{0.27 \lesssim z \lesssim 0.68}$

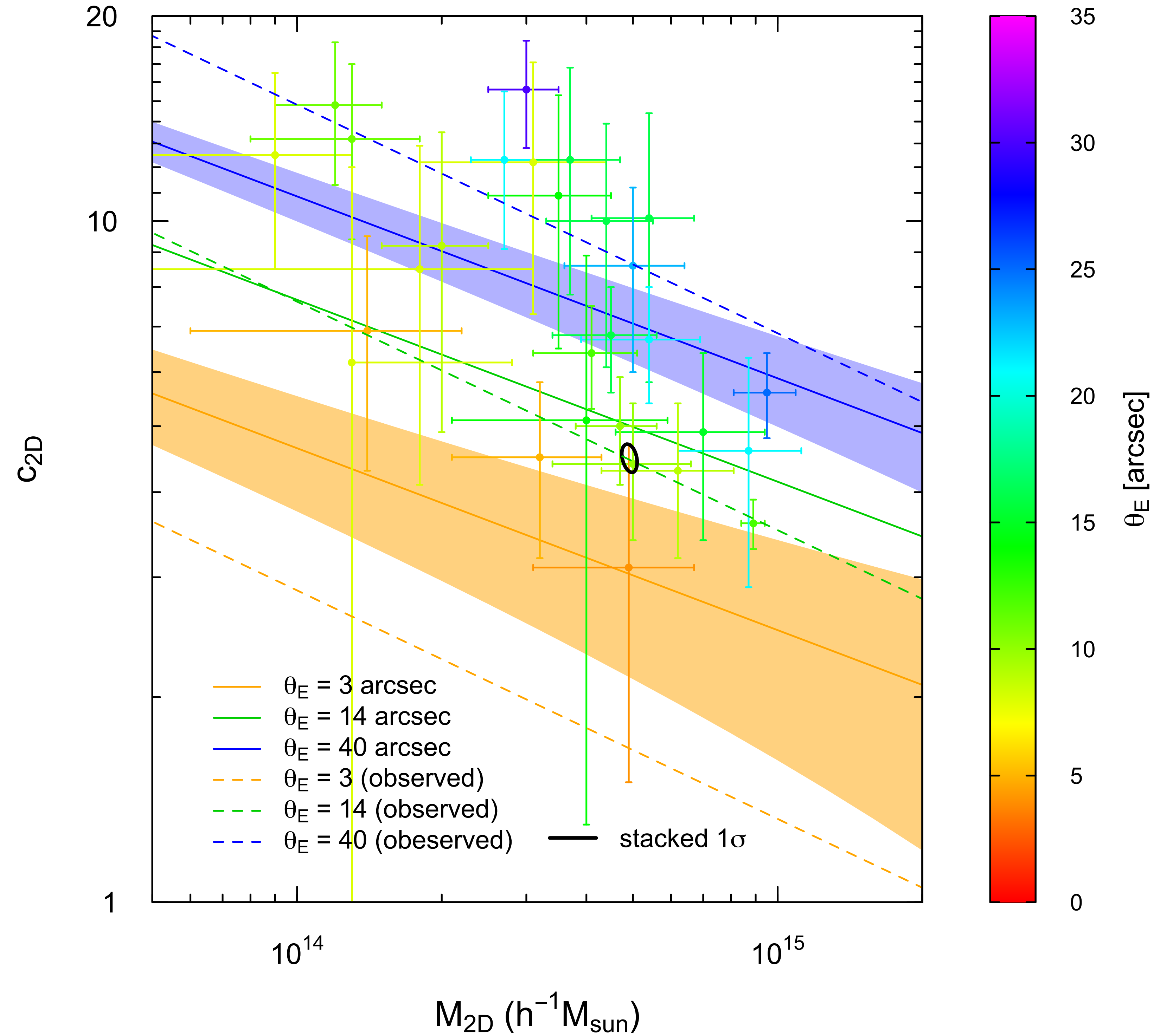
- ***Sereno et al. 2015*** re-analyzed the shear profiles of these 28 clusters with the additional constraints on the *effective* Einstein radii

- For a direct comparison, we construct a **$\boldsymbol{\theta_E}$-selected sample** (with $\theta_E \geq 3$ *arcsec*) within $0.25 \leq z \leq 0.70$

- For each *all-sky* realization, we propose a fitting function to the ***c–M–$\boldsymbol{\theta_E}$*** **relation** as follows:

$$c(M, \theta) = A\, \theta^B \left(\frac{M}{8 \times 10^{14} h^{-1} M_\odot} \right)^C$$

7. Comparison with observations: the SGAS clusters



8. Conclusions

- **MAPLENS**: full cosmological distribution of Einstein radii combining halo abundance + *Monte Carlo* samples
- The sampled Einstein radii distributions recovered with **MAPLENS account for the projection effects**
- The **SGAS sample**, although slightly over-concentrated, is **consistent within errors** with the theoretical predictions of MAPLENS for a strong lensing selected sample of halos in the redshift range $0.25 \leq z \leq 0.70$
- We **did not find any significant disagreement** between the observed $c - M$ relation for the **CLASH sample** and the theoretical relations of MAPLENS, after accounting for **projections and selection effects** ($\langle c_{obs}/c_{sim} \rangle = 0.87 \pm 0.15$)
- We **did not find statistical evidences** for claiming that the Einstein radii distribution of the **10000 SDSS clusters** exceed the theoretical expectations of the ΛCDM cosmological model
- Given the large uncertainties of the method used by Zitrin et al. 2012, it could be interesting to carefully **re-analyze the SDSS sample**, particularly focusing on the strongest lenses in their sample.

After accounting for projection and selection effects

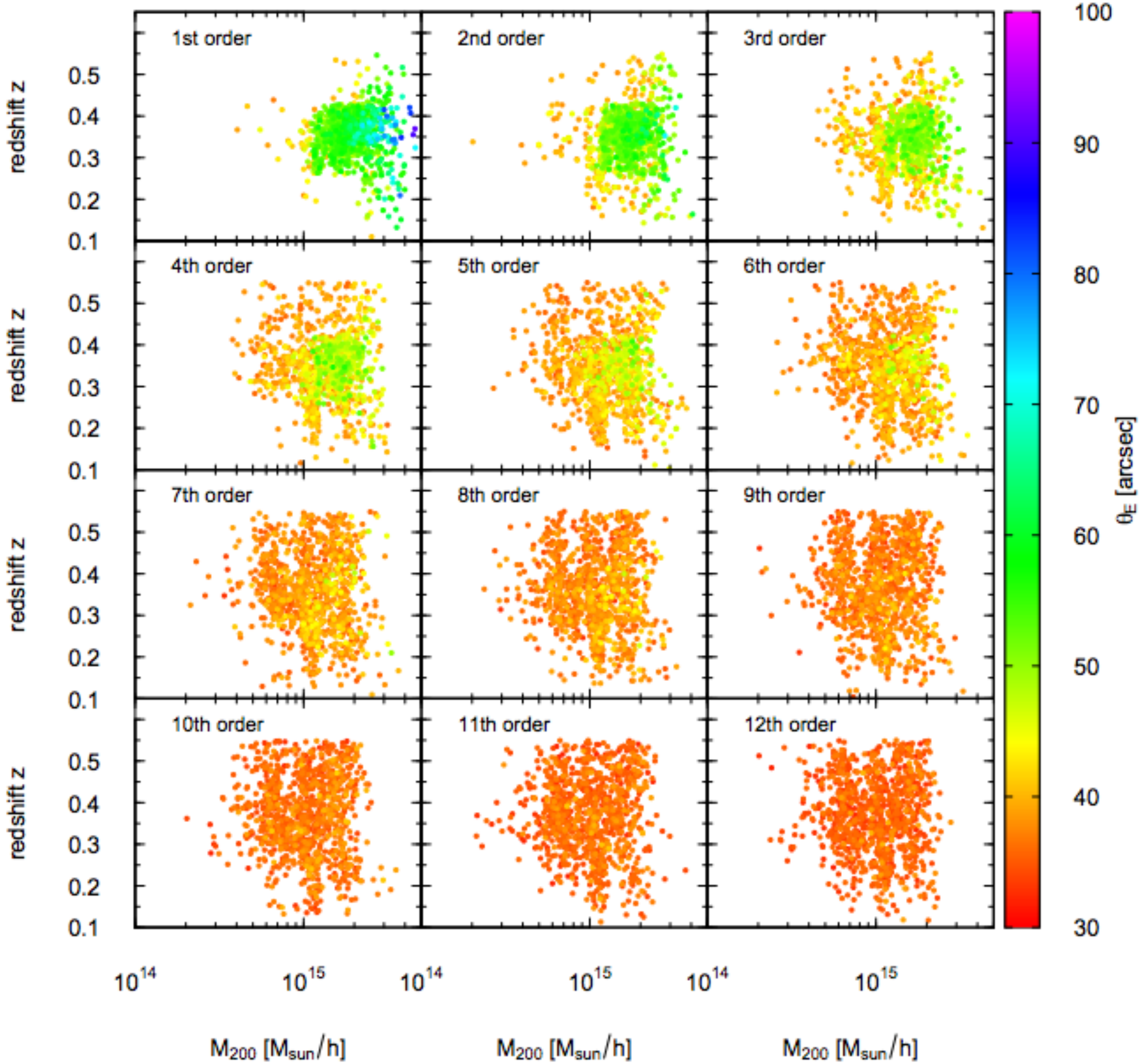
Arc statistics and over-concentration problems

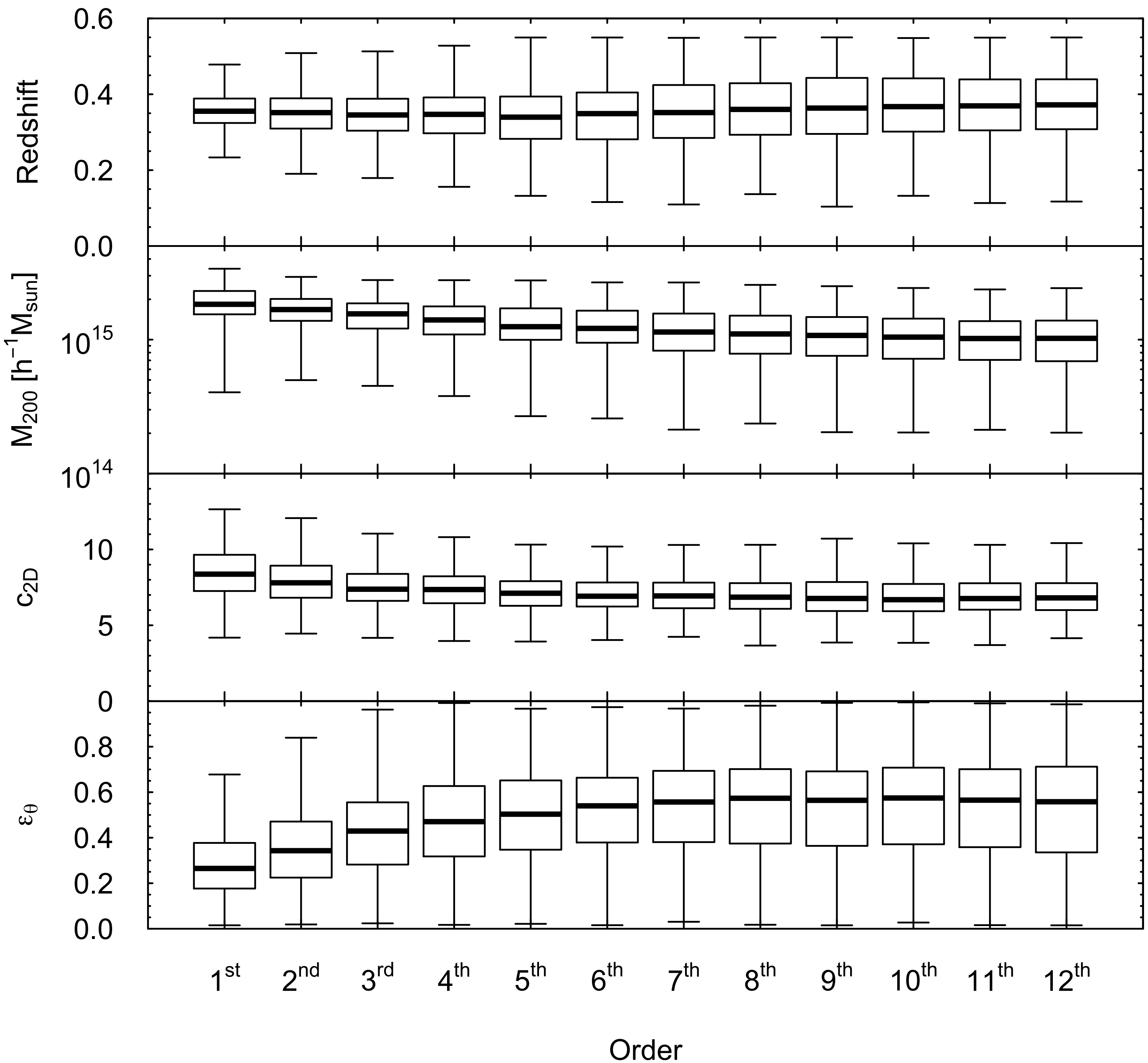
can be mostly solved

- ☑ **Strong lensing as cosmological probes**
- ☑ ***c-M* relation to study cosmological parameters**

- ***MultiDark* simulation** (*Prada et al. 2011*)
  - Dark-matter only with WMAP7 cosmology ($\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $\Omega_m = 0.0469$, $\sigma_8 = 0.82$, $h = 0.7$, $n=0.95$)
  - 2048^3 particles
  - $1\ h^{-1}Gpc$ cubic box

- ***MUSIC-MultiDark*** resimulated clusters (*Sembolini et al. 2013*)
  - Mass limited sample selected from the *MultiDark* simulation
  - 282 more massive objects ($>10^{15} h^{-1} M_\odot$) resimulated
  - Radiative and non-radiative physics
  - *8x* more resolution ($m_{dm} = 9.01 \times 10^8\ h^{-1}M_\odot$; $m_{gas} = 1.09 \times 10^8\ h^{-1}M_\odot$)
  - Gravitational softening: $6\ h^{-1}kpc$


1st order
2nd order
3rd order
4th order
5th order
6th order
7th order
8th order
9th order
10th order
11th order
12th order
redshift z
0.5
0.4
0.3
0.2
0.1
10^{14}
10^{15}
M_{200} [M_{sun}/h]
θ_E [arcsec]
100
90
80
70
60
50
40
30



0.6
0.4
0.2
0.0
Redshift
10^15
10^14
M200 [h^-1 Msun]
10
5
0
c2D
0.8
0.6
0.4
0.2
0.0
εθ
1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th 11th 12th
Order